Title: Stochastic parametrisation models for GFD
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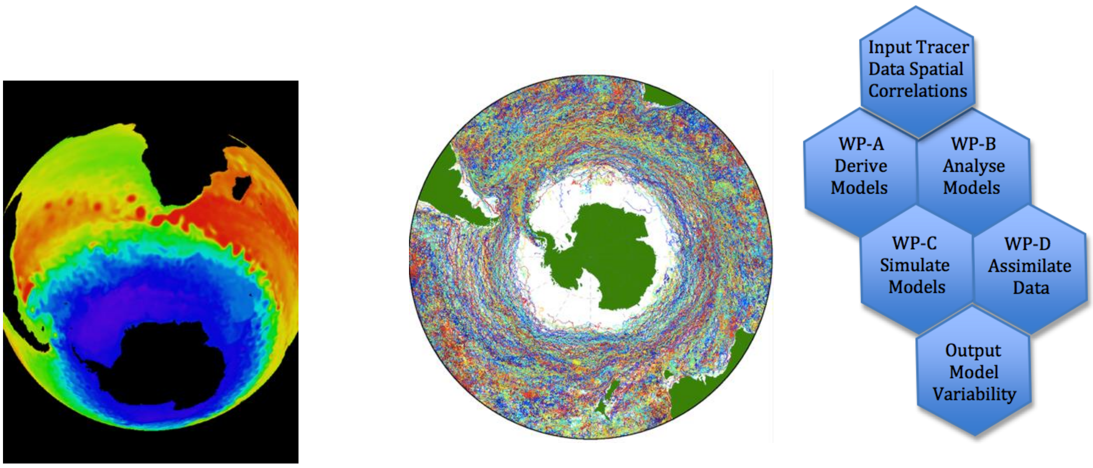
In next-generation weather and climate models, stochastic parameterisation should be an important element in providing reliable estimates of model uncertainty. A fundamental conclusion of Berner, Jung & Palmer [2012] is that

> "*a posteriori* addition of stochasticity to an already tuned model is simply not viable [satisfactory]. This in turn suggests that stochasticity must be incorporated at a very basic level within the design of physical process parameterisations and improvements to the dynamical core."

This talk responded to the workshop's challenge of "How do we improve the physical basis for model uncertainty schemes?" It proposed an *a priori* introduction of stochasticity for GFD models at various levels of approximation, by introducing the methodology of Holm [2015] as a potential framework for quantifying model *transport error*. In turn, the stochastic representation of model transport error would introduce stochasticity into the parameterisations of subgrid scale processes.

This methodology introduces Stochasticity into Partial Differential Equations (SPDEs) for the model, via Variational Principles (SVPs), with corresponding implications for Numerical Modelling, Stochastic Data Analysis, and Geophysical Fluid Dynamics (SGFD). The motivation for introducing stochasticity was illustrated by comparing the relative resolutions of numerical simulations and satellite data for tracers on the surface of the ocean; and the methodology was sketched as a series of interconnected hexagons in the following Figure. The left and centre panels of the Figure illustrate the difference in scales between the numerical resolution and the satellite observations for this problem, e.g., for estimating the spread of floating tracers such as plastic containers (or, "rubber duckies") in the Southern Ocean. The rightmost panel in the Figure shows the closely integrated tasks in formulating the methodology for stochastic estimation of model transport error.



In this methodology, the transport stochasticity is introduced via the correlation eigenfunctions for the advection data being analysed, by multiplying each eigenvector of the correlation matrix for the tracer data (called an empirical orthogonal function, or EOF) with a stochastic amplitude in the Stratonovich sense, and then taking the sum over these stochastic products as the deviation from the drift velocity. The drift velocity itself is then

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obtained via the known Hamilton's variational principal for deterministic fluid dynamics, but with variations of the velocity and advected quantities which are constrained to satisfy the stochastic EOF approximation of the satellite tracer data. This methodology of stochastically constrained variational principles is complementary to the customary practice in weather forecasting in which data is assimilated using variational principles. However, this proposed methodology is to be used in formulating the model for the dynamical core, rather than assimilating the observed data. The task of the methodology is to learn from stochastic assimilation of data (tracers) the spatial correlation features of the observed advected quantities. These quantities are needed as input into a constrained variational principle to derive the stochastic fluid motion equations, whose transport will predict statistics such as the variability of the advected data which, by constrauction, will be consistent with the observations.

The talk outlined this methodology, then illustrated it by deriving several new stochastic GFD models for predicting the evolution of climate and weather variability, based on observations of tracer data. The new feature of these potential dynamical core motion equations is that they contain stochastic perturbations which multiply both the solution velocity *and* its spatial gradient. Remarkably, these stochastic GFD models still preserve fundamental fluid properties such as Kelvin's circulation theorem and PV conservation. Indeed, as illustrated by the Kelvin circulation theorem for three-dimensional incompressible stochastic fluid motion, these fundamental mathematical structures in fluid dynamics retain their deterministic forms. However, their transport velocities are augmented by advection along the stochastic Lagrangian particle paths obtained from the spatial correlations of the tracer data. As a mathematical bonus, the equivalent Ito forms of these Stratonovich equations contain symmetric, second-order, derivative operators which tend to regularize the solutions of the new stochastic GFD equations, without any additional viscosity. This is apparently because the stochasticity in these new motion equations multiplies the gradients of the solutions, so its effects are enhanced in the vicinity of strong gradients during the evolution of the flow.

The areas of relevance of the new approach in matters of potential interest for ECMWF are:
(A) New stochastic parameterisation models for GFD derived using variational methods;
(B) Mathematical analysis of the stochastic transport equations in these models;
(C) Development of numerical methodology for stochastic GFD;
(D) Stochastic data assimilation using nonlinear particle filtering.

Key references:

J. Berner, T. Jung and T. N. Palmer [2012] Systematic Model Error: The Impact of Increased Horizontal Resolution versus Improved Stochastic and Deterministic Parameterizations, Journal of Climate, 25: 4946--4962.

D. D. Holm, [2015] Variational Principles for Stochastic Fluid Dynamics, Proc Roy Soc A, 471: 20140963.