Hybrid Variational-Ensemble Data Assimilation

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Abstract

Hybrid Variational-Ensemble data assimilation refers to a methodology through which the respective advantages of traditional variational and ensemble data assimilation approaches are combined to produce an analysis that is superior to that produced by either pure form. Traditional four-dimensional variational (4D-Var) assimilation has been the workhorse of many leading operational NWP centres for over a decade, and benefits from full-rank, four-dimensional forecast error covariances modelled via multivariate balance constraints and a linear model to evolve covariances in time. However, covariance models are imperfect and typically only model climatological balances. The linear model can be expensive to develop and maintain, and may suffer from poor scalability on modern HPC platforms. Ensemble data assimilation attempts to circumvent the covariance modeling effort by making explicit use of flow-dependent forecast error information provided by ensemble prediction systems. However, ensemble-based error covariances typically suffer from significant sampling error due to the relatively small ensemble size (20-200) affordable in an NWP context, so covariance modelling in the form of covariance localization and inflation is still required. The hybrid approach merges the two sources of covariance information to ameliorate the low-rank, ensemble sampling issue whilst at the same time smoothly introducing flow-dependence covariances to the 4D-Var algorithm. The hybrid variational/ensemble approach is particularly attractive for those operational centres that have already developed sophisticated variational data assimilation systems as well as ensemble prediction systems for probabilistic NWP. With these building blocks, the transition to hybrid variational/ensemble data assimilation is low cost, low risk and provides a smooth transition to the emerging world of ensemble data assimilation for operational NWP.

This paper provides a brief description of the so-called 'alpha control variable' approach to hybrid variational-ensemble data assimilation approach, including details of the application of traditional variational covariance modelling approaches to model ensemble covariance localization. A hybrid 4D-Var/Ensemble Transform Kalman Filter (ETKF) algorithm was implemented in operational global NWP at the Met Office in July 2011. Selected results from final trials of this implementation are presented. Plans to further couple the data assimilation and ensemble prediction systems at the Met Office are briefly outlined.

1. Introduction

Modern data assimilation systems use short-range forecast error information to optimize the detailed fit of the analysis to available observations. Forecast error covariances typically used within current-generation four-dimensional variational (4D-Var) data assimilation systems (Rabier et al. 2000, Rawlins et al. 2007, Huang et al. 2009) are typically based on the same climatological, modelled estimates used in previous generation 3D-Var systems (Lorenc et al. 2000, Wu et al. 2002, Barker et al. 2004). Use of the nonlinear model within 4D-Var, either directly as in "full-fields" 4D-Var (e.g. Sun and Crook 1998, Zou et al. 1997), or as a base state for a linear "perturbation forecast" (PF) model within "incremental" 4D-Var (Courtier et al. 1994), does provide a flow-dependent evolution of analysis increments through the time window. However, most 4D-Var systems still make use of a static error covariance matrix specified at the start of the time window, and so the analysis increment is somewhat blind to the current forecast "errors of the day" (Lorenc 2003). It is therefore reasonable to assume that better use of observations within both 3D-Var and 4D-Var will result from making use of the flow-dependent background error covariances.

Ensemble Kalman filter (EnKF) data assimilation techniques have received enormous attention in recent years as potential alternatives to variational data assimilation systems for NWP (e.g. Houtekamer and Mitchell 1998, Whitaker and Hamill 2002, Tippet et al. 2003, Snyder and Zhang 2003, Zupanski 2005, Tong and Xue 2005, Hunt et al. 2007, Miyoshi and Yamane 2007, Whitaker et al. 2008). Ensemble filters implicitly evolve flow-dependent forecast error covariances through the integration of ensembles of nonlinear forecasts. The ability of the ensemble to resolve details of the error covariance structure is proportional to the ensemble size, which is limited by practical constraints; typically 20-200 members are affordable for operational NWP. Lorenc (2003) lists several problems resulting from the limitation of finite ensemble size. Firstly, the number of observations that can be successfully assimilated using ensemble-based forecast error covariance estimates scales with ensemble size. Assimilation of high-density observations, such as radar and hyperspectral radiometers can lead to spurious increments in nearby data-sparse regions. One solution is to thin the data to reduce the number of degrees of freedom being analysed. Secondly, sampling error in resolving the forecast error probability density function will also manifest as spurious analysis increments. The usual solution is to apply empirical "covariance localization" (e.g. via use of a Schur/Hadamard product – Hamill et al. 2001) to eliminate weak, (hopefully) spurious covariances. Localization also has the benefit of increasing the number of degrees of freedom, and hence reducing the need for thinning, by decoupling analysis increments situated at large distance. Unfortunately, localization also tends to destroy balance (e.g. geostrophic, hydrostatic, cyclostrophic) that may be present in the true forecast errors (Lorenc 2003). Thirdly, low-rank ensemble-based forecast error covariances tend to underestimate error variance (spread), especially if model error has not been adequately represented in the ensemble (Anderson 2001). Solutions include the use of multiplicative/additive "covariance inflation" of ensemble-based error-estimates, and the inclusion of perturbations (e.g. stochastic physics) within the forecast integrations themselves (e.g. Mitchell et al. 2002, Bowler et al. 2008).

In contrast, the modelled forecast error covariances typically used with variational data assimilation do not suffer from such sampling problems. They do, however, suffer from a range of other practical problems. Background error estimates for 3/4D-Var are typically computed off-line from lagged forecast differences (Parrish and Derber 1992) or ensemble perturbations (Fisher 2003) averaged over an extended time-period ranging from a few weeks to several years. This time averaging removes any flow-dependent detail beyond a crude seasonal dependence. Frequently, the error covariance estimates are not recalculated with model upgrades. Secondly, background error covariances are typically specified not in physical space, but in an esoteric "control variable" space (e.g. power spectra of the eigenvector projection of the vertical component of unbalanced temperature forecast error, e.g. Ingleby 2001). This has the desired practical effect of preconditioning and diagonalizing the prescribed background error covariance, but makes their visualization and interpretation difficult. Finally, even if flow-dependent information were available, typical assumptions made within the definition of control variables (e.g. isotropy, homogeneity) would render the assimilation system blind to these details. Except for that flow-dependence that can be retrieved from the use of the nonlinear trajectory as base state within the linear model in 4D-Var, the error covariances have very little knowledge concerning the quality of the forecast against which they are attempting to fit observations. Clearly, both variational and ensemble estimates of forecast error are sub-optimal in practical applications.

The scientific advantages and disadvantages need to be weighted against practical considerations, for which all techniques have strengths/weaknesses. As ensemble prediction becomes mainstream due to the requirement for probabilistic forecast products, the major cost of ensemble data assimilation (the forecast update step) is already ‘paid for’, and could conceivably be completed prior to the start of the assimilation step. In contrast, the major costs of incremental 4D-Var are the tangent linear and adjoint models, integrated iteratively within the assimilation step, and run essentially for data assimilation purposes only. Thus, although the overall costs may be similar, 4D-Var places a much larger burden on the assimilation step, potentially delaying the completion of the analysis (and subsequent forecast) step: an important consideration for operational NWP. It has frequently been argued that ensemble data assimilation systems are easier to maintain than variational systems (e.g. Kalnay et al. 2007). Whilst it is true that linear/adjoint models require significant resources to develop, this is only an issue for those models for which adjoints do not yet exist (many operational centers maintain adjoint models with relatively low maintenance costs). It is also inevitable that the complexity of ensemble data assimilation systems will increase as they begin to include the features already contained within current variational schemes; e.g. outer loop treatment of nonlinearities, correlated observation errors, complex quality control, observation operators for high-density non-traditional observations (e.g. radiances, refractivities). Memory scalability and redundant recomputations of analysis increments are still sub-optimal features in ensemble data assimilation (Hunt et al. 2007, Anderson and Collins 2007). The scalability of 4D-Var is dependent on the numerical scheme used within the linear forecast model, which is often based on the numerics of the corresponding nonlinear model. Frequently, the low-resolution linear application within 4D-Var has not been considering in the design of the nonlinear model, and hence scalability may be compromised.

Hybrid (variational/ensemble) data assimilation approaches have been investigated in recent years that attempt to combine the best of both variational and ensemble frameworks. Barker (1999) performed initial studies of a hybrid variational/ensemble system using the Met Office’s operational 3D-Var system (Lorenc et al. 2000), and an ensemble prediction system based on the Error Breeding technique (Toth and Kalnay 1997). Results indicated that a flow-dependent response to observations could be achieved at very low cost through the introduction of additional control variables within the variational cost function. As the Met Office did not at the time have a strategic requirement for an ensemble prediction system, only a very crude ensemble – low-resolution and only two members – was possible. Perhaps unsurprisingly, preliminary pre-operational trials indicated only a neutral impact overall, but recommended further studies using a larger ensemble size. Hamill and Snyder (2000) presented positive results from an alternative 3D-Var-ensemble hybrid requiring perturbed observations and multiple analyses. Etherton and Bishop (2004) found a similar result in a barotropic vorticity, perfect model context, and also showed that in the presence of model error, the optimal hybrid possessed a much smaller component of ensemble-based error. This they attributed to the climatological 3D-Var component of forecast error being a more accurate representation of model error than the ensemble-based covariances. However, the small amount (~10%) of flow-dependent covariance information retained was still sufficient to significantly reduce analysis/forecast error compared to the pure 3D-Var results. Buehner (2005) tested a hybrid approach in a quasi-operational 3D-Var setting. Impacts were rather small – the impact of model error and sampling error in a real-world situation may dominate improvements due to the ability of the hybrid to model flow-dependence. The equivalence of the original Met Office approach (Barker 1999, Lorenc 2003) to the Hamill and Snyder (2000) and Buehner (2005) hybrid has been demonstrated in Wang et al. (2007).

The representation of flow-dependent errors via additional control variables has been revisited by Wang et al. (2008a, b) using the Weather Research and Forecasting (WRF) model's "WRFDA" system (Barker et al. 2012). Major differences from the original Met Office study include a) Application in a regional model, and b) Ensemble perturbations source changing from an error breeding system to an Ensemble Transform Kalman Filter (ETKF – Bishop et al. 2001, Wang and Bishop 2003). Wang et al. (2008a) assessed the impact of the WRF-based hybrid in a very low-resolution (200km), reduced observation network (radiosondes only), non-cycling Observation System Simulation Experiment (OSSE) framework. Results indicated that the hybrid produced better forecasts than both the deterministic 3D-Var control and the ensemble mean analysis, especially in data-sparse areas. A second paper (Wang et al. 2008b) relaxed the OSSE assumptions (perfect model and known observation errors) by retesting with real radiosonde observations. The impact of the flow-dependent analysis increments was reduced, but still positive. Hamill et al. (2011) illustrate the positive impact of a similar hybrid 3D-Var algorithm versus both traditional 3D-Var as well as a 60 member EnKF on tropical cyclone track forecasts for the 2010 hurricane season.

In section 2, the specification of flow-dependent, ensemble-based error covariances in a variational assimilation system via the 'alpha control variable' method is described, including the use of existing variational data assimilation techniques to represent spatial ensemble covariance localization. In section 3, results from the initial implementation of a hybrid 4D-Var/ETKF algorithm within the Met Office's global NWP system are briefly described. Further details can be found in Clayton et al. (2012). The hybrid is only a first stage in developing a full, two-way coupling between data assimilation and an ensemble prediction system (EPS) – the current hybrid still relies on deterministic 4D-Var and a separate ensemble perturbation update mechanism (e.g. the ETKF). Plans for an extension to the hybrid concept, which attempts to address the issues of linear model scalability and maintenance, is briefly described in Section 4.

2. Background

2.1. The Alpha Control Variable (ACV) Method

The implementation of the hybrid approach in a variational framework proceeds as described in Lorenc (2003), and is briefly reviewed here. In the following, it is assumed that a set $\delta\mathbf{X}_f$ of N short-range ensemble forecast perturbation states $\delta\mathbf{x}_{fn}$ (member minus mean, $1 \le n \le N$) is available from a previous cycle of the EPS:

$$\delta\mathbf{X}_f = \left(\delta\mathbf{x}_{f1}, \delta\mathbf{x}_{f2}, ..., \delta\mathbf{x}_{fN}\right) \tag{1}$$

The standard climatological increment within 3/4D-Var is given by $\delta\mathbf{x}_{clim} = \mathbf{B}^{1/2}\mathbf{v} = \mathbf{U}\mathbf{v}$, where $\mathbf{B} = \mathbf{U}\mathbf{U}^{\mathbf{T}}$ is the standard background error covariance, modelled by spatial and variable transforms through the operator U. The vector of standard control variables **v** contains, for example, normalized, spectral modes of meteorological fields known to have relatively uncorrelated cross-covariances – see e.g. Lorenc et al. (2000). In the hybrid, flow-dependent ensemble perturbations $\delta\mathbf{x}_{fn}$ are introduced via their element-wise (Schur) products $\circ$ with three-dimensional scalar weighting fields $\boldsymbol{\alpha}_n$:

$$\delta\mathbf{x}_{flow-dep} = \sum_{n=1}^{N} \delta\mathbf{x}_{fn} \circ \alpha_n = \mathbf{U}_\alpha \mathbf{v}_\alpha \tag{2}$$

The covariances of the weighting fields $\boldsymbol{\alpha}_n$ are modelling in a similar way to their climatological counterparts through an 'alpha control variable transform' $\mathbf{U}_\alpha$, with control variables $\mathbf{v}_\alpha$ constrained by an additional term J_e in an expanded variational cost function

$$J = J_b + J_e + J_o = \frac{W_b}{2}\delta\mathbf{x}_0{}^T\mathbf{B}^{-1}\delta\mathbf{x}_0 + \frac{W_\alpha}{2}\mathbf{a}^T\mathbf{A}^{-1}\mathbf{a} + \frac{1}{2}\sum_{i=0}^{I}\left[\mathbf{H}_i\delta\mathbf{x}(t_i) - \mathbf{d}_i\right]^T \mathbf{R}_i^{-1}\left[\mathbf{H}_i\delta\mathbf{x}(t_i) - \mathbf{d}_i\right] \tag{3}$$

where J_b/J_o are the standard background/observation cost functions, $\delta\mathbf{x}_0 = \delta\mathbf{x}_{clim} + \delta\mathbf{x}_{flow-dep}$ is the total analysis increment, **B**/**R** the usual background/observation error covariance matrices, **H** the linearized observation operator acting on the increment $\delta\mathbf{x}_t = \mathbf{M}_{0\to t}\delta\mathbf{x}_0$, and $\mathbf{d}_i = \mathbf{y}_{oi} - H(\mathbf{x}_i)$ is the innovation vector difference between observation $\mathbf{y}_{oi}$ and full model state represented in observation space through the potentially nonlinear operator H. The weights W_b and W_α fix the relative contributions of climatological and flow-dependent increments, and can be related through the conservation of total error variance (Wang et al. 2008a). It is important to reiterate that with the exception of the J_e term, all other operators in Eq. (3) are part of the standard 3/4D-Var algorithm. The code changes required to implement the hybrid are therefore relatively minor, and independent of technique (3/4D-Var), and observation network.

The alpha covariance matrix $\mathbf{A} = \mathbf{U}_\alpha\mathbf{U}_\alpha^\mathbf{T}$ in Eq. (3) performs the role of covariance localization by limiting the influence of the flow-dependent covariances to within a specified distance of the observation (Wang et al. 2007). Within the variational system, it is convenient (but not essential) to make use of components of the pre-existing control variable transform **U** in the covariance localization model $\mathbf{U}_\alpha$. Two examples of this technique are given below which lead to a highly computationally-efficient three-dimensional spatial covariance localization through the use of spectral transforms and empirical orthogonal function decomposition.

2.2. Horizontal Covariance Localization Via Spectral Transform

Fig. (1a) shows example empirical covariance localization functions typical of those used within EnKF algorithms. Assuming isotropy and homogeneity, these correlation functions can be efficiently represented in spectral space by low wavenumber (~T21) power spectra (Fig. 1b).

The impact of spectral horizontal covariance localization within a 3D-Var context is demonstrated in Fig. (2) for a single temperature observation and an artificially small ensemble size of two (similar to the initial Met Office experimentation with one perturbation in Barker 1999). For clarity, full-weight is given to the ensemble component of the hybrid covariance (W_e=1, W_b=∞) in Figs. (2b, 2c, 2e, 2f). For comparison the standard 3D-Var response (W_e=∞, W_b=1) is shown in Figs. (2a, 2d). The impact of spectral covariance localization can be seen by comparing Figs. (2b, 2c) and (2e, 2f). Without localization (Figs. 2b, 2e) the multivariate increment is completely dominated by sampling noise. With localization applied (Figs. 2c, 2f), the increment response is confined to the vicinity of the observation, but still retains an element of anisotropy (the localization is isotropic, but the single ensemble perturbation $\delta\mathbf{x}_f$ is not).

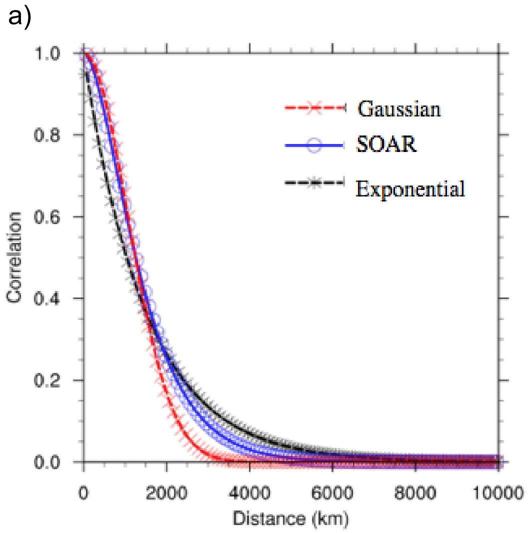

Figure 1a) Example empirical covariance localization: Gaussian, SOAR, and exponential functions with 1500km localization radius, and b) Corresponding power spectra.

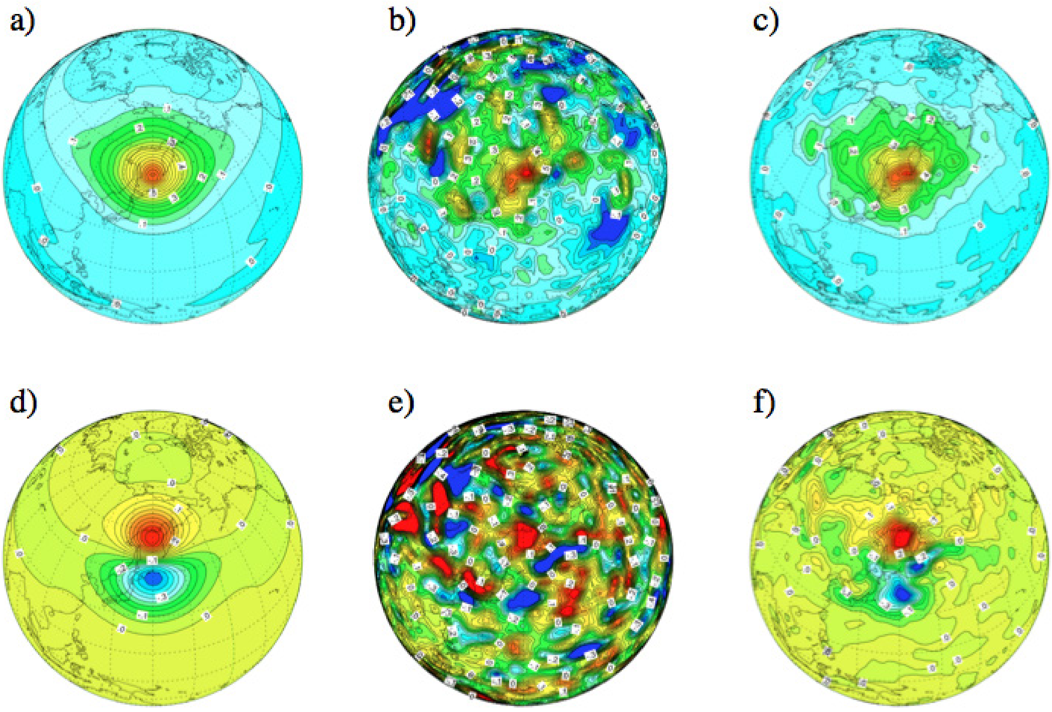

Figure 2. Temperature (above) and u-wind component (below) analysis increment response due to a single temperature observation minus background forcing of 1degK at 50N, 150E, 500hPa: standard, climatological 3D-Var response (left), raw ensemble covariance (centre), and localized ensemble covariance (right).

2.3. Vertical Covariance Localization

A second example of empirical covariance localization modelling within the variational hybrid is demonstrated through the use of empirical orthogonal function (EOF) decomposition to represent vertical covariance localization. As an example, a vertically-dependent covariance localization function is defined as

$$\rho(k,k_c) = \exp\left[-(k-k_c)^2 / L_c^2\right] \tag{4}$$

for model levels k and k_c. The width of the correlation function is made to vary by making the vertical correlation lengthscale L_c a function of k_c; e.g. $L_c = 20k_c / 41$, taken from tuning of a 41-level version of the WRFDA AFWA application of the hybrid (Barker et al. 2012). The vertical correlation functions are shown for representative levels in Fig. (3a). As in the horizontal covariance localization case above, it is possible to make use of EOF routines within the existing **U** transform to efficiently represent vertical localization. Individual and cumulative eigenvalues corresponding to the localization function (Eq. (4)) are shown in Fig. (3b) – note the 41-level grid-space correlations can be represented accurately by only 9 EOF modes, indicating a significant data compression. The combination of single, low wavenumber (T21) spectral and truncated (T9) EOF decompositions represents a very significant reduction (compare with a 300x300 grid and 41 model levels) in the number of additional control variables $\mathbf{v}_\alpha$ required for the additional term in the hybrid cost function (Eq. 3).

In this section, the basic formulism of the hybrid has been laid out, and the relative flexibility and efficiency of the particular form of covariance localization demonstrated. In the following section, the hybrid is demonstrated in a full operational NWP context at the Met Office.

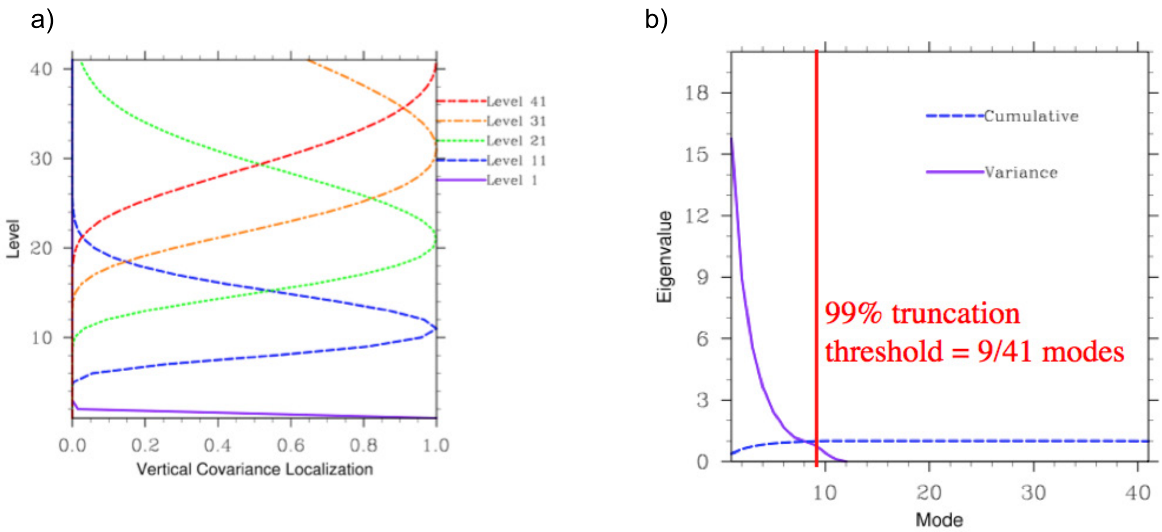

Figure 3 a) Vertical covariance localization functions as a function of selected model levels of the 41-level WRFDA application of the hybrid (Barker et al. 2012), and b) Corresponding eigenvalues calculated via standard EOF decomposition.

3. Met Office Hybrid 4D-Var/ETKF Operational Implementation

Current Met Office atmospheric data assimilation capabilities are based on an incremental four-dimensional variational data assimilation scheme operational in a global domain since 2004 and a regional North Atlantic and Europe (NAE) domain since 2006. The system is capable of assimilating a wide range of conventional and satellite-remotely sensed observations, and has contributed significantly to improvements to global and UK performance metrics in recent years (Lorenc and Rawlins 2005). More recently, the "Met Office Global and Regional Ensemble Prediction System" (MOGREPS) was implemented operationally in 2008, following several years of development. MOGREPS provides short-range (0-72hr) probabilistic estimates of forecast uncertainty for a variety of Met Office customers. An extended 15-day version (MOGREPS-15) runs at ECMWF, providing medium-range ensemble forecasts as part of the Met Office's contribution to THORPEX. Initial condition perturbations are created via an Ensemble Transform Kalman Filter (ETKF) algorithm, with additional spread being introduced through dynamically and physically based perturbations within the forecast model step itself (Bowler et al. 2008, Bowler et al. 2009).

In the Met Office system, 4D-Var and MOGREPS are coupled through the use of 4D-Var's deterministic analysis as the control to which ETKF perturbations are added to provide the ensemble of analyses for the next cycle (see Fig. 4). A world-first hybrid variational/ensemble data assimilation algorithm was implemented on 20 July 2011, introducing two-way coupling between global 4D-Var and MOGREPS. This is represented by the red 'Ensemble Covariances' connection in Fig. 4, which

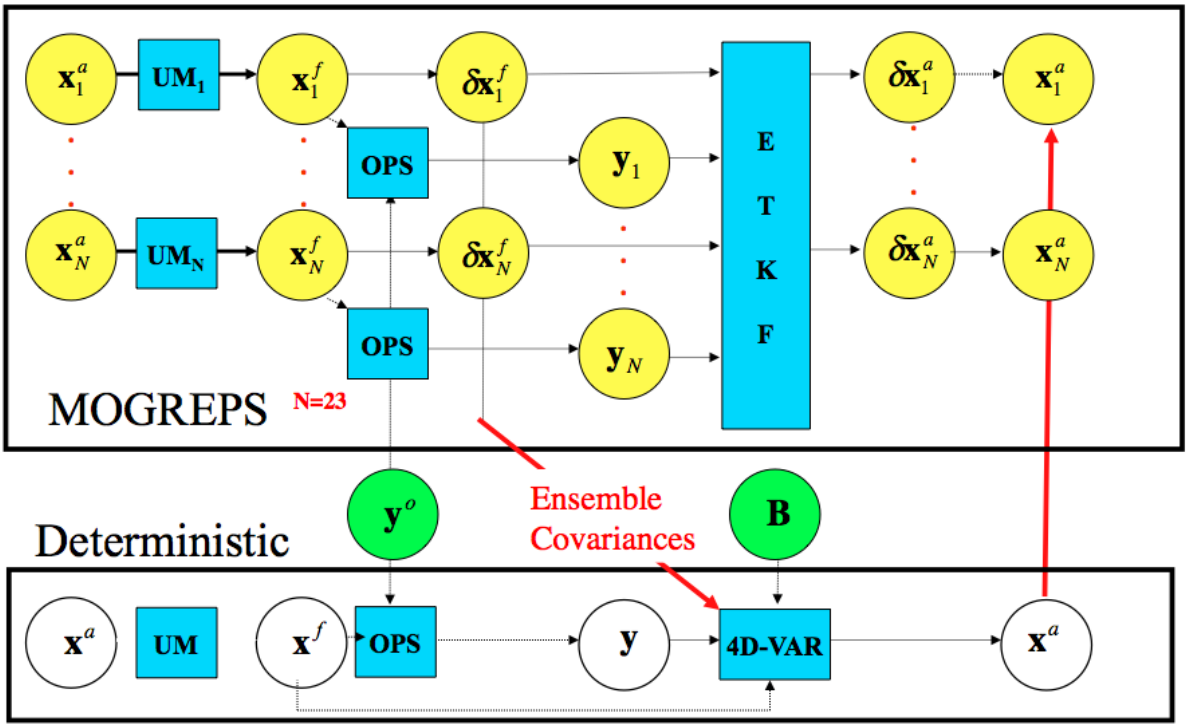

Figure 4. Sketch of the interactions between MOGREPS (upper box) and high-resolution deterministic NWP (lower box) systems. UM=Unified model, OPS=Observation Preprocessing System, ETKF=Ensemble Transform Kalman Filter. The red arrow denotes the coupling supplying ensemble perturbations as estimates of flow-dependent forecast error to the data assimilation, and 4D-Var analysis to which ETKF-updated ensemble perturbations are added for the next cycle of ensemble forecasts.

indicates the provision of flow-dependent ensemble perturbations to 4D-Var. These are incorporated into VAR using the alpha control variable technique described in section 2a above, using a Gaussian horizontal localization function, but a different vertical localization scheme to that presented in section 3c.

In high-resolution pre-operational trials, the hybrid system gave significant performance improvements relative to non-hybrid controls. Fig. 5 shows the changes to RMS error for the fields used in the Met Office's "NWP Index"; i.e., the fields considered most important to customers of global NWP products. For both trial periods, errors versus radiosonde and surface observations were consistently improved. Tropical wind errors against Met Office analyses were significantly increased, but we believe this is an artifact of the verification measure, and not a true reflection of forecast skill. When verifying against independent (ECMWF) analyses, this signal disappears, and the results become more consistent with the scores against observations. For verification against observations and ECMWF analyses, the average RMS error reduction across the two trial periods was 0.9%. Further details of the Met Office global NWP 4D-Var/ETKF hybrid implementation can be found in Clayton et al. (2012).

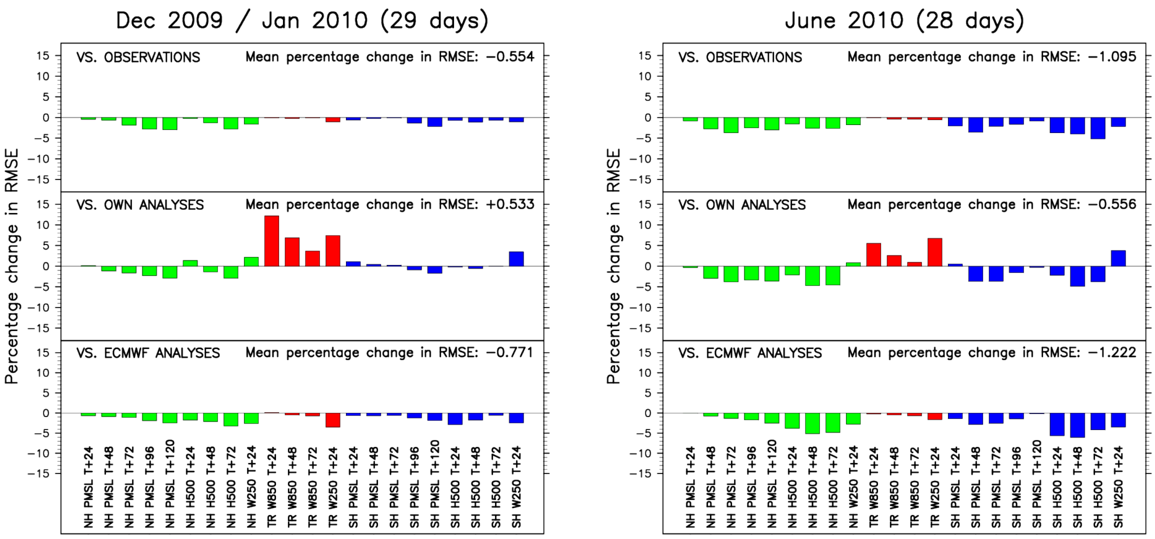

Figure 5. Changes to RMS error relative to non-hybrid controls for the fields used within the Met Office's NWP index.

4. Conclusions and Future Work

The implementation of a hybrid variational/ensemble algorithm in operational global NWP in July 2011 represents a significant milestone in the development of ensemble data assimilation capabilities at the Met Office, since efforts to develop the 'alpha control variable' approach began in 1997 (Barker 1999). However, the hybrid represents only an initial stage in the coupling between data assimilation and ensemble prediction systems. Short-term development plans for the global hybrid include increasing the ensemble size from the current (relatively small) 24 members, and more sophisticated (e.g. variable-dependent, adaptive) covariance localization (e.g. Bishop et al. 2011). In 2012, a new, 2.2km component of the ensemble (MOGREPS-UK) will be implemented, thus permitting the testing of hybrid variational/ensemble data assimilation for convective-scale UK data assimilation system in the future.

Looking further ahead, more radical changes to the basic data assimilation algorithm are envisaged. As discussed above, apart from the use of hybrid covariances, the global 4D-Var algorithm is essentially unchanged. Thus, although the hybrid permits a smooth transition to ensemble-based flow-dependent covariances, the long-term challenges for 4D-Var scalability, maintainability and flexibility remain. A review of alternative ensemble-based data assimilation algorithms was undertaken in 2010/2011 to assess potential alternatives to 4D-Var. In summary, the Met Office ensemble data assimilation strategy going forward involves a) Continuing efforts to further improve the efficiency of 4D-Var in the short/medium-term, and b) The development of a '4D-Ensemble-Var' algorithm for the medium/long-term that removes the need for the expensive linear PF-model and its adjoint completely. The new algorithm – similar to the "En4DVar" algorithm of Liu et al. (2008), and the "En-4D-Var" algorithm of Buehner et al. (2010)[1] - is a natural successor to the current hybrid, by extending the use of ensemble perturbations to model the evolution of forecast error throughout the 4D-Var time window (Fig. 6).

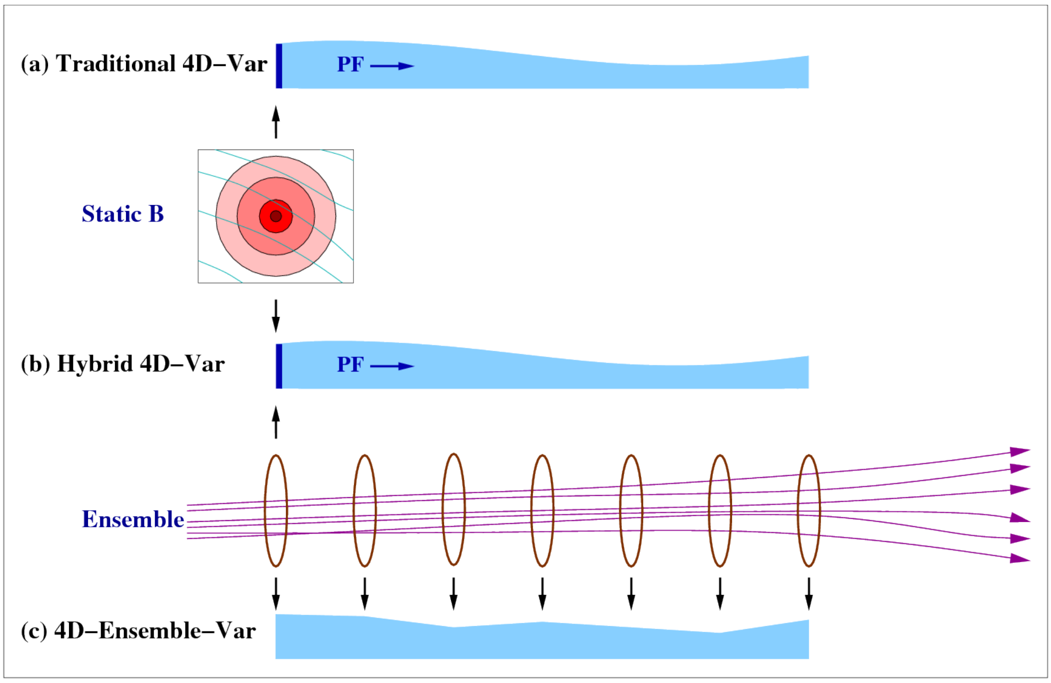

Figure 6. Schematic relationship between a) Traditional 4D-Var (making no use of the ensemble, and modelling covariance evolution via the linear PF model, using static initial covariance), b) Hybrid 4D-Var (using a combination of static and ensemble covariances at the start of the time window in combination with the PF model), and c) 4D-Ensemble-Var (using the ensemble throughout the time-window instead of the PF model).

As in all ensemble data assimilation algorithms, the bulk of the computational cost of 4D-Ensemble-Var is in the integration of the ensemble forecasts. The analysis step (assimilation) is relatively cheap - a similar number of operations to 3D-Var, although with significantly increased memory and I/O costs. The computational cost savings from removing the PF/adjoint model can be reinvested in a

[1] We prefer the name "4D-Ensemble-Var" because the key feature is the 4-dimensional use of the ensemble; it also is more consistent with the 4DEnKF terminology of Hunt et al. (2007).

larger ensemble to reduce ensemble sampling error. Results from a similar technique in Buehner et al. (2010) indicate an ensemble size of 100-200 members may be sufficient to match 4D-Var performance.

Ensemble data assimilation algorithms are typically less tied to particular models than their variational counterparts. Increased flexibility will be strategically important during the development of the next-generation dynamical cores. Reduced model/application-dependence also opens up the possibility of truly coupled data assimilation between earth system model components (i.e. cross-covariances between atmosphere, land, ocean, etc). Over the next two years, the 4D-Ensemble-Var algorithm will be developed and tested within the current VAR software framework. This permits both a clean intercomparison between alternative techniques, as well as ensuring that general developments benefit all flavours of 4D-Var under consideration within a single software system.

It should be noted that the 4D-Ensemble-Var algorithm still requires a separate mechanism to update the ensemble perturbations, separately from the data assimilation. In the current hybrid, this role is performed by the ETKF. This separation is suboptimal because the ensemble mean (data assimilation) and perturbations (ETKF) are updated using different covariance models. In the 4D-Ensemble-Var project, an 'Ensemble of 4D-Ensemble-Vars' will be developed to address this inconsistency, in a similar way to the ECMWF's strategy to develop an 'Ensemble of traditional 4D-Vars'. The 4D-Ensemble-Var approach promotes increased flexibility, relying on covariance localization techniques and larger ensemble sizes to make maximum use of the raw ensemble covariances. The ECMWF approach requires fewer ensemble members, instead relying on the continued use of sophisticated (but more core-specific) linear/adjoint/covariance models to treat sampling error, allowing the ensemble to define only a subset of flow-dependent forecast error parameters (e.g. variances, lengthscales, etc).

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