

Progress (...) on forward modelling L1 and L2 bending angles

Ian Culverwell, Met Office



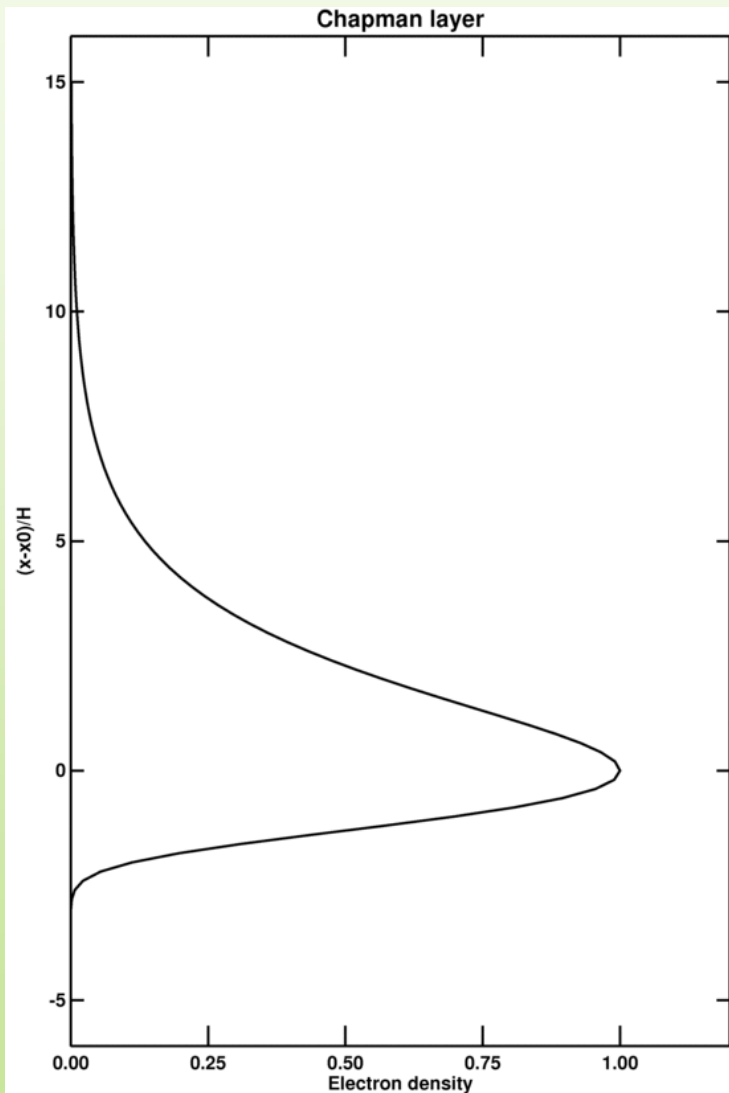
Sean Healy, ECMWF



Motivation

- How to do ionospheric correction when L2 signal drops out?
- Different (better?) tropospheric retrievals if L1 and L2 both used?
- Some information on ionospheric parameters?
- Residual ionospheric error?

Model ionosphere: electron density



Single Chapman Layer (Chapman 1931)

$$n_e(r) = n_e^{\max} \exp\left[\frac{1}{2}(1 - u - e^{-u})\right],$$

where $u = (r - r_0) / H$.

3 parameters:

$$n_e^{\max} = \text{TEC} / \sqrt{(2\pi e) H}$$

r_0 = peak height

H = *ionospheric scale height*

Model ionosphere: bending angle

$$\begin{aligned}\alpha_{\text{Li}}(a) &= -2a \int_a^\infty d \log n / dx \, dx / \sqrt{(x^2 - a^2)}, \quad x=nr \\ &\approx (k_4 / f_{\text{Li}}^2) 2a \int_a^\infty dn_e / dx \, dx / \sqrt{(x^2 - a^2)}, \quad k_4 = 40.3 \text{ m}^3\text{s}^{-2} \\ &\approx (k_4 / f_{\text{Li}}^2) 2a [2r_0 / (r_0+a)^{3/2}] \int_a^\infty dn_e / dr \, dr / \sqrt{(r - a)} \\ &= (k_4 / f_{\text{Li}}^2) n_e^{\text{max}} [4er_0^2 a^2 / H(r_0+a)^3]^{1/2} \cdot Z(\ell)\end{aligned}$$

where

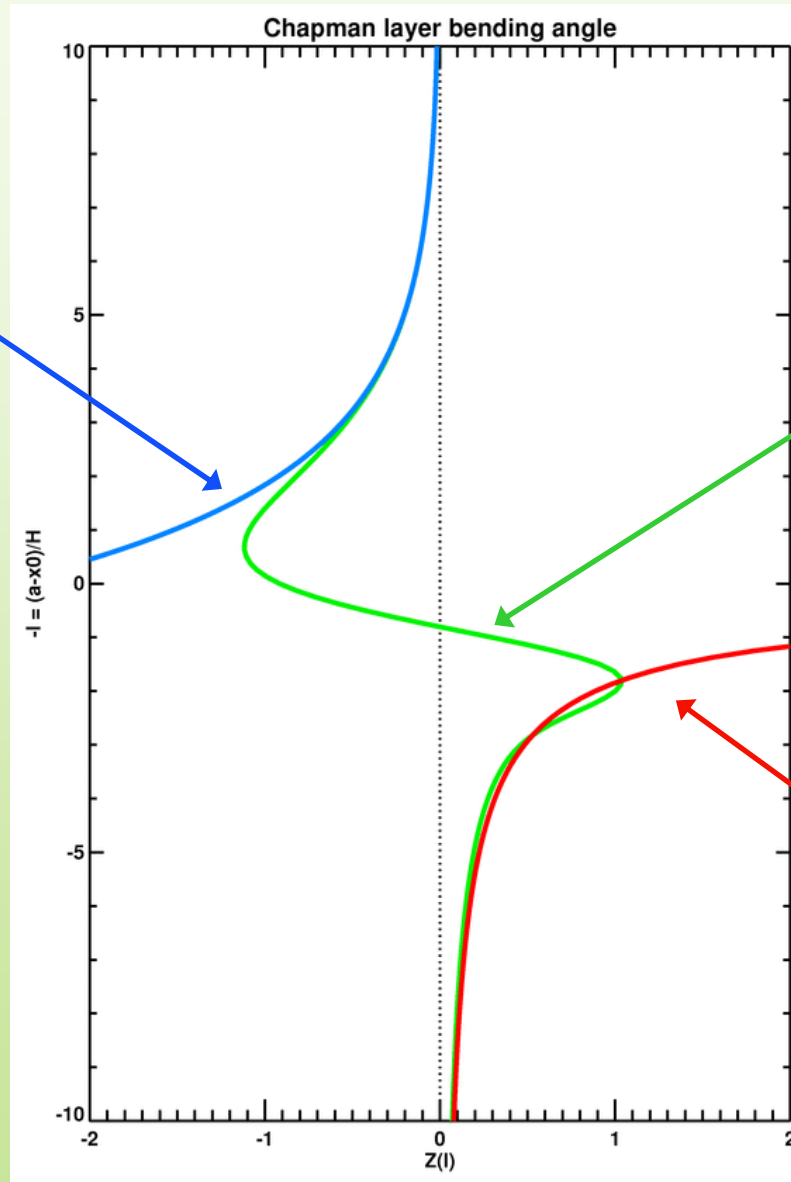
$$Z(\ell) = \int_{-\ell}^\infty (\mathbf{e}^{-u} - 1) \exp[1/2(1 - u - \mathbf{e}^{-u})] / \sqrt{(u + \ell)} \, du$$

is just a dimensionless, $O(1)$ function of

$$\ell = (r_0 - a) / H$$

Model ionosphere: bending angle $Z(\ell)$

$$-\sqrt{(2\pi)} e^{\ell/2}$$



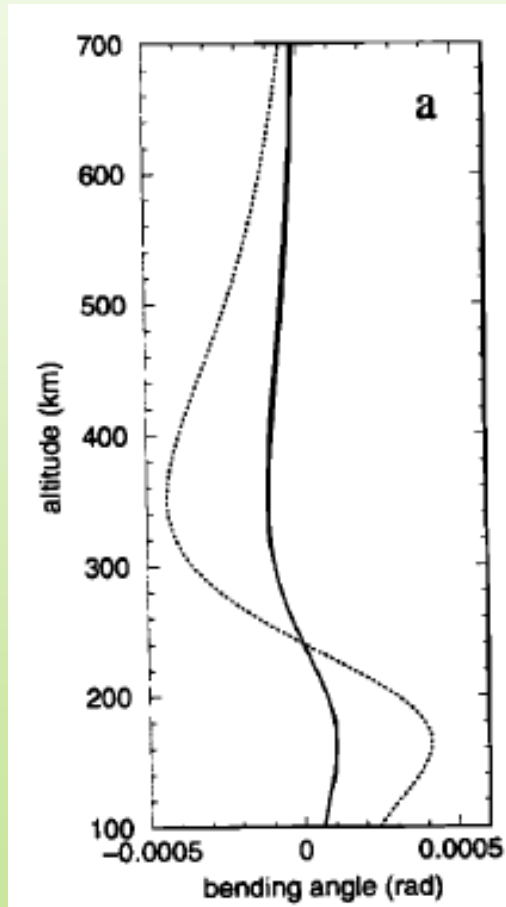
Actual \approx

$$\sqrt{(2\pi\theta) \frac{\sum_{i=0}^3 p_i \theta^i}{\sum_{i=0}^5 q_i \theta^i}}$$

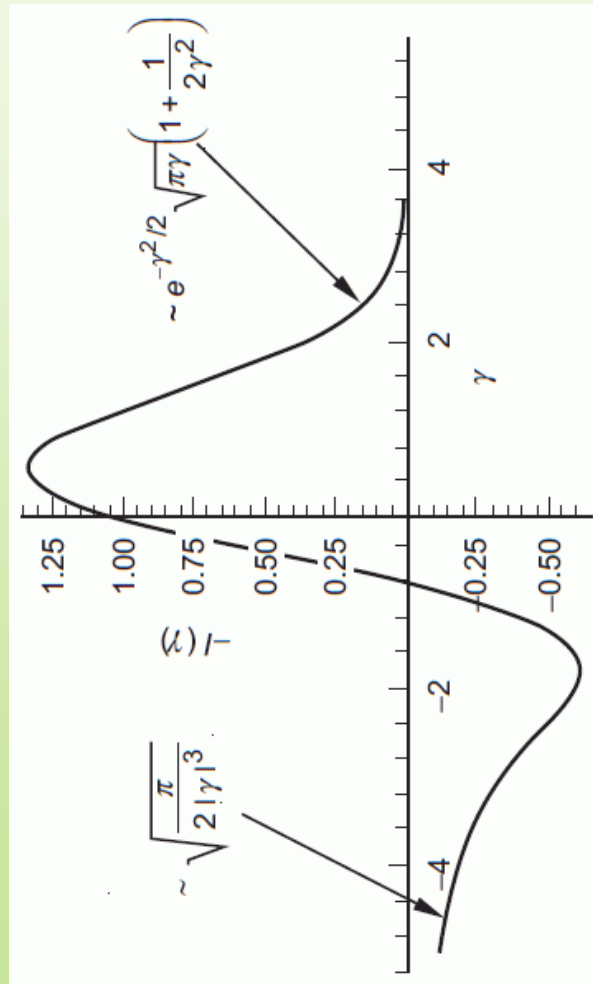
$$\sqrt{(2\pi/\ell^3)}$$

Model ionosphere: bending angle $Z(\ell)$

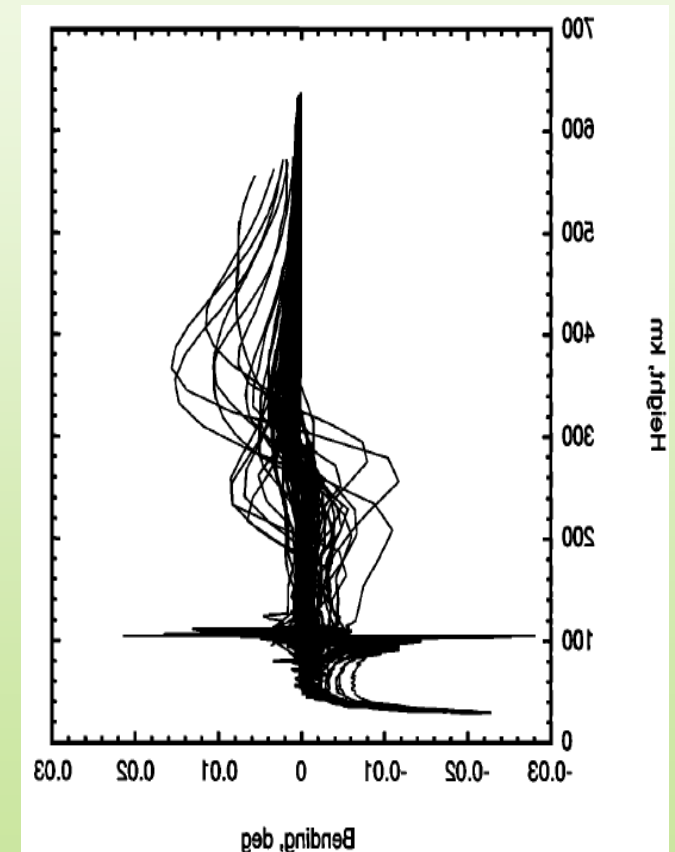
Schreiner et al, 1999,
Chapman layer



Melbourne, 2004,
Gaussian n_e



Hajj & Romans,
1998, GPS/MET



Direct modelling of L1 and L2 in ROPP

- The Radio Occultation Processing Package:
 - A collection of Fortran 95 code, build and test scripts, data files and documentation designed to aid users wishing to process, quality-control and assimilate radio occultation data into their NWP models.
 - Provided by ROM SAF (EUMETSAT).
 - The following features will be available in ROPP8.0 (Dec 2014).

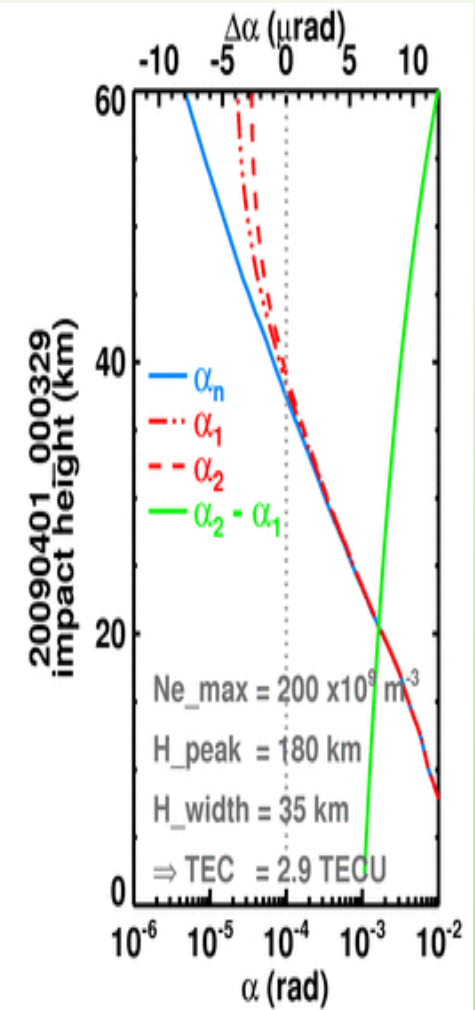
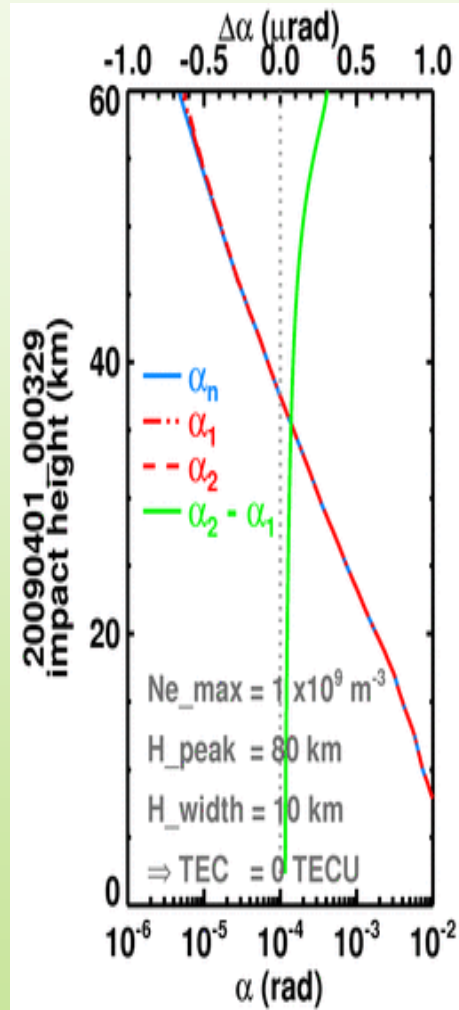
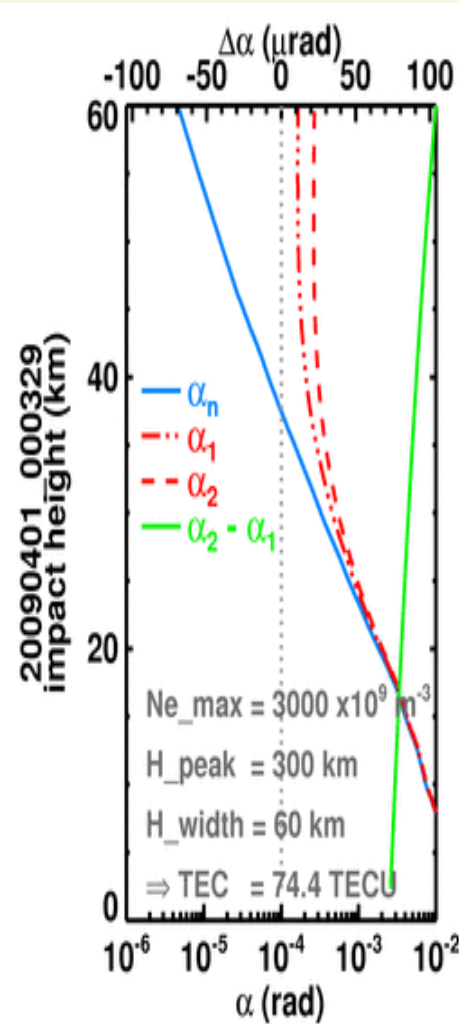
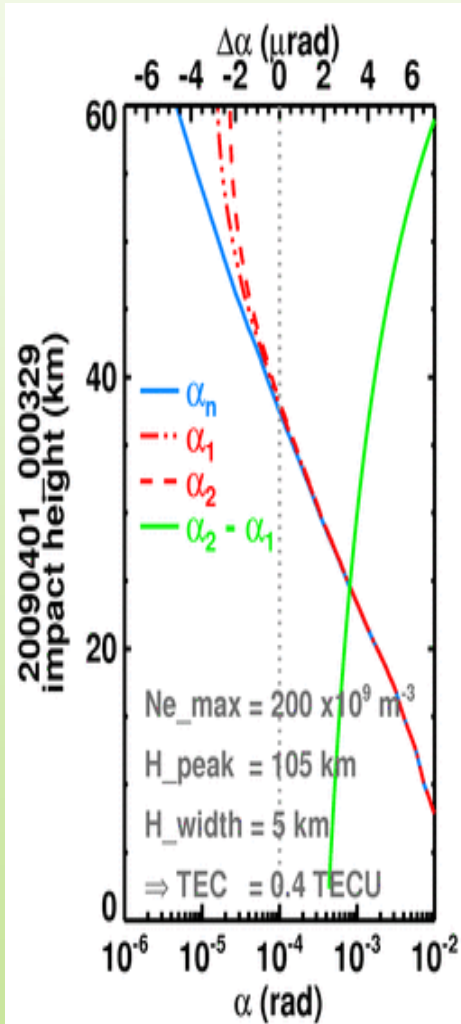
Examples of forward modelled L1 and L2 using ropp_fm

E-layer

F-layer

D-layer

F1-layer

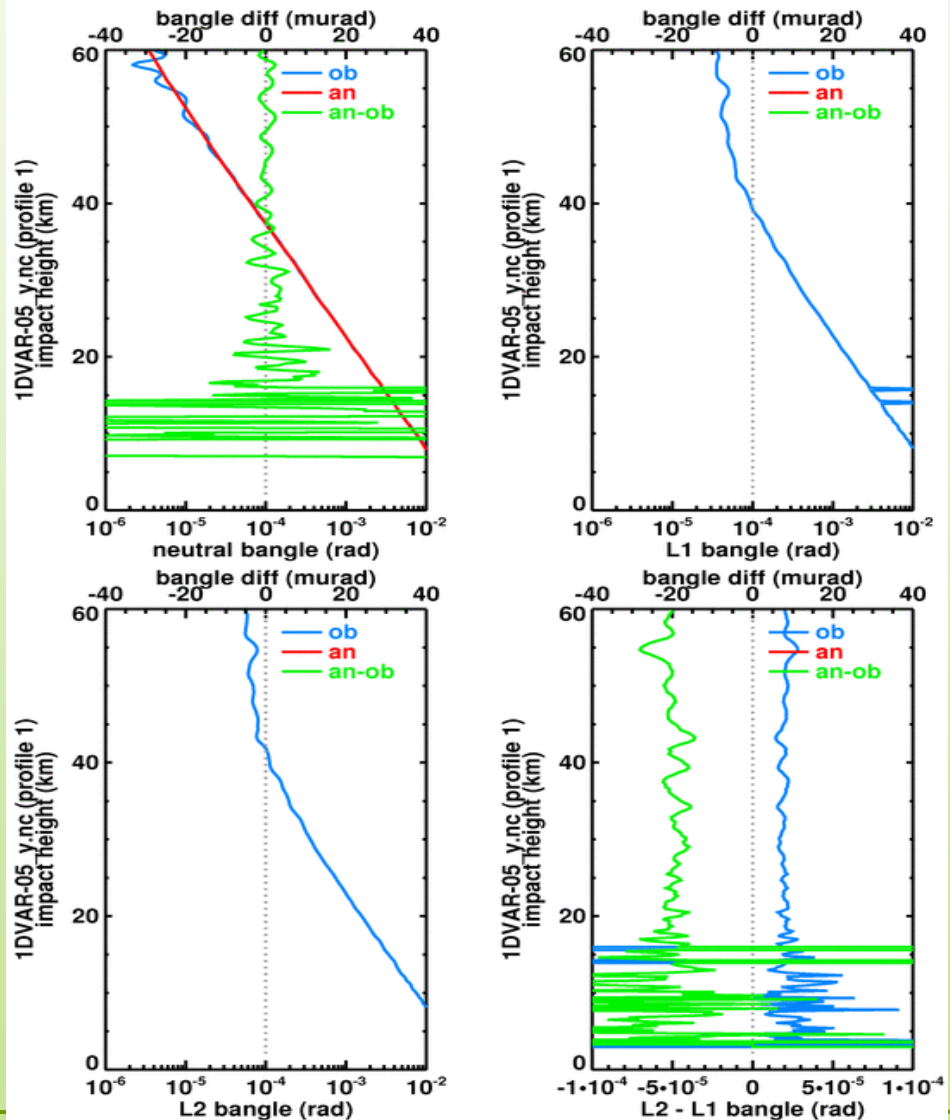


Direct modelling of L1 and L2 in ropp_1dvar

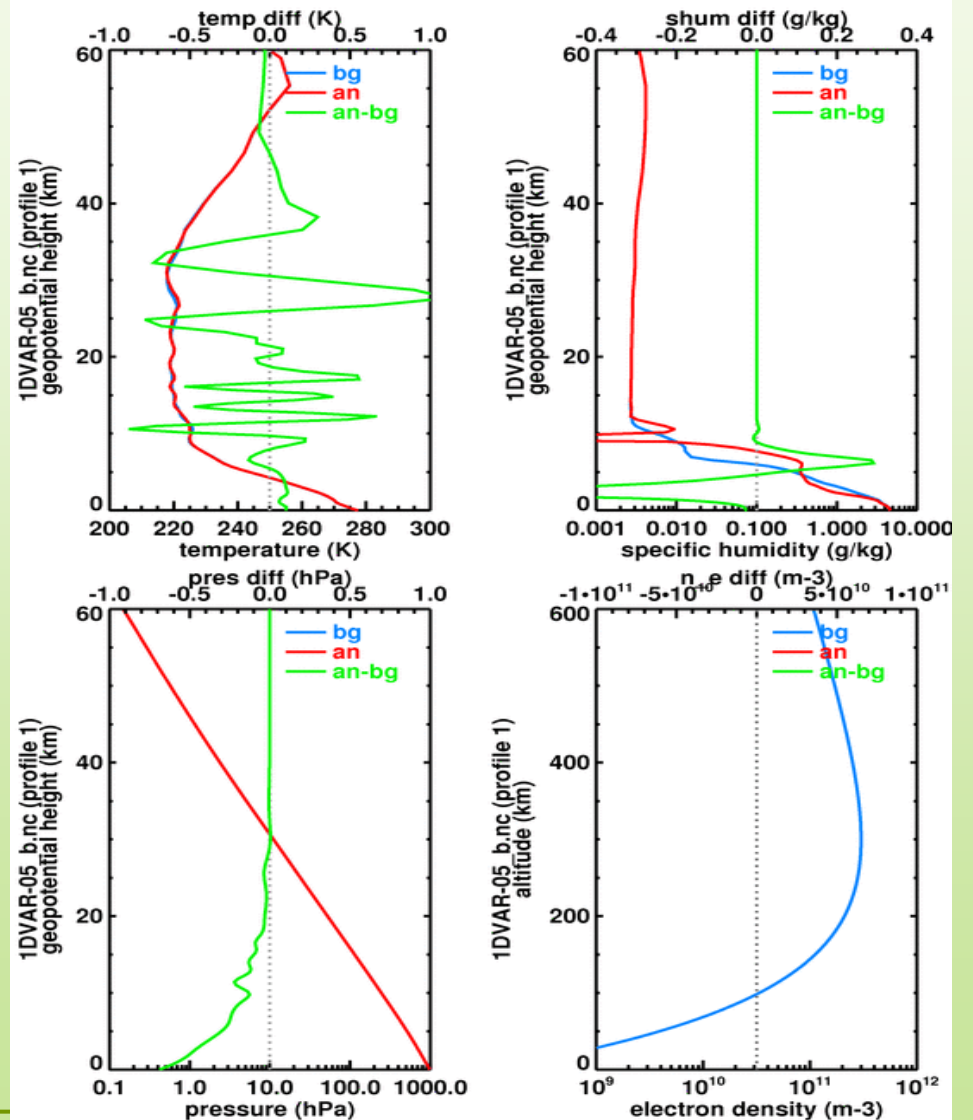
- Minimise $2J(\mathbf{x}) = (\mathbf{x} - \mathbf{b})^T \mathcal{B}^{-1} (\mathbf{x} - \mathbf{b}) + (\mathcal{H}(\mathbf{x}) - \mathbf{o})^T \mathcal{R}^{-1} (\mathcal{H}(\mathbf{x}) - \mathbf{o})$
- $\mathbf{x} = \{T, q, p^*, n_e^{\max}, r_0, H\}$; $\mathbf{o} = \{\alpha_{L1}, \alpha_{L2}\}$
- \mathcal{R}
 - $\sigma(\alpha_{L1}) = \max(\alpha_n, 10 \mu\text{rad})$; $\sigma(\alpha_{L2}) = \max(\alpha_n, 30 \mu\text{rad})$
 - Assume α_{L1} and α_{L2} errors to be uncorrelated
 - Needs some experimentation
- \mathcal{B}
 - $\sigma(n_e^{\max}) = 2e11 \text{ m}^{-3}$; $\sigma(r_0) = 150 \text{ km}$; $\sigma(H) = 25 \text{ km}$
 - Assume $\{n_e^{\max}, r_0, H\}$ errors to be uncorrelated from each other and from those of $\{T, q, p^*\}$
 - Needs some experimentation

Example 1: ropp_1dvar retrieval based on neutral bending angle

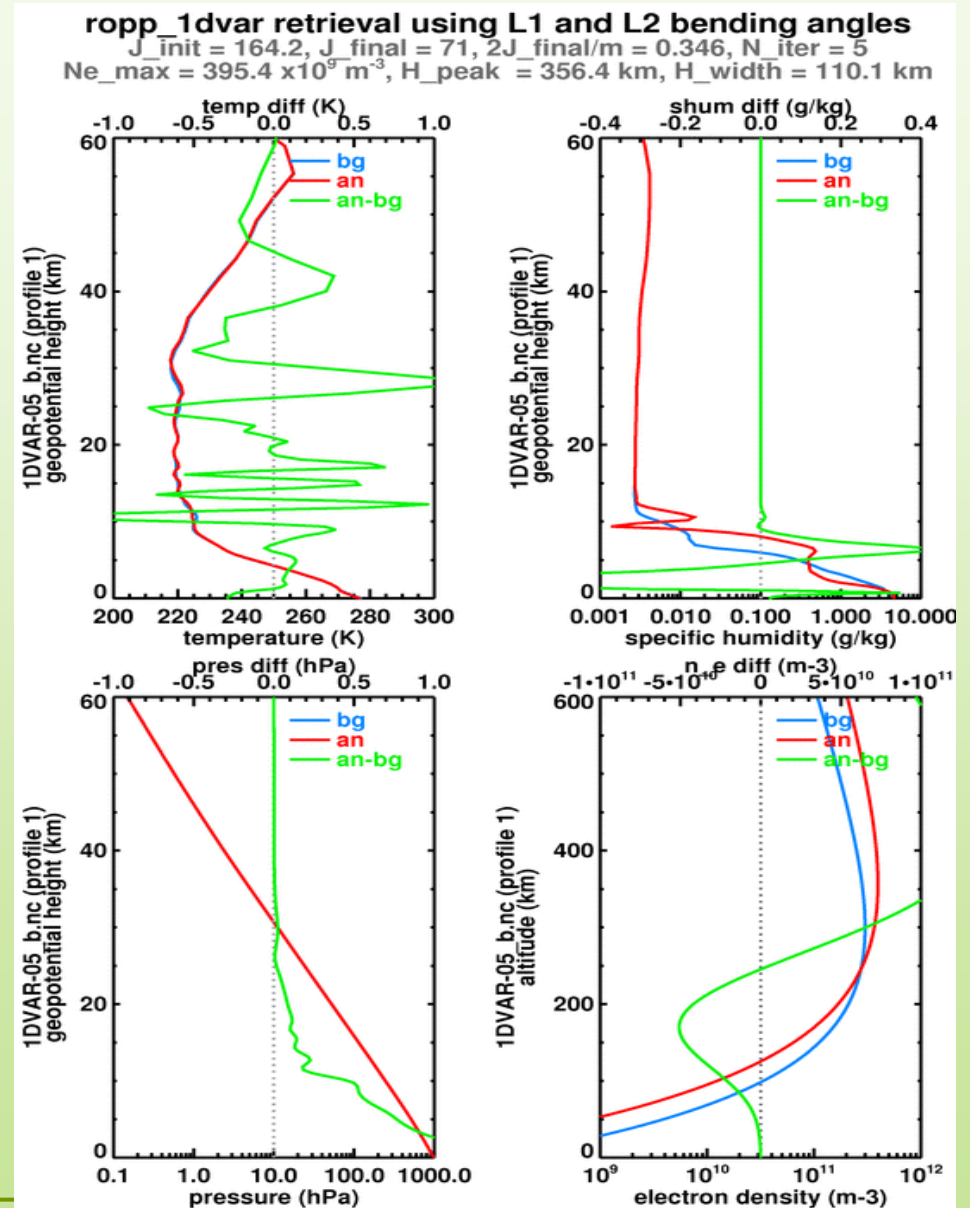
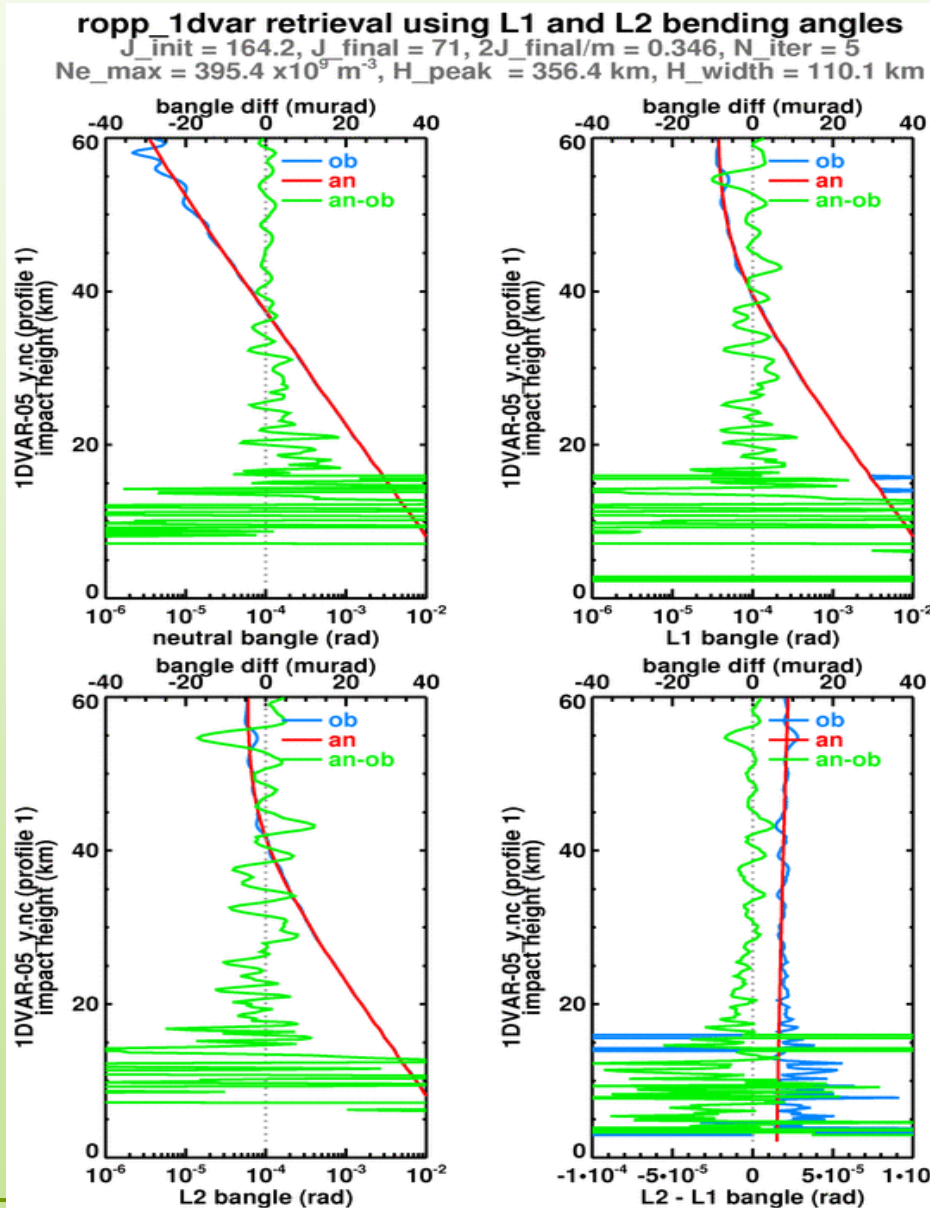
ropp_1dvar retrieval using neutral bending angle
 $J_{init} = 78.6$, $J_{final} = 46.6$, $2J_{final}/m = 0.469$, $N_{iter} = 4$
 $Ne_{max} = ropp_MDFV$, $H_{peak} = ropp_MDFV$, $H_{width} = ropp_MDFV$



ropp_1dvar retrieval using neutral bending angle
 $J_{init} = 78.6$, $J_{final} = 46.6$, $2J_{final}/m = 0.469$, $N_{iter} = 4$
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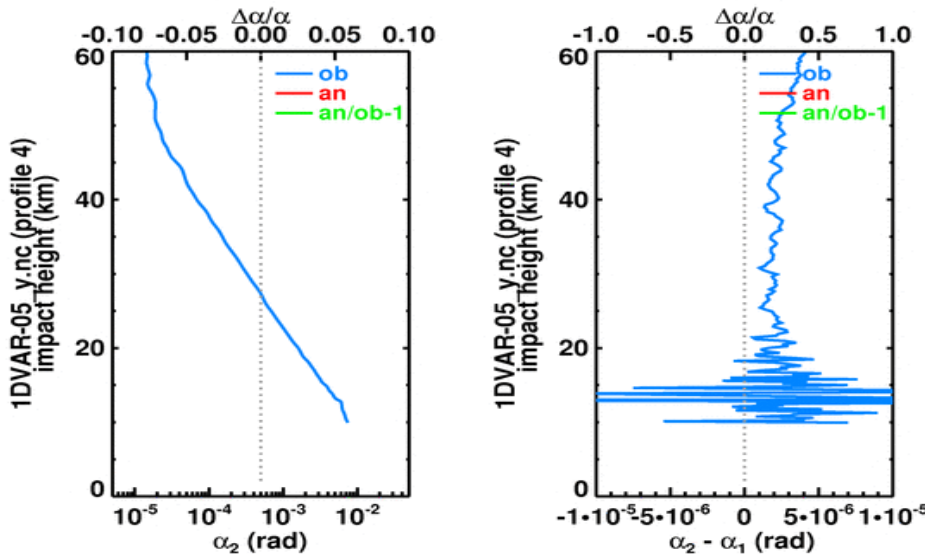
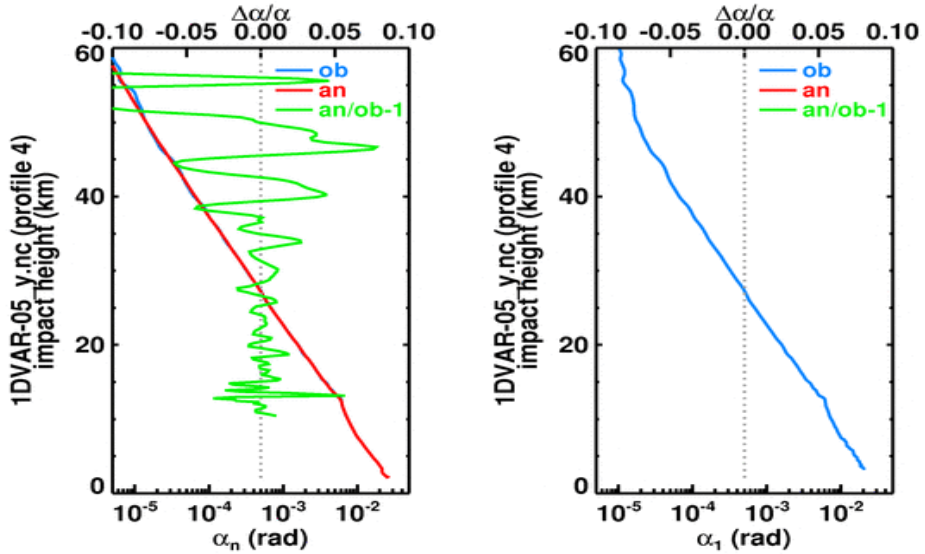


Example 1: ropp_1dvar retrieval based on L1 & L2 bending angles

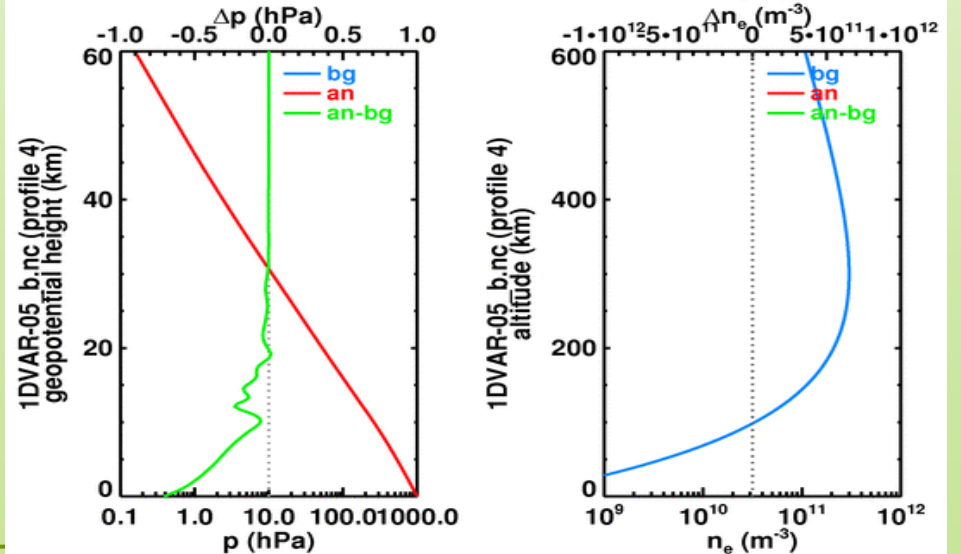
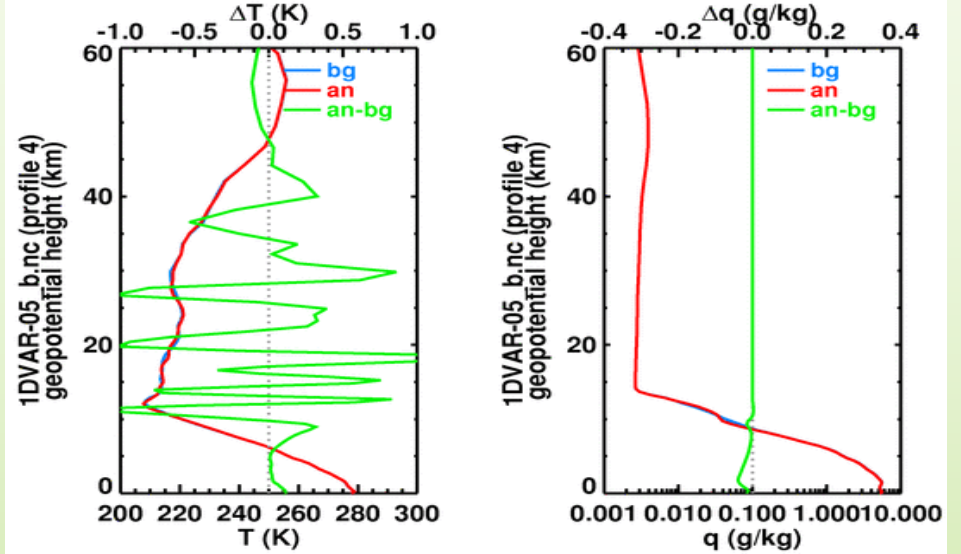


Example 2: ropp_1dvar retrieval based on neutral bending angle

ropp_1dvar retrieval using neutral bending angle
 $J_{init} = 154.5$, $J_{final} = 89.9$, $2J_{final}/m = 0.961$, $N_{iter} = 5$
 $Ne_{max} = ropp_MDFV$, $H_{peak} = ropp_MDFV$, $H_{width} = ropp_MDFV$



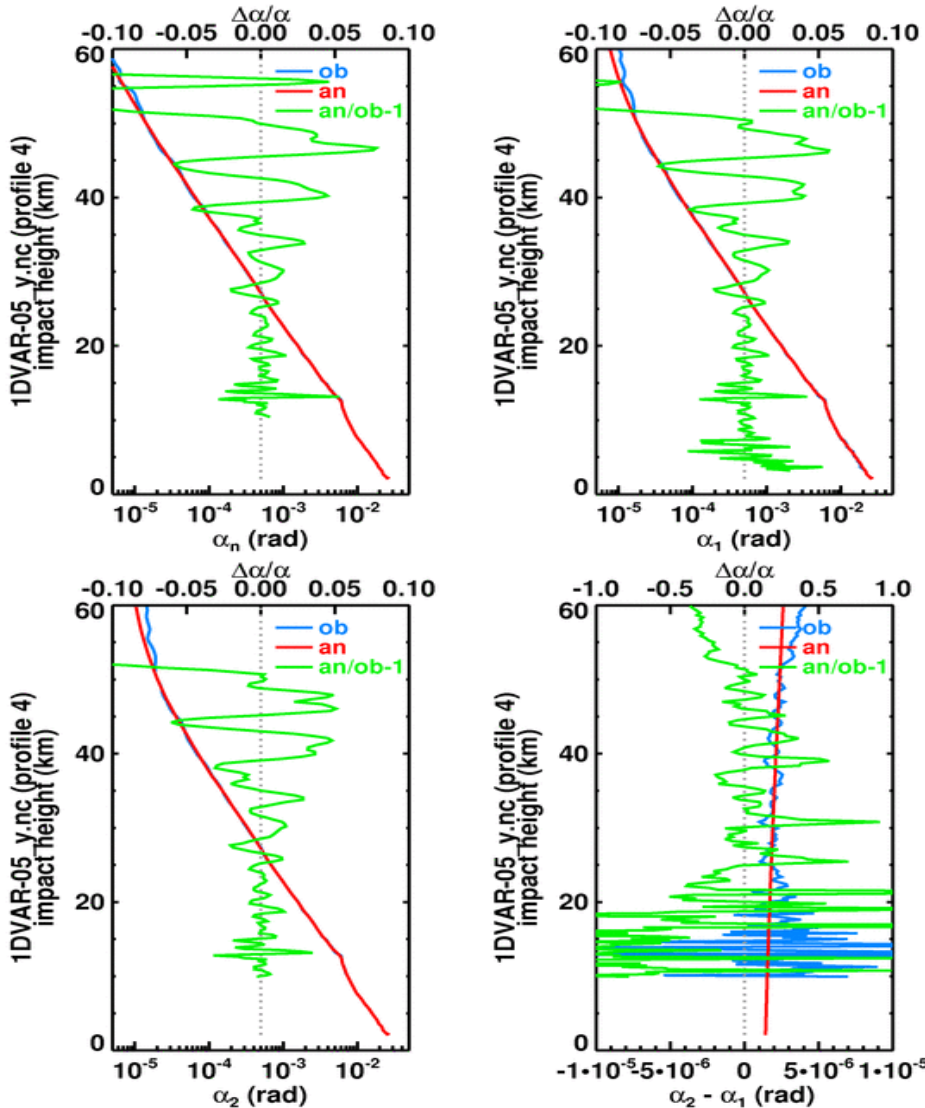
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Example 2: ropp_1dvar retrieval based on L1 & L2 bending angles

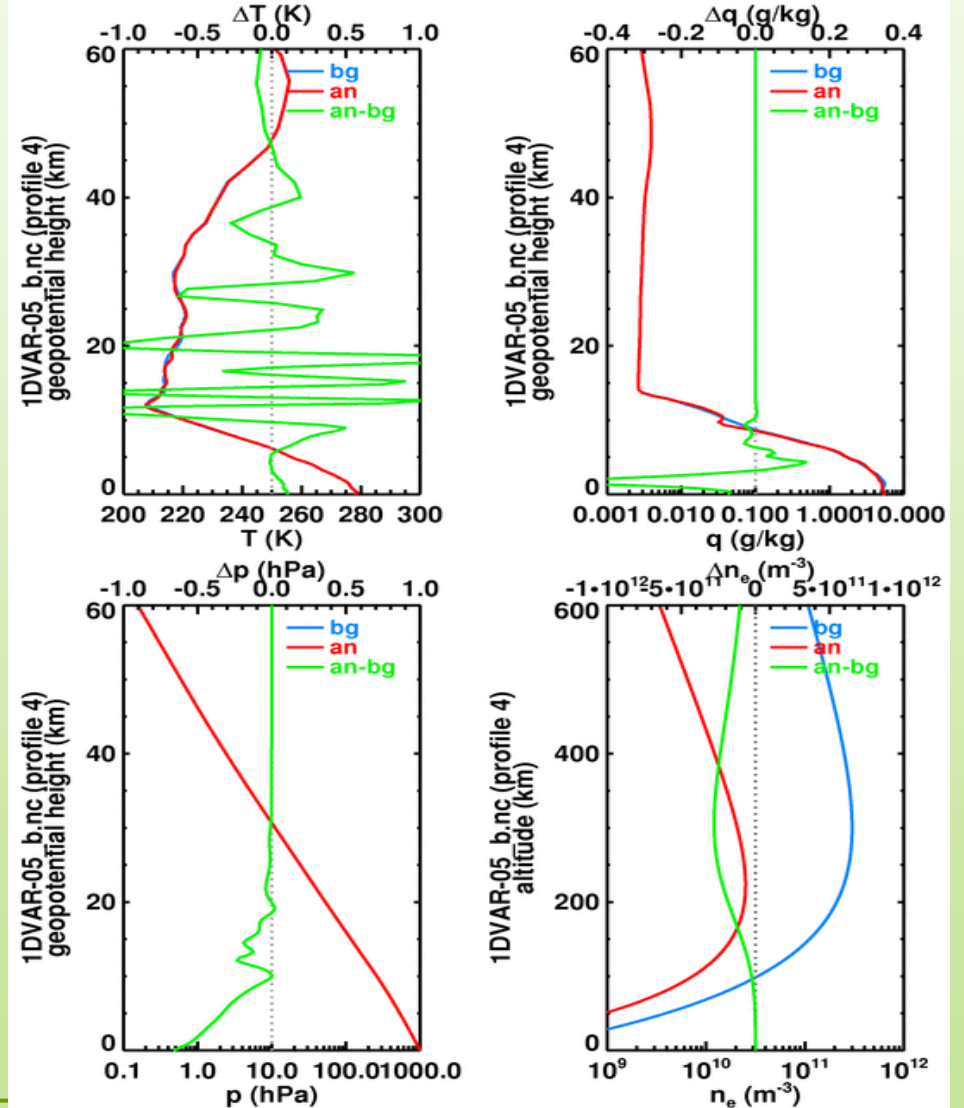
ropp_1dvar retrieval using L1 and L2 bending angles

J_init = 258.3, J_final = 114, 2J_final/m = 0.533, N_iter = 4
 Ne_max = 25.1 x 10⁹ m⁻³, H_peak = 222.7 km, H_width = 75.4 km



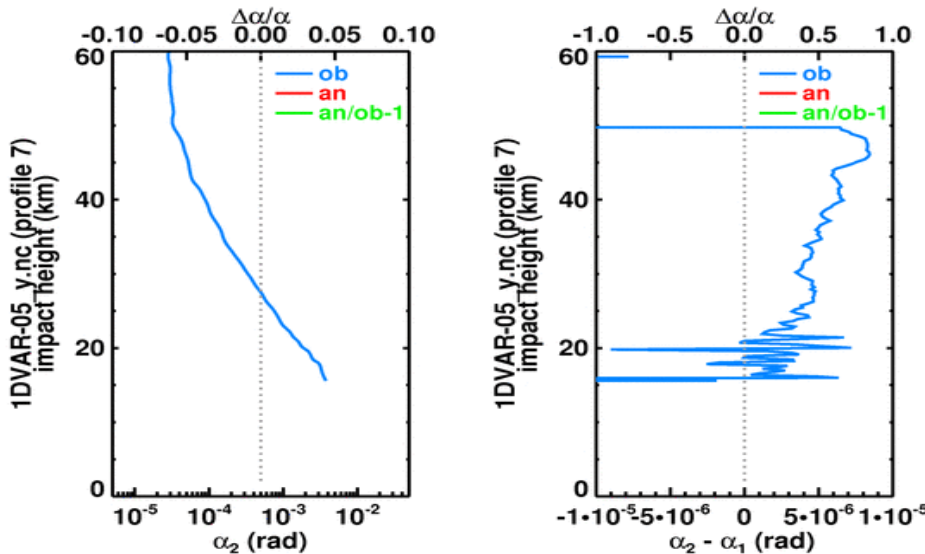
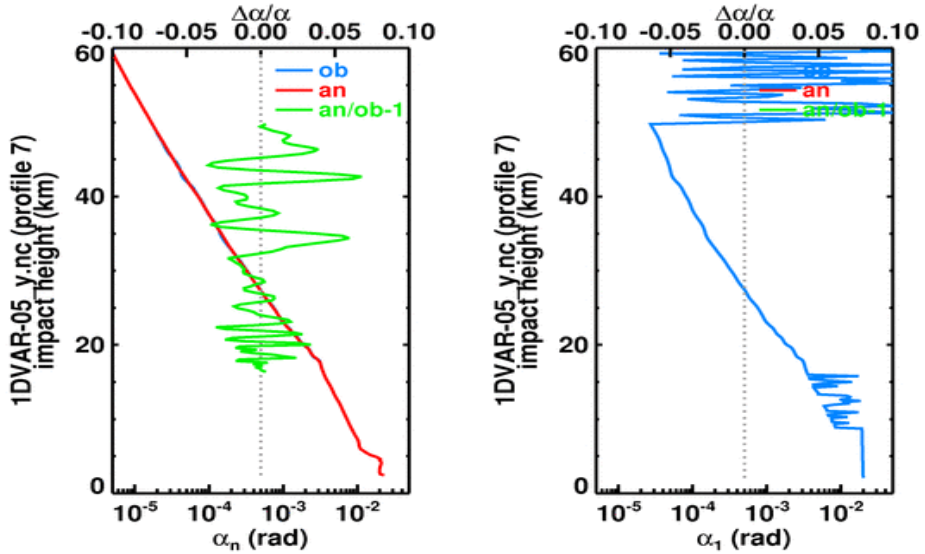
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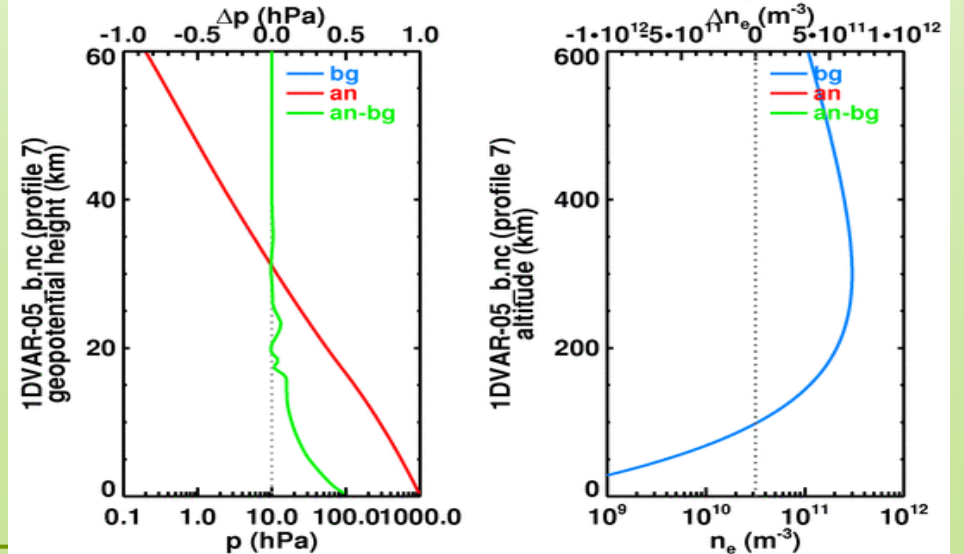
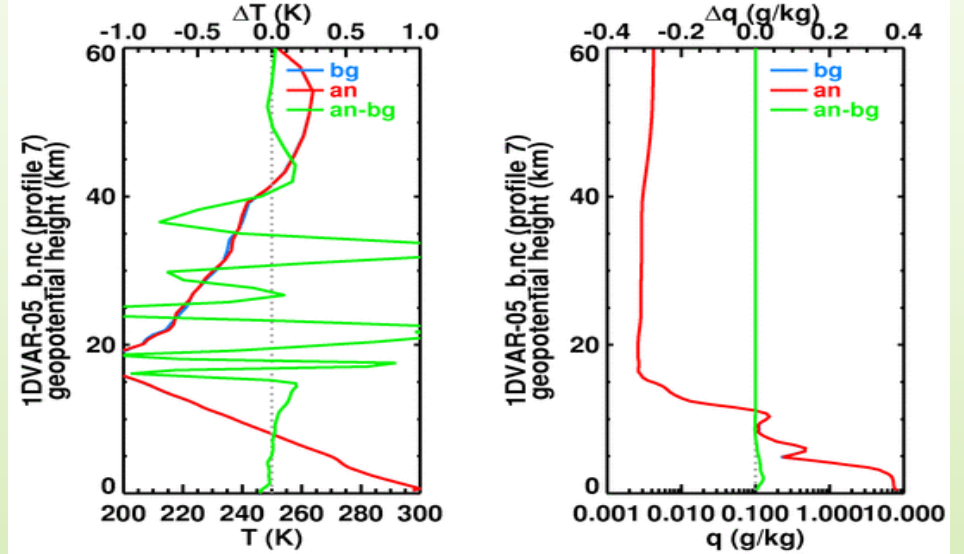


Example 3: ropp_1dvar retrieval based on neutral bending angle

ropp_1dvar retrieval using neutral bending angle
 $J_{init} = 140.4$, $J_{final} = 87.5$, $2J_{final}/m = 1.446$, $N_{iter} = 4$
 $Ne_{max} = ropp_MDFV$, $H_{peak} = ropp_MDFV$, $H_{width} = ropp_MDFV$



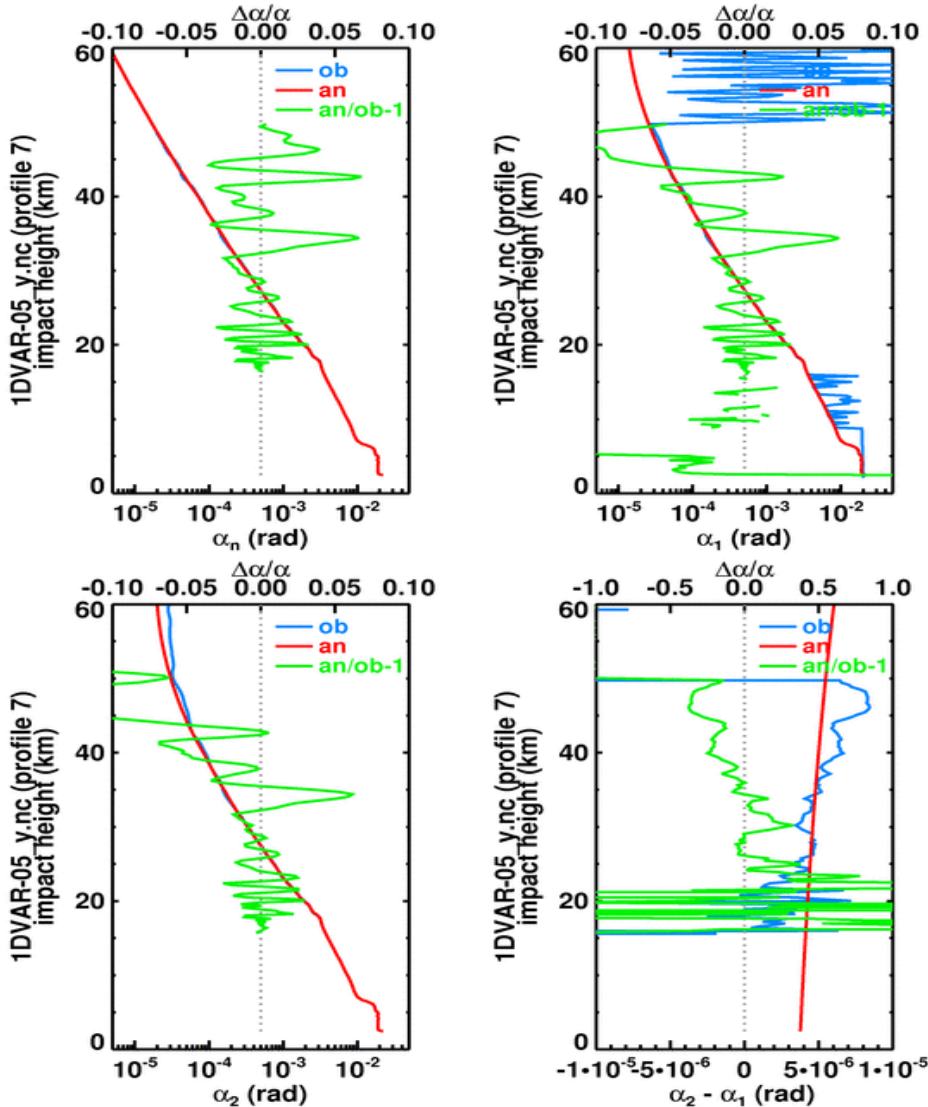
ropp_1dvar retrieval using neutral bending angle
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 $Ne_{max} = ropp_MDFV$, $H_{peak} = ropp_MDFV$, $H_{width} = ropp_MDFV$



Example 3: ropp_1dvar retrieval based on L1 & L2 bending angles

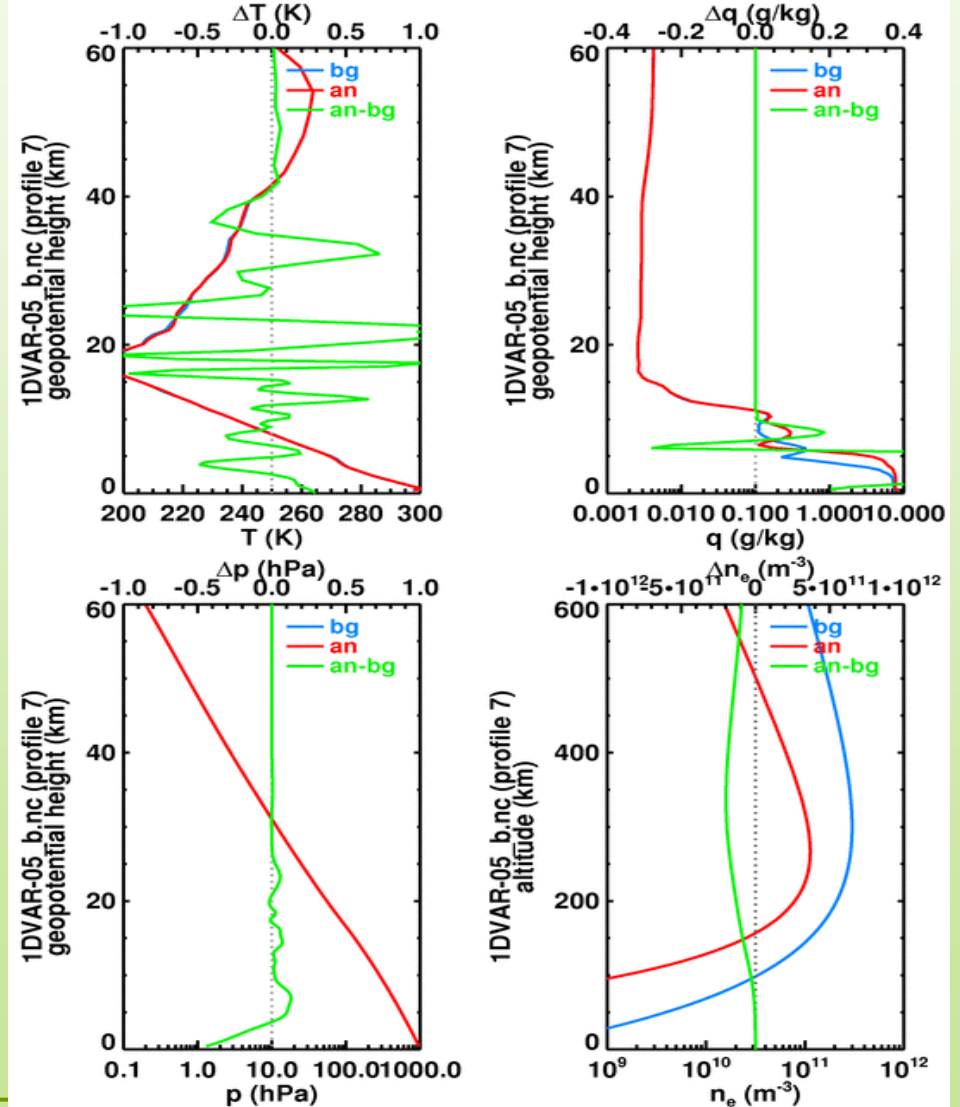
ropp_1dvar retrieval using L1 and L2 bending angles

$J_{init} = 483.9$, $J_{final} = 232.4$, $2J_{final}/m = 1.387$, $N_{iter} = 6$
 $Ne_{max} = 113.2 \times 10^9 \text{ m}^{-3}$, $H_{peak} = 266.8 \text{ km}$, $H_{width} = 66.8 \text{ km}$



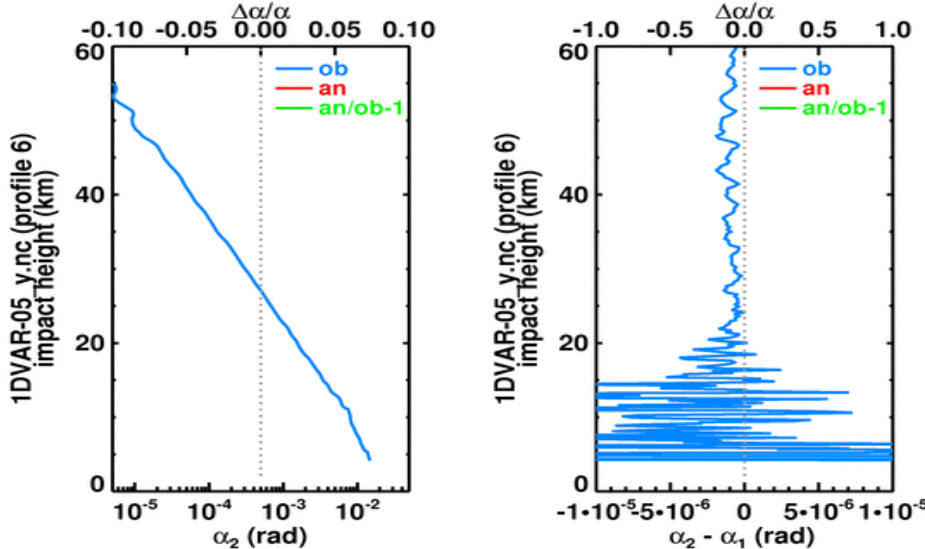
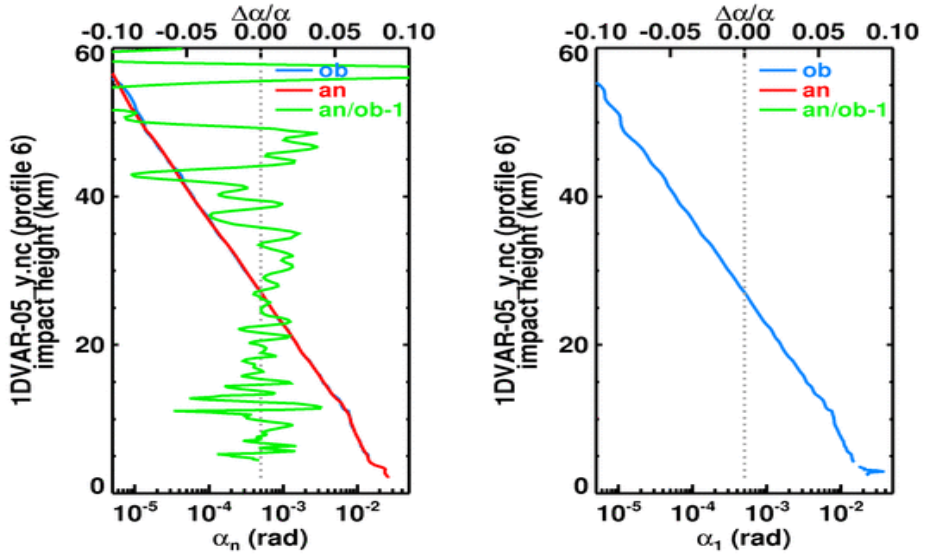
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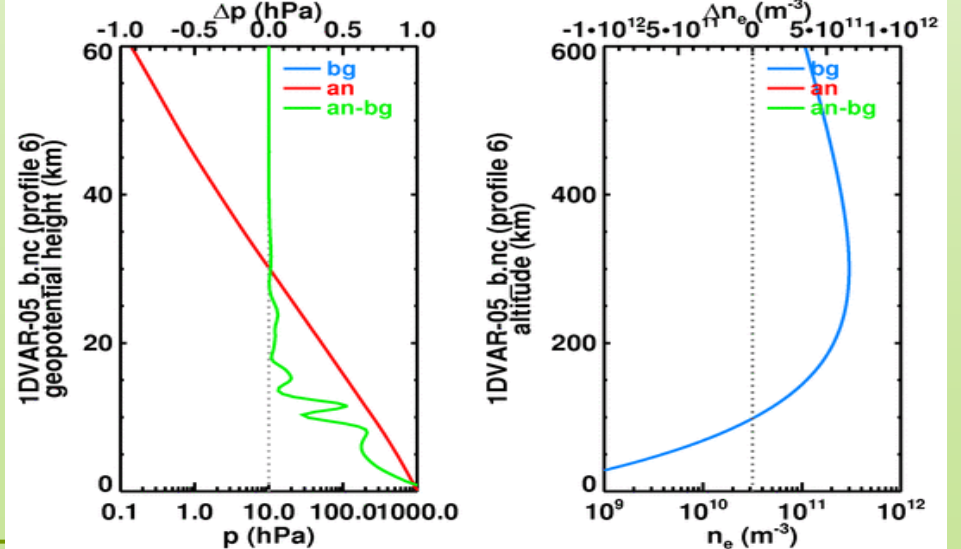
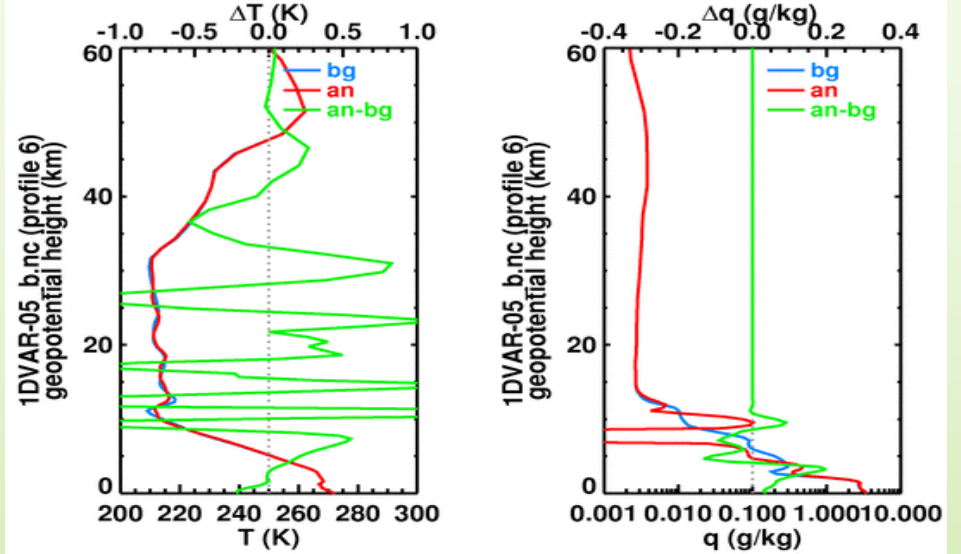


Example 4: ropp_1dvar retrieval based on neutral bending angle

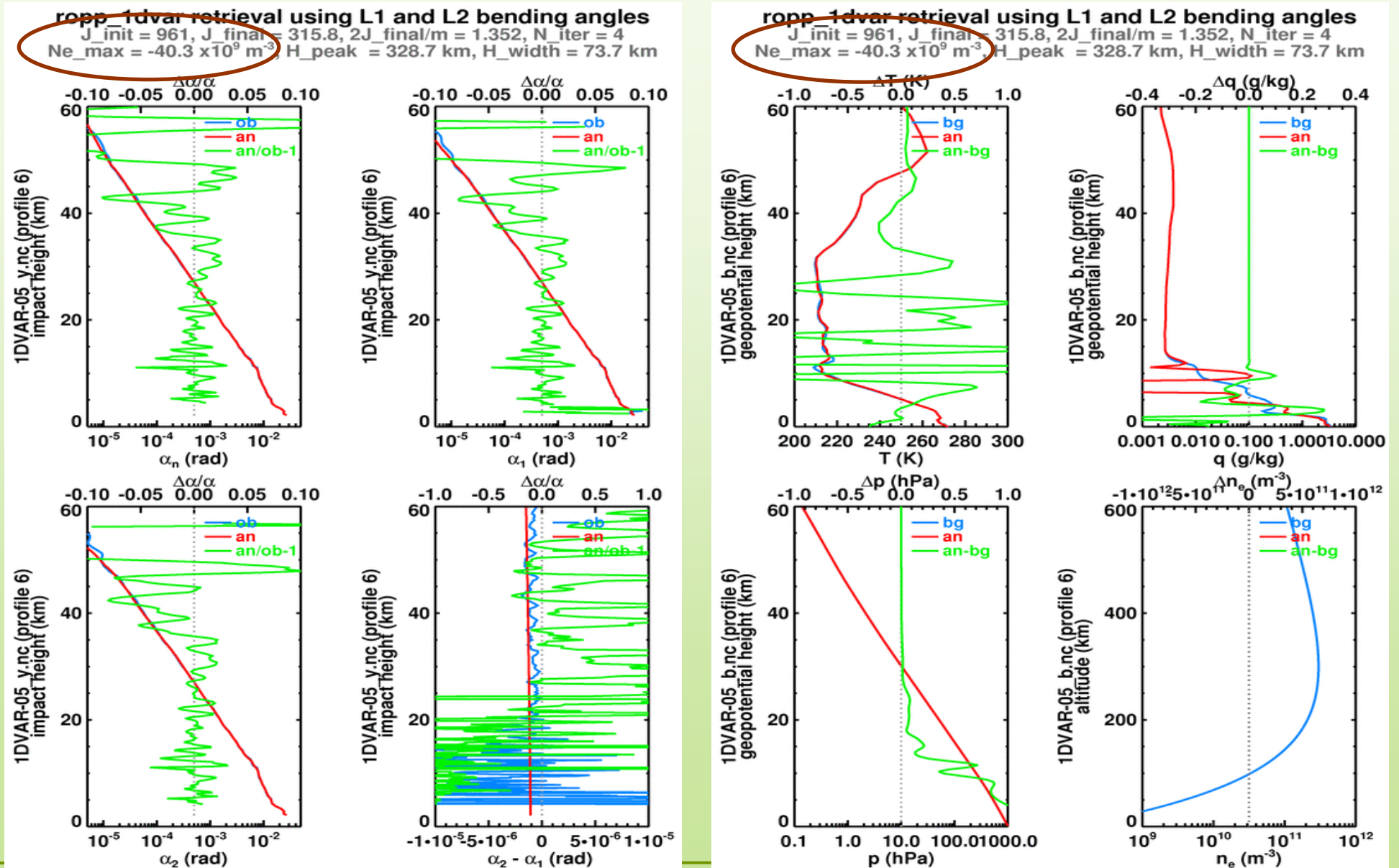
ropp_1dvar retrieval using neutral bending angle
 $J_{init} = 527, J_{final} = 251.5, 2J_{final}/m = 2.206, N_{iter} = 4$
 $Ne_{max} = ropp_MDFV, H_{peak} = ropp_MDFV, H_{width} = ropp_MDFV$



ropp_1dvar retrieval using neutral bending angle
 $J_{init} = 527, J_{final} = 251.5, 2J_{final}/m = 2.206, N_{iter} = 4$
 $Ne_{max} = ropp_MDFV, H_{peak} = ropp_MDFV, H_{width} = ropp_MDFV$

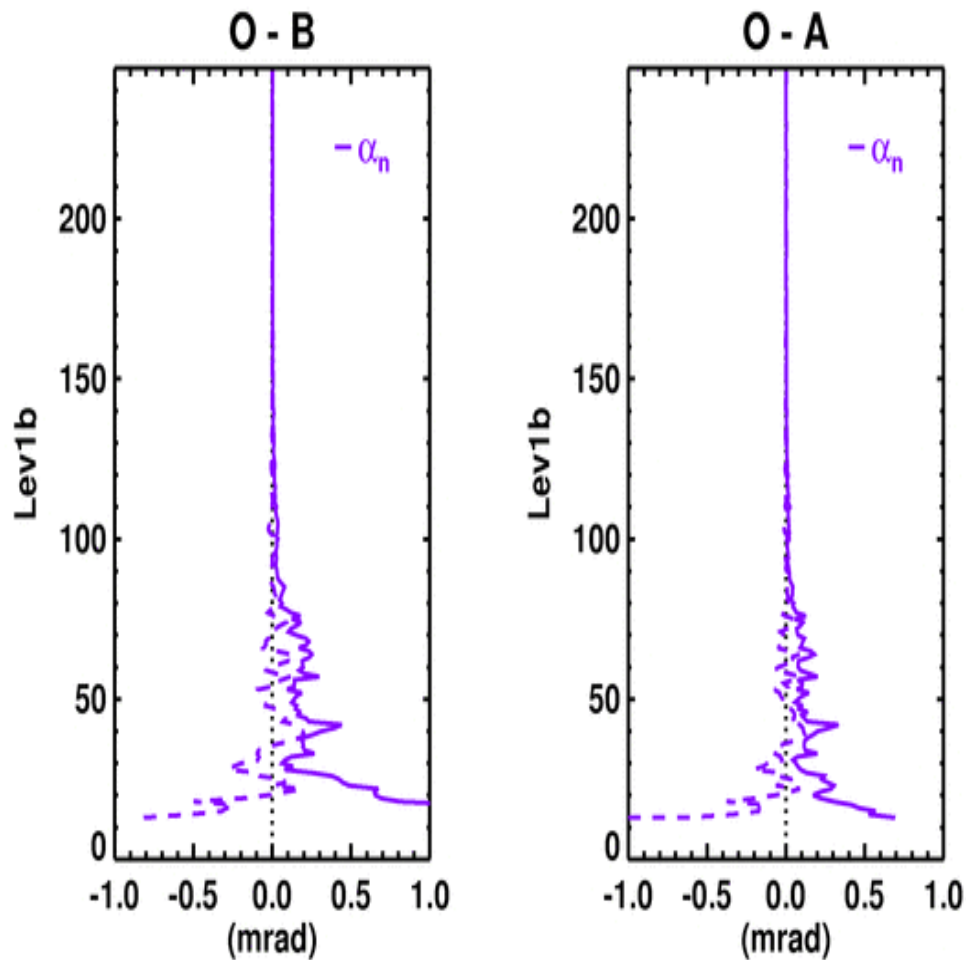


Example 4: ropp_1dvar retrieval based on L1 & L2 bending angles

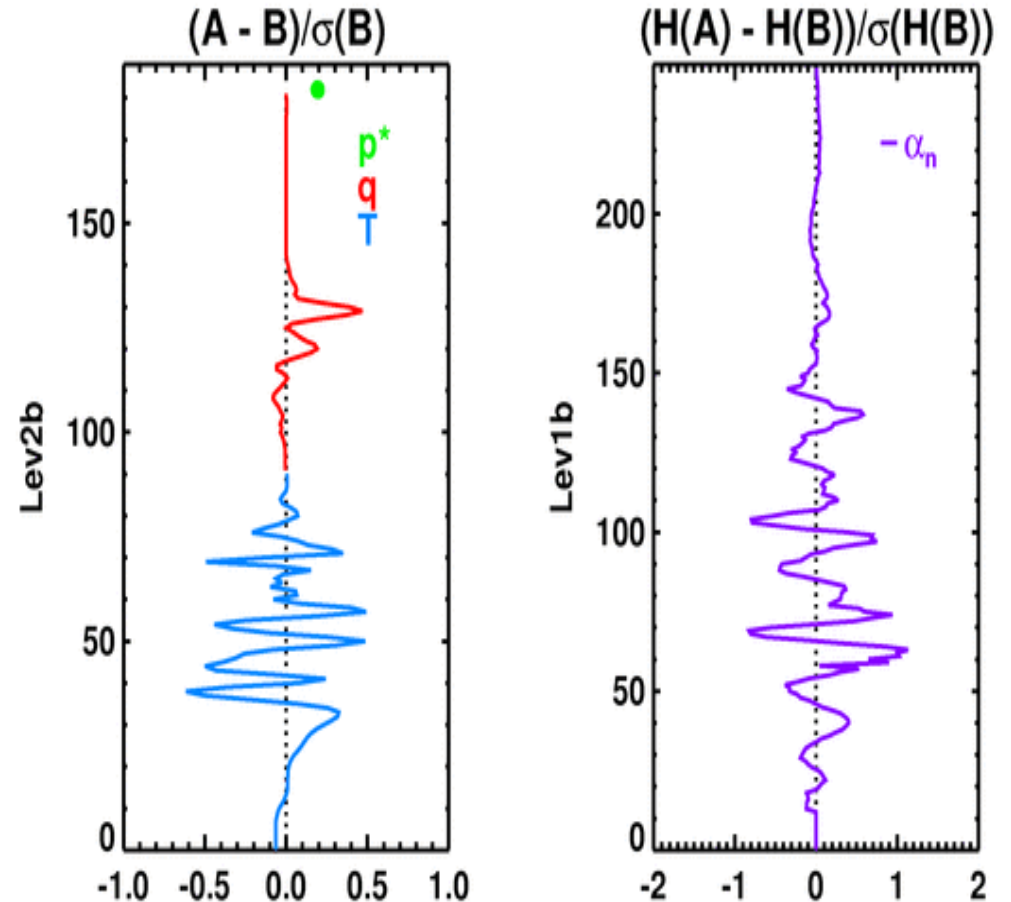


Direct modelling of L1 and L2 in ROPP: α_n stats

ropp_1dvar retrieval using neutral bending angle

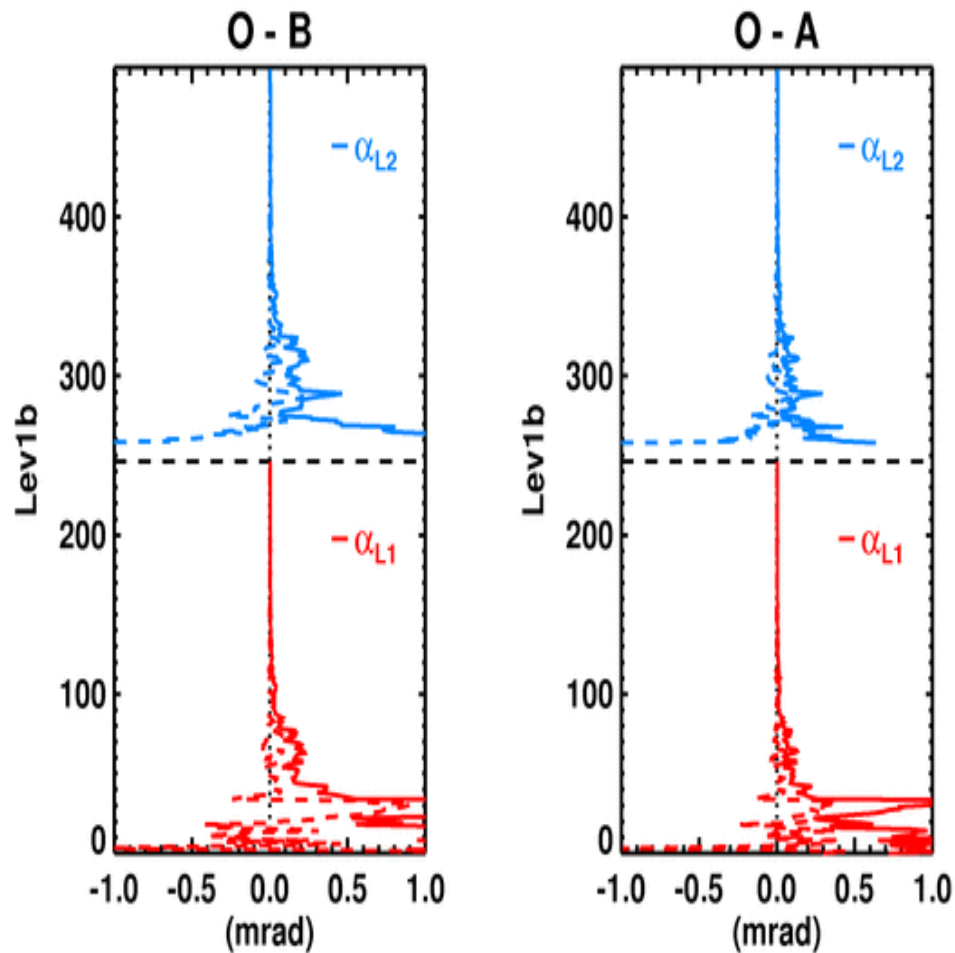


ropp_1dvar retrieval using neutral bending angle

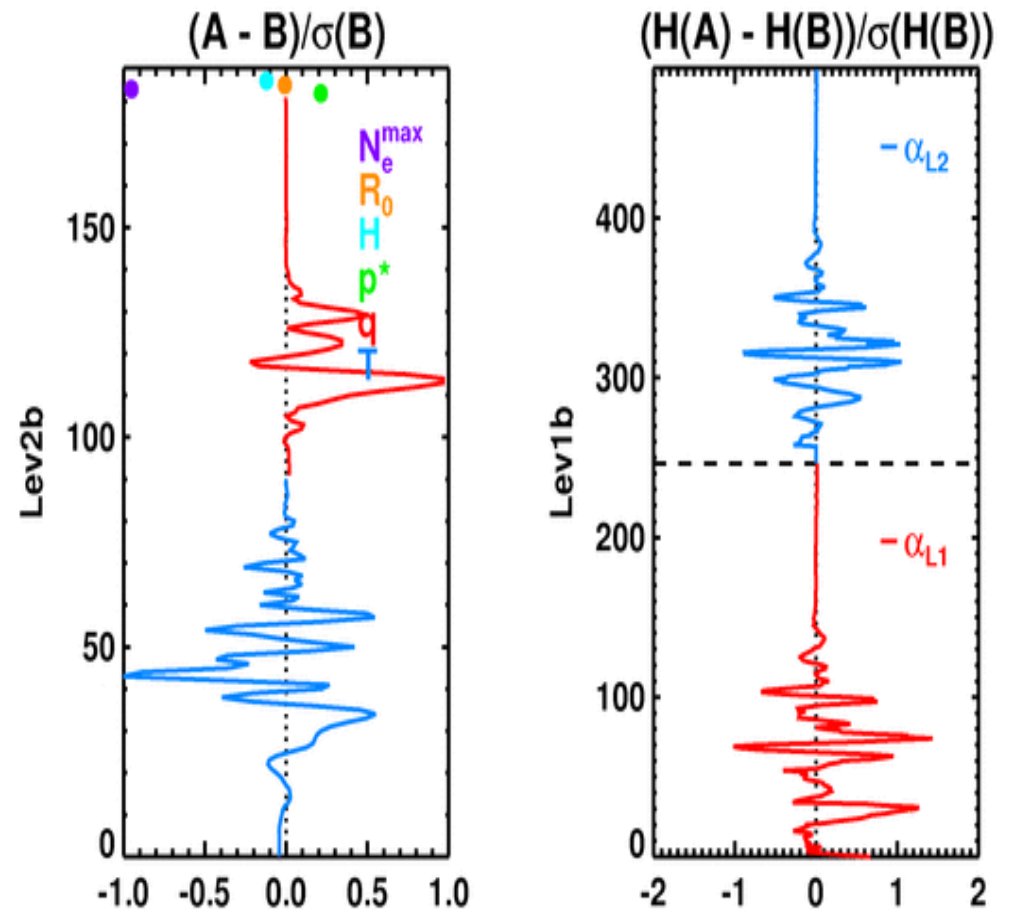


Direct modelling of L1 and L2 in ROPP: α_{L_i} stats

ropp_1dvar retrieval using L1 and L2 bending angles

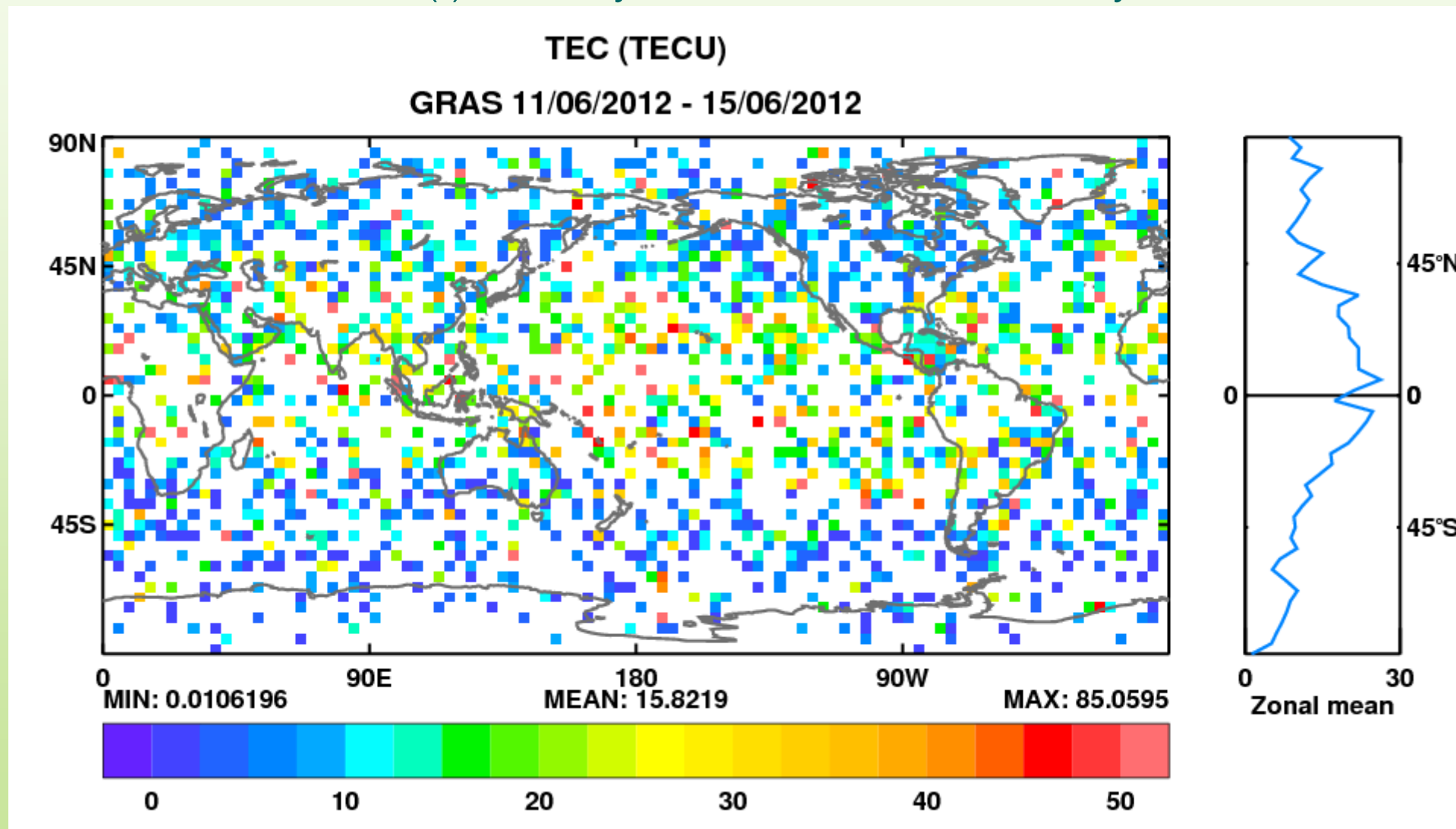


ropp_1dvar retrieval using L1 and L2 bending angles



“Retrieved ionospheric parameters”

Fit “normal” L1-L2 to $Z(\ell)$ for every GRAS occultation for 5 days in June 2012



Direct modelling of L1 and L2 in ROPP: summary

- Can forward model α_{L1} and α_{L2} by assuming simple Chapman layer ionosphere
- Allows extrapolation of α_{L1} and α_{L2} based on whole profiles of data
- Retrievals using α_{L1} and α_{L2} are possible in principle BUT
 - Bigger impacts on q and tropospheric T ?
 - Not always capturing curvature of L1 and L2 at top?
 - Require $n_e^{\max} < 0$ to get decent fit in some cases!
- Backgrounds/errors/covariances of n_e^{\max} , r_0 and H **need some experimentation.**
- Errors/covariances of α_{L1} and α_{L2} **need some experimentation.**

Residual ionospheric correction

- Working on a model for the residual error that remains after standard ionospheric correction (Vorob'ev and Krasil'nikova, 1994)

- $\alpha(a) = -2a \int_a^\infty d \log n / dx \, dx / \sqrt{(x^2 - a^2)} \approx$

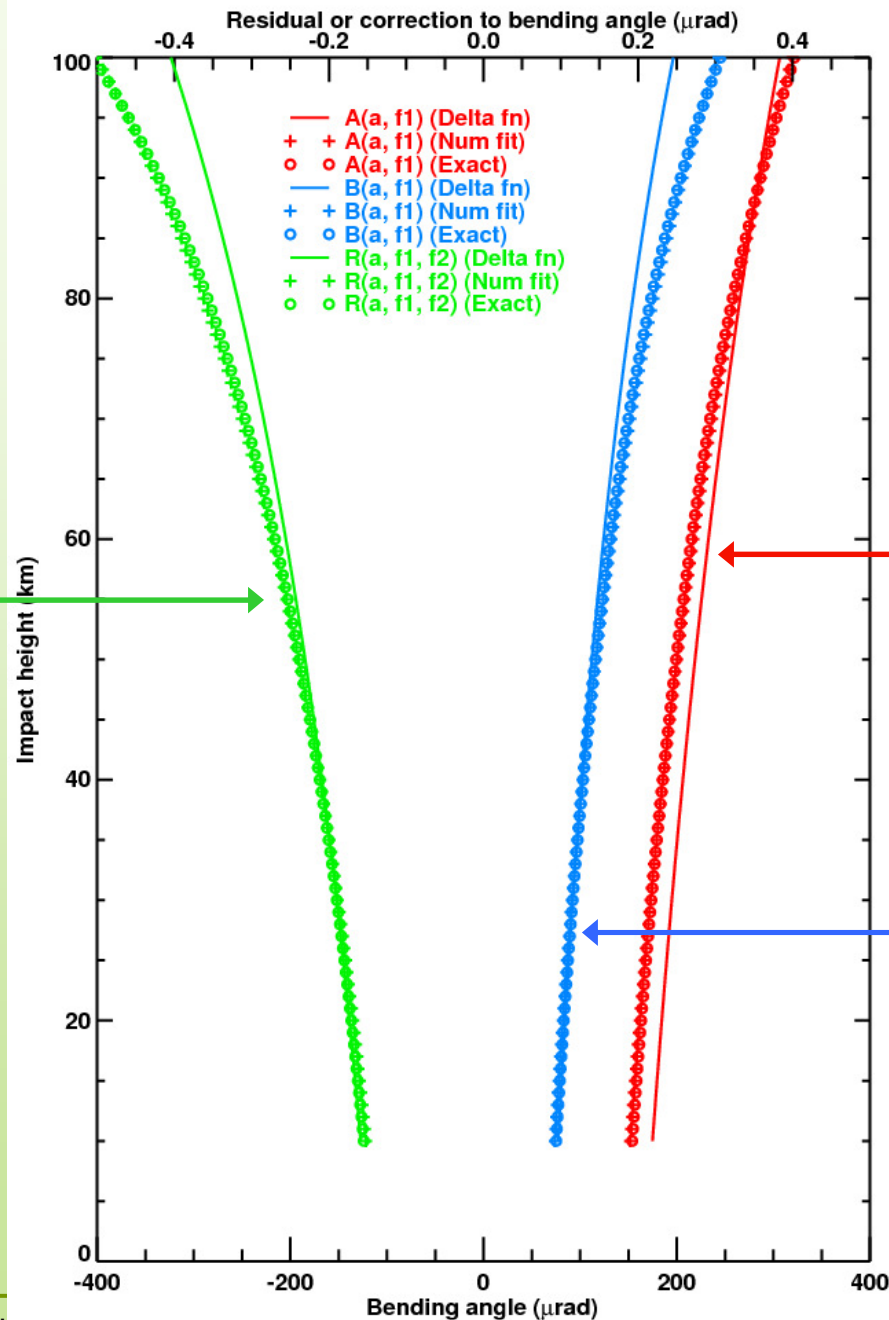
$$-2a \int_a^\infty N' \, dr / \sqrt{(r^2 - a^2)} \quad \text{Basic bending angle, } \propto \text{TEC}/f^2$$

$$+ 2a \int_a^\infty NN' \, dr (2r^2 - a^2) / (r^2 - a^2)^{3/2} \quad \text{1st order correction, } \propto \text{TEC}^2/f^4$$

- Implies that residual after standard ionospheric correction $\propto (\alpha_{L1} - \alpha_{L2})^2 \times$ some *slowly varying function of height*, which we can estimate/calculate analytically for a Chapman layer.
- We are working with Julia Danzer (GFZ/ROM SAF VS) to test this.
- Could have implications for RO-based climatological mean temperatures.

(cf climate change trend ~ few μrad per decade)

Residual



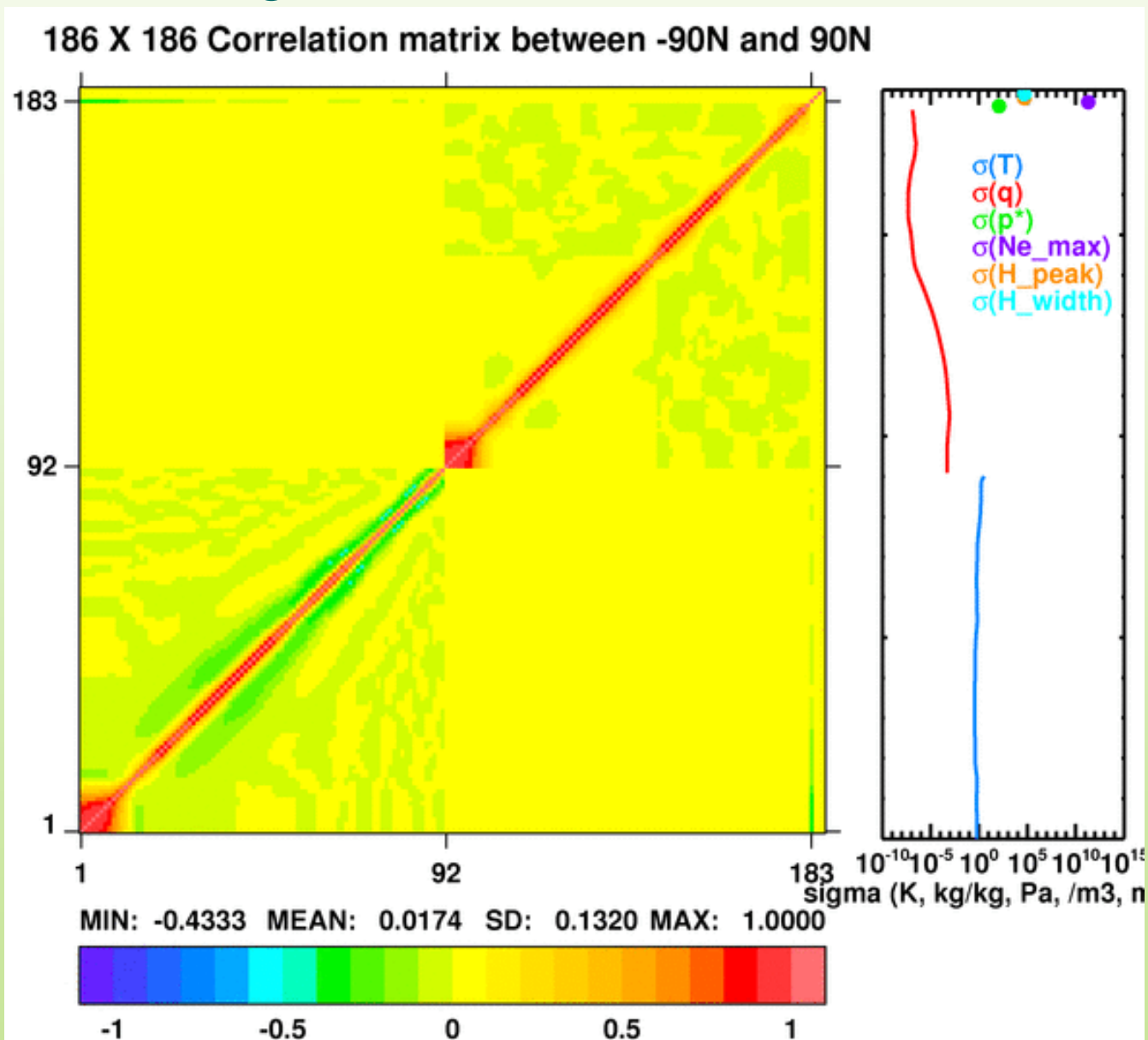
α_{L1}

α_{L1} correction

Conclusions

- Calculated bending angle produced by a spherically symmetric Chapman layer ionosphere.
- Incorporated in ROPP forward model and 1dvar system.
- Expressed residual bending angle, after standard ionospheric correction, in terms of $(\alpha_{L1} - \alpha_{L2})^2$.

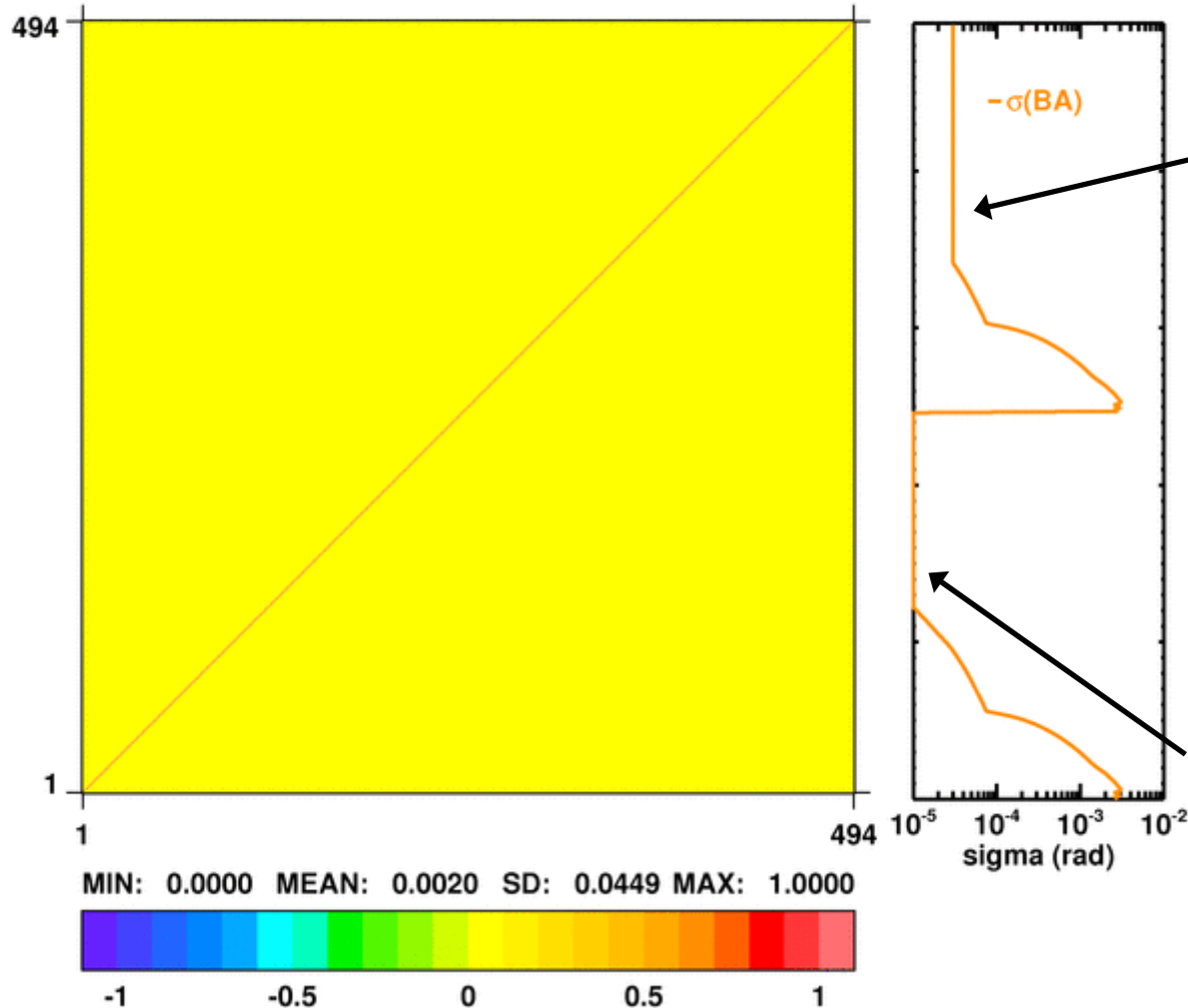
Background error covariance matrix



Bending angle error covariance matrix

Bending angle packed observation correlation matrix for ROPP

494 X 494 Correlation matrix between -90N and 90N

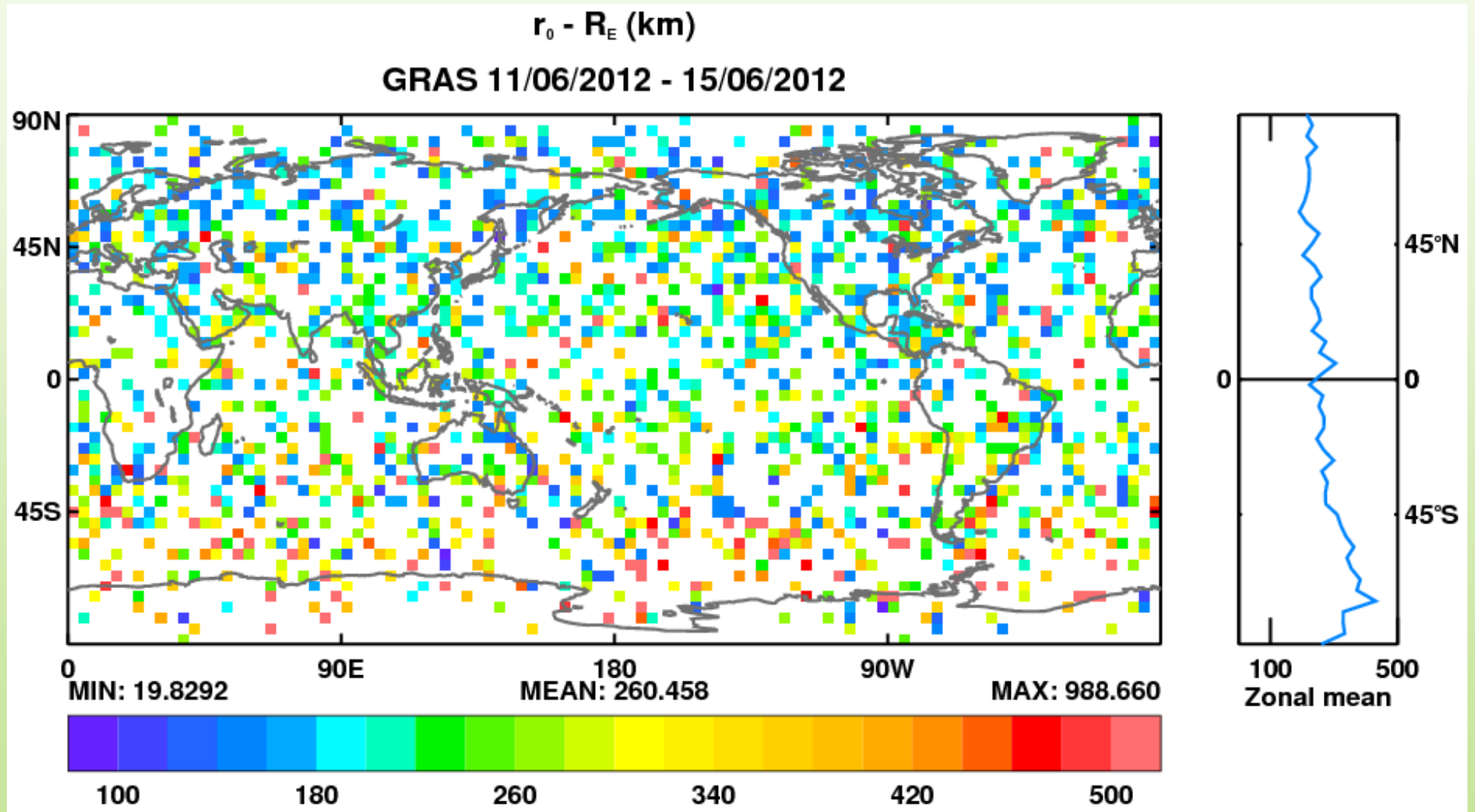


$\sigma_2^{\min} = 30 \mu\text{rad}$

cf $\sigma_n^{\min} = 6 \mu\text{rad}$

$\sigma_1^{\min} = 10 \mu\text{rad}$

Fit “normal” L1-L2 to $Z(\ell)$ for every GRAS occultation for 5 days in June 2012



Fit “normal” L1-L2 to $Z(\ell)$ for every GRAS occultation for
5 days in June 2012

