

# Impact and diagnosis of model error in the ECMWF Ensemble of Data Assimilations

Massimo Bonavita

*ECMWF, Reading, UK  
Massimo.Bonavita@ecmwf.int*

## ABSTRACT

The ECMWF Ensemble of Data Assimilations (EDA) is a Monte Carlo system which aims to estimate the analysis and background errors of the deterministic ECMWF 4D-Var assimilation system. The underlying idea is to represent, in a stochastic way, all relevant sources of uncertainty present in the deterministic analysis cycle and derive from the ensemble sample statistics estimates of the errors of the operational assimilation system. An important source of uncertainty is due to the fact that the forecast model is not perfect. In the EDA this is accounted for through the use of a model error parameterization scheme.

Two different model error parameterizations have been developed at ECMWF and their impact on the EDA sample error statistics is evaluated by comparing the estimated error distribution with the perceived error distribution. Results show that both model error parameterizations have a positive, synergistic effect on the ability of the EDA to represent the *variance* of the background error. On the other hand, a preliminary analysis of the effect of model error parameterizations on the EDA estimates of background error *covariances* has given mixed results, pointing to the need for closer inspection of the issue.

## 1. Introduction

The ECMWF Ensemble of Data Assimilations (EDA) is a system of  $N$  ( $N=10$  in June 2011) independent, reduced resolution, assimilation cycles which differ by perturbing observations, sea-surface temperature fields and model physics. A theoretical analysis shows that if the perturbations are drawn from the true distributions of observation and model error, then the spread of the EDA about the control (unperturbed) analysis will be representative of the analysis error (Isaksen *et al.*, 2010). This is the first system implemented at ECMWF which provides estimates of analysis uncertainty, taking observational information and non-linear model dynamics into account. The use of EDA perturbations has already proved to have a beneficial impact on the representation of initial uncertainties in the ECMWF Ensemble Prediction System (EPS) and on the estimation of flow-dependent background errors in the deterministic 4D-Var assimilation system (Isaksen *et al.*, 2010).

The ability of the EDA to correctly capture the analysis and background errors of the reference analysis relies on an accurate and, as far as possible, complete representation of all the relevant sources of uncertainty in the deterministic analysis cycle: observation error, errors in the boundary conditions and model error. This is done with physically plausible Monte Carlo realizations of the error sources, known as stochastic parameterizations, in a similar way as to what is done in the context of ensemble prediction; not through a specified covariance matrix (as is usual in Kalman Filter theory and Weak-constraint 4D-Var; see Tremolet, 2007; Tremolet, 2011, in this proceedings). Because the EDA is only used for error estimation a stochastic parameterization of model error is

sufficient. Proper characterization of the model error covariance matrix in 4D-Var is a very difficult and yet unsolved problem.

Stochastic parameterizations have been used in the ECMWF EPS system for many years (Buizza et al., 1999; Palmer *et al.*, 2009) showing a positive impact in terms of the forecast ensemble reliability and probabilistic skill scores. However to design a stochastic parameterization that improves the EDA sample error statistics by representing the true sources of model error is arguably a more stringent test. In an assimilation system the effects of using a certain model error representation accumulate in time due to the cycling of information in the analysis, so any shortcomings in the formulation become more apparent. EDA also provides a means to confront the model error scheme with observational reality, both directly and through the EDA sampled background error statistics which are then used in the deterministic analysis. A further distinction is that in an EPS context one is typically concerned with the verification and use of univariate probability distributions (pdf) at a given lead time and location (i.e., what is the chance of rain tomorrow in Reading?). On the other hand, one of the main uses of an EDA is to diagnose spatial and multivariate *covariances*, both to have a climatological estimate of background errors (Fisher, 2003; Belo Pereira and Berre, 2006) and, more recently, to derive flow-dependent estimates (Varella *et al.*, 2011). Finally, background errors (i.e., forecast errors at typically 6-12 hours lead times) have been shown (Wei and Toth, 2003) to span a much larger portion of the error space than errors at longer forecast lead times, since they have not condensed yet on the dominant modes of instability of the system. This obviously makes their estimation a more challenging problem.

This paper presents the results of an initial investigation of the impact of model error parameterizations in the context of the ECMWF EDA system. In section 2 we briefly describe the model error schemes developed at ECMWF and the setup of the EDA experiments designed to test their impact. In section 3 we describe the impact of model error parameterizations on the climatology of EDA variances. In section 4 we present diagnostics in both observation and model space of the model error impact on EDA variances. In section 5 we show preliminary results of the impact of model error parameterizations on EDA covariances. Conclusions and suggestions for further research are offered in section 6.

## 2. Experimental setup

The first model error parameterization developed and implemented at ECMWF (Buizza et al., 1999), known as ‘stochastic physics’, consisted in the application of spatially piecewise homogeneous random perturbations to the total forecast tendencies of the physical parameterized processes. This technique has been revised significantly in September 2009 (details in M. Leutbecher’s contribution to these proceedings). We will refer to this as Stochastically Perturbed Parameterization Tendencies (SPPT).

While the SPPT technique aims at representing model uncertainty arising from un-/mis-represented physical parameterizations processes, it has been recognized that the dynamical part of the forecast model can also be the source of non negligible model error. In particular, energy dissipation due to unresolved scales, numerical dissipation in the semi-Lagrangian semi-implicit integration scheme,

implicit energy dissipation in the mountain wave drag and convection parameterization schemes, are deemed to be significant sources of energy dissipation which need to be represented. The a Stochastic Kinetic Energy Backscatter (SKEB) scheme (also known as SPBS; see Shutts, 2005 and Berner et al, 2008) aims to simulate these types of errors by introducing random vorticity perturbations on the resolved scales of the flow, whose magnitude is proportional to the estimated total rate of energy dissipation.

From this short description it should be apparent that the two model error parameterizations to some extent target different aspects of the model uncertainties and should thus have partially complementary effects.

To investigate the impact of the different model error schemes, three 10 member EDA experiments have been run at T399L91 resolution (this is the same resolution at which the EDA system is run operationally at ECMWF). A baseline experiment (fgk7) has been run without any model error parameterization; a second experiment (fi8s) has only the SPPT scheme activated (this is the current EDA operational configuration); the third experiment (fgk6) has both the SPPT and SKEB schemes active. All experiments are using the Integrated Forecasting System cycle 36R4. The experiments have been run for the month of April 2010 and results presented here have been computed on a 21 day period (20100405-20100425).

### 3. Average impact on EDA variances

The first aspect to consider is the time-averaged effect the different model error representations have on the EDA sample statistics. The globally averaged vertical profiles of temperature and vorticity spread of the EDA short range (t+12h) forecasts (Fig.1) show that both model error parameterizations act to increase the EDA background standard deviation. The incremental effect of the SKEB parameterization is small in terms of temperature spread but comparable to the SPPT impact for the wind field. Using both parameterizations results in an increment of around 25% of the temperature spread and of around 40% of the vorticity spread with respect to the baseline configuration.

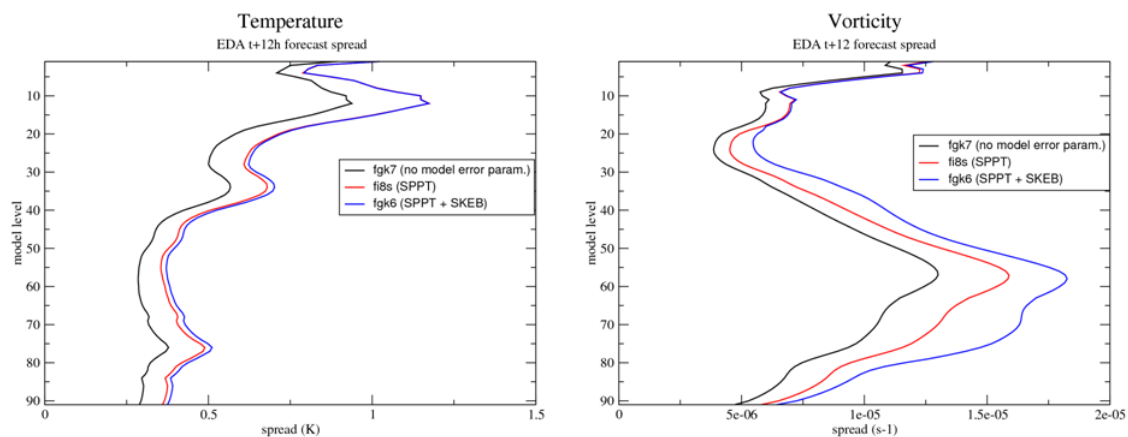


Figure 1: Average vertical profiles of the EDA short range (t+12h) temperature (left plot) and vorticity (right plot) forecast spread for an EDA without model error parameterization ( fgk7, black lines); an EDA experiment with the SPPT parameterizations (fi8s, red lines); an EDA experiment with both the SPPT and SKEB parameterizations active (fgk6, blue line). Mean values over a 21 day period (20100405-20100425).

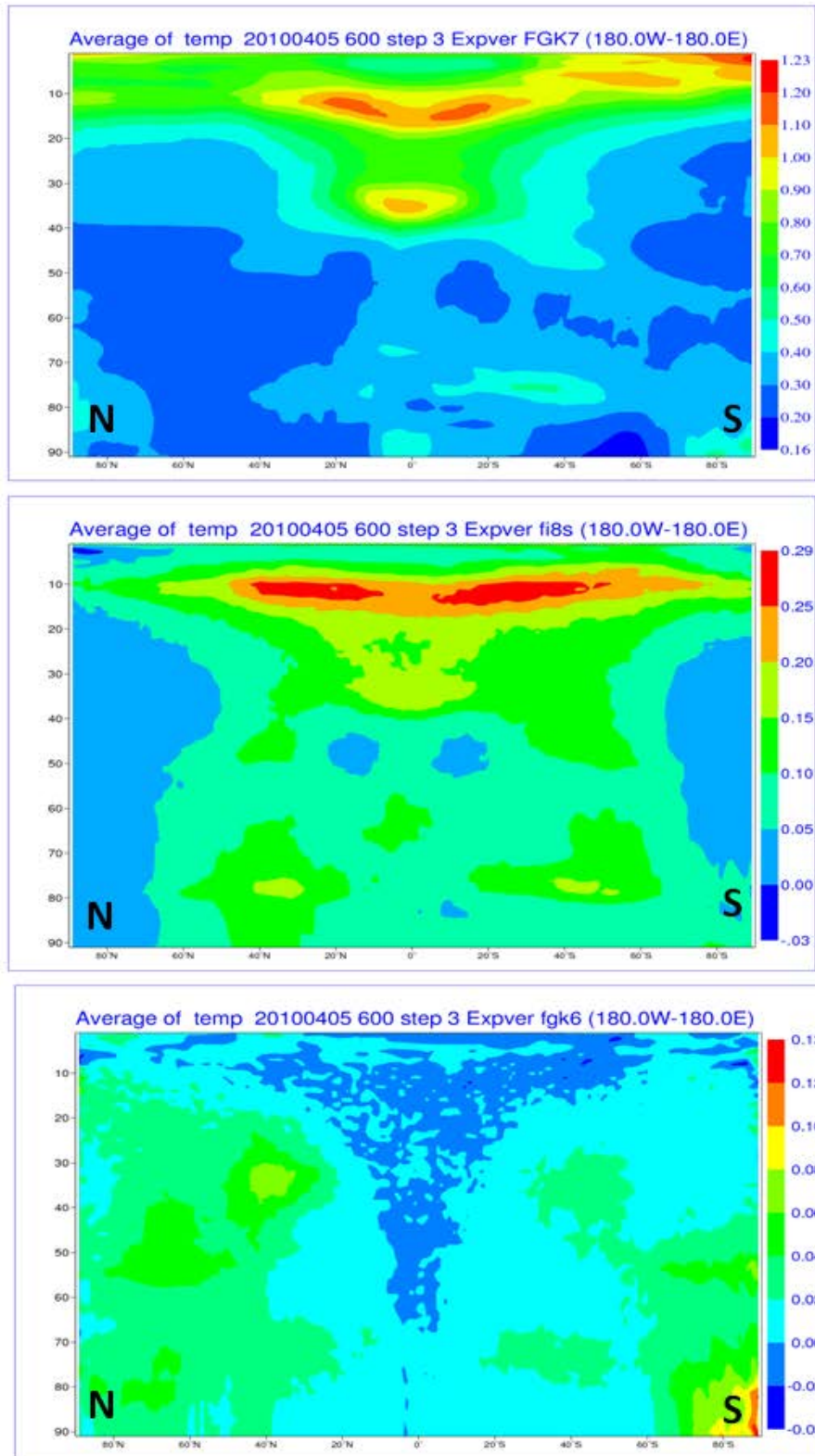


Figure 2: Longitudinal average of the EDA short range ( $t+12h$ ) temperature forecast spread for an EDA without model error parameterization (first line); increase in temperature spread due to the use of SPPT (second line); further increase in temperature spread due to the use of SKEB (third line). Mean values over a 21 day period (20100405-20100425). Latitudes on the x axis from 90N (left) to 90S (right). Model levels on the y axis from 1 (top) to 91(bottom). Unit: K.

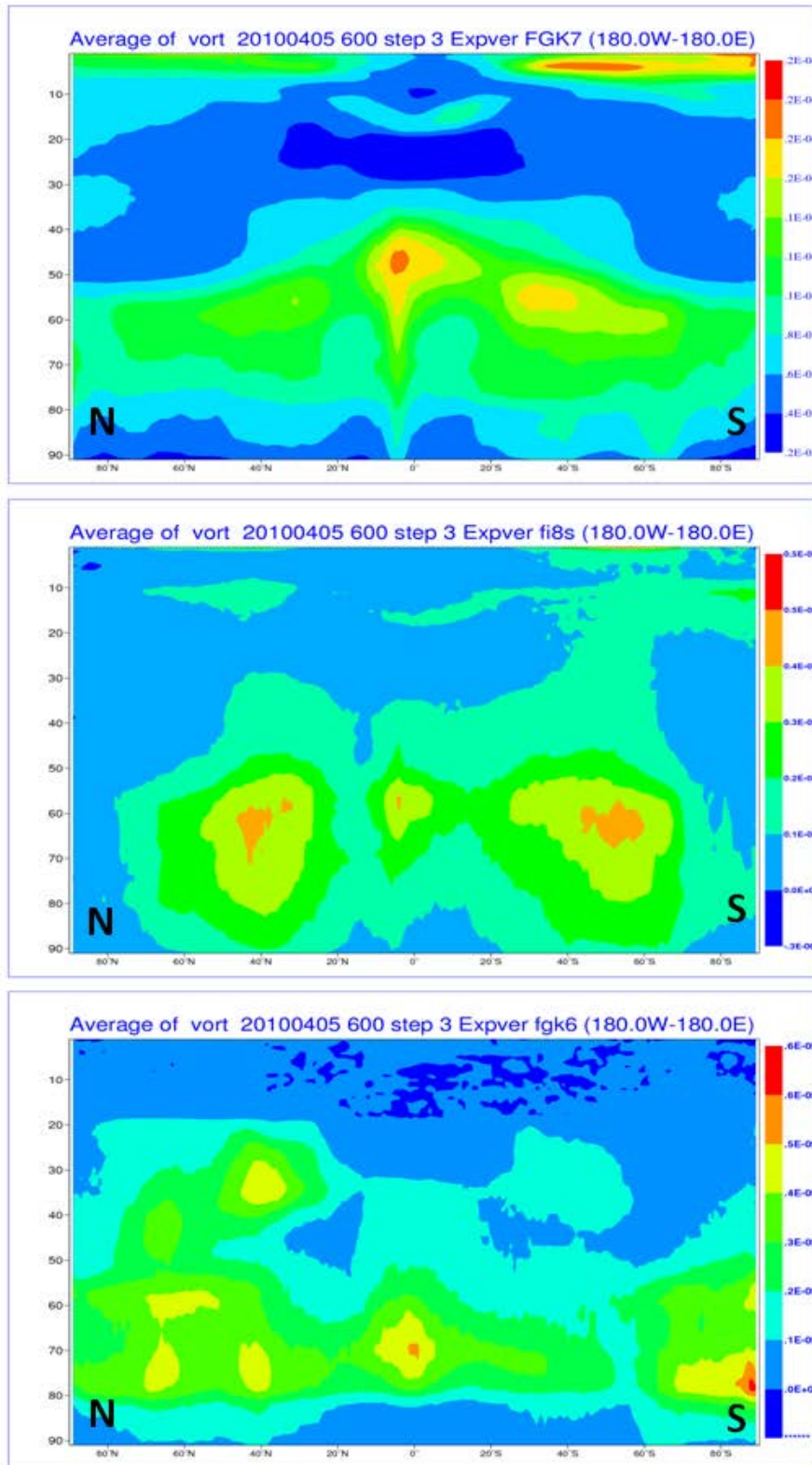


Figure 3: Longitudinal average of the EDA short range ( $t+12h$ ) vorticity forecast spread for an EDA without model error parameterization (first line); increase in vorticity spread due to the use of SPPT (second line); further increase in vorticity spread due to the use of SKEB (third line). Mean values over a 21 day period (20100405-20100425). Latitudes on the x axis from 90N (left) to 90S (right). Model levels on the y axis from 1 (top) to 91(bottom). Unit:  $s^{-1}$ .

In Fig. 2 meridional cross sections of the longitudinally and time averaged spread of the EDA temperature  $t+12h$  forecasts are shown. The first line refers to the EDA experiment where no model error parameterization is used (fgk7). The second line shows the additional spread which is introduced in the EDA by the use of the SPPT scheme. The third line shows the additional spread which is introduced in the EDA by the use of the SKEB scheme. Fig.3 presents the same plots for the vorticity fields.

The main effect of the SPPT parameterization in the lower stratosphere and in the troposphere is visible in the extra tropics between model levels 40 (~100 hPa) and 50 (~210 hPa) and in the boundary layer, below model level 75 (~800 hPa); in the tropics in the middle troposphere (in the 300-700 hPa layer). The position of these relative maxima suggests that the main sources of model uncertainties that the SPPT scheme captures are those related to the convective parameterization (particularly in the tropics), the gravity wave drag and the radiative parameterizations. This is confirmed by a geographical map of the additional temperature spread induced by the SPPT at model level 49 (~200 hPa; Fig.4, first line) and at model level 78 (~850 hPa; Fig.4, second line). The increase in EDA spread is particularly apparent in the cyclonic tracks of both hemispheres, downwind of significant mountain ranges, in the ITCZ, and in the oceanic regions with extensive stratocumulus decks. The effect of the SKEB scheme is, on the other hand, smaller by a factor of three for the EDA temperature spread, consistent with the fact that SKEB acts on the temperature field only indirectly (through the mass-wind balance and the cycling). It is however comparable in magnitude to that of the SPPT scheme for the wind field (Fig.3, second and third line) but with a different distribution in height and latitude: there are relative maxima in the extra-tropics between model levels 30-40 (~50-100 hPa layer) and in both the tropics and in the extra-tropics between model levels 70-80 (~700-900 hPa). A geographical map of the additional vorticity spread induced by the SKEB at model level 30 (~50 hPa; Fig.5, first line) and at model level 78 (~850 hPa; Fig.5, second line) confirms that the scheme mainly captures model uncertainty linked to the gravity wave drag parameterizations and, to a lesser extent, convection. This is not surprising, considering the fact that in synoptic situations of intense flow impinging on perpendicular mountain ranges, the background forecasts tend to show considerable errors (Fig. 6 shows an example).

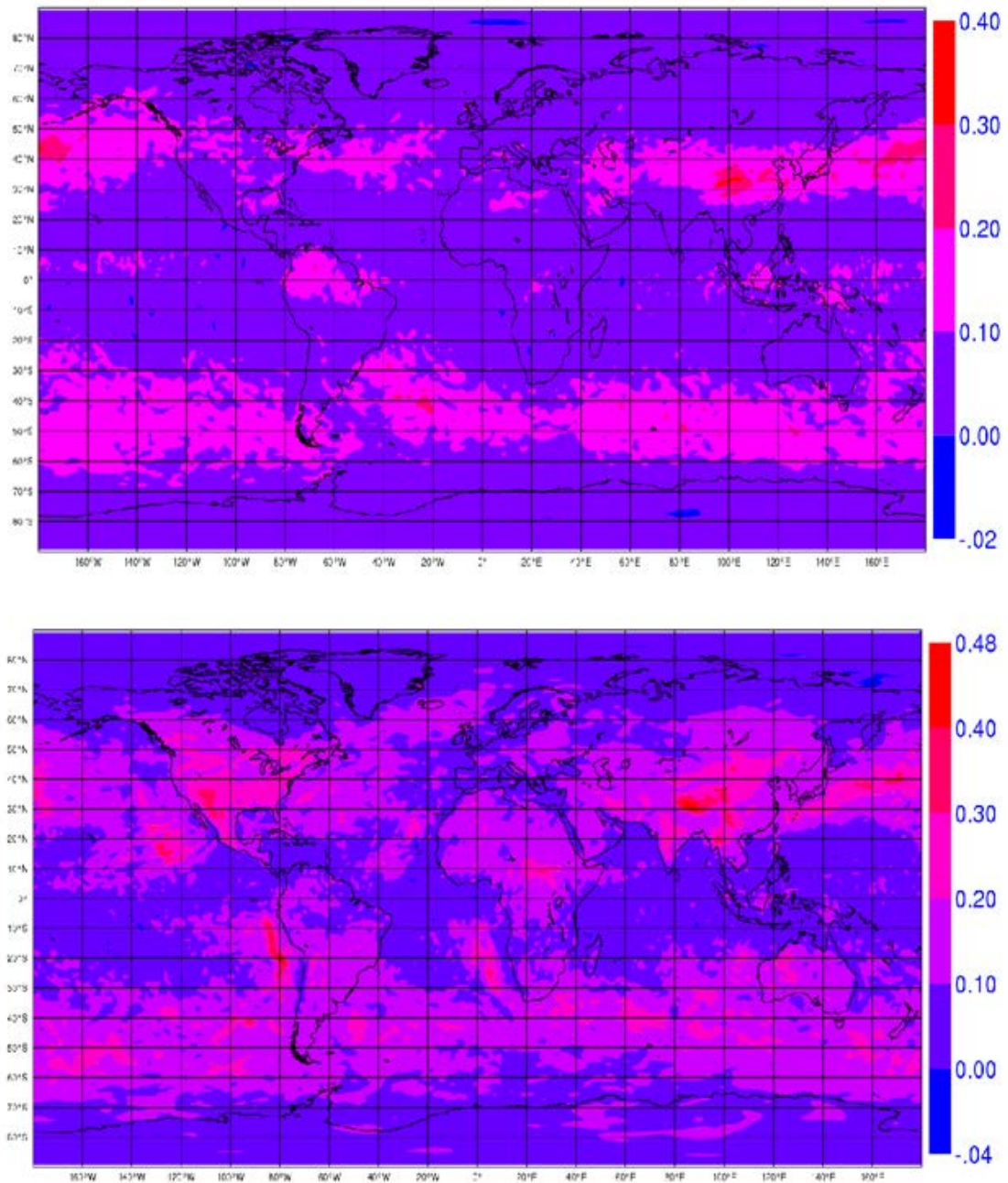


Figure 4: Map of the time-averaged (21 day period, 20100405-20100425) increase of EDA short range ( $t+12h$ ) temperature forecast spread due to the use of the SPPT scheme: model level 49 (~200 hPa) on the first line, model level 78 (~850 hPa) on the second line. Unit: K.

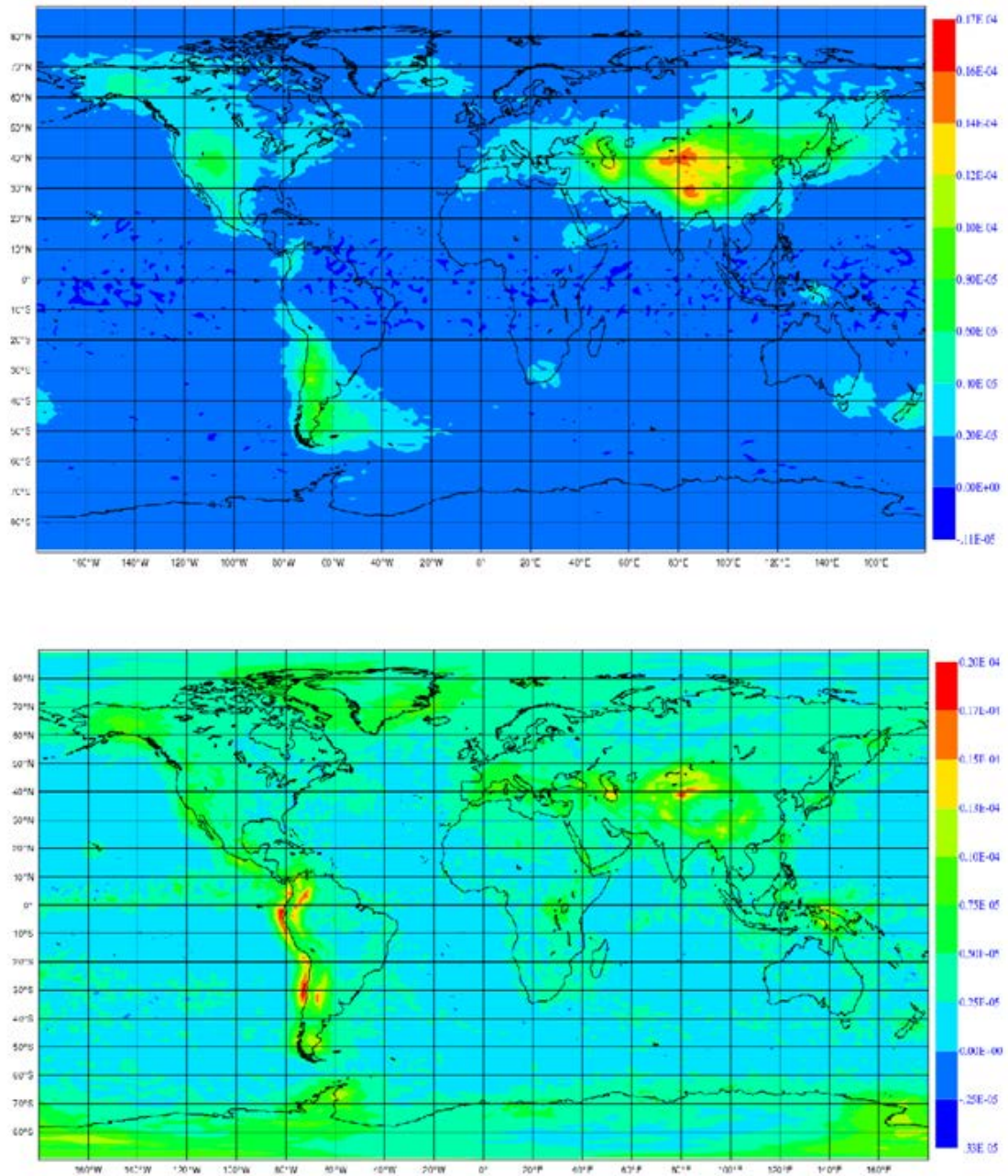


Figure 5: Map of the time-averaged (21 day period, 20100405-20100425) increase of EDA short range ( $t+12h$ ) vorticity forecast spread due to the use of the SKEB scheme: model level 30 ( $\sim 50$  hPa) on the first line, model level 78 ( $\sim 850$  hPa) on the second line. Unit:  $s^{-1}$ .



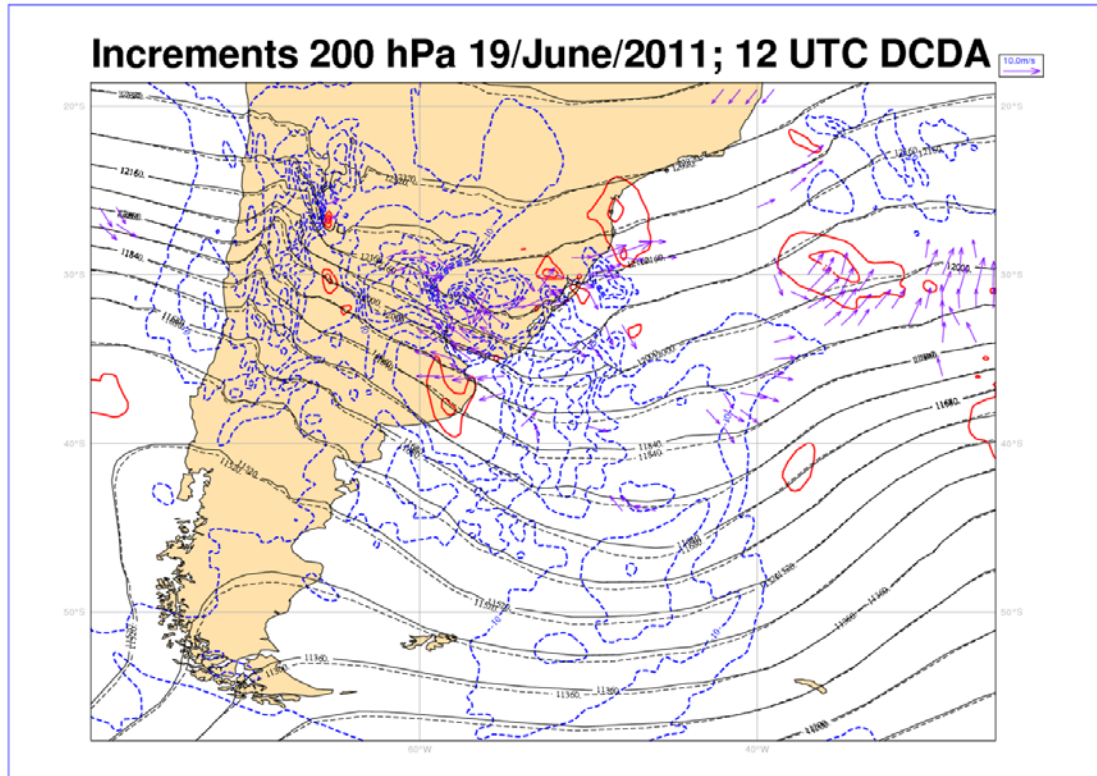


Figure 6: Map of the 200 hPa geopotential height background (solid black lines), analysis (dashed black lines), and analysis increments (dashed blue lines are negative increments, solid red lines positive) on the 19 June 2011, 12 UTC. Purple arrows show 200 hPa wind analysis increments.

#### 4. Impact on the statistical distribution of EDA variances

EDA background spread is used in the ECMWF deterministic analysis as a proxy for background error variances. It is then important to evaluate how the use of model error parameterizations affects the statistical distribution of EDA spread.

At a very basic level the EDA background spread should satisfy the following equality (Mitchell and Houtekamer, 2000):

$$\langle \mathbf{d}, \mathbf{d}^T \rangle = \mathbf{H} \mathbf{B} \mathbf{H}^T + \mathbf{R} \quad (1)$$

where  $\mathbf{d} \equiv \mathbf{y} - \mathbf{H} \bar{\mathbf{x}}^b$  is the innovation vector computed with respect to the ensemble mean background ( $\mathbf{y}$  is the observation vector,  $\bar{\mathbf{x}}^b$  the ensemble mean observation prior). As a special case of this relationship, consider only the EDA variances:

$$diag(\langle \mathbf{d}, \mathbf{d}^T \rangle) = diag(\mathbf{H} \mathbf{B} \mathbf{H}^T) + diag(\mathbf{R}) \quad (2)$$

Equation (2) states that for a statistically consistent EDA the observed innovation variance should equal the predicted innovation variance, i.e., the sum of the EDA background forecast spread in observation space and the observation error variance. Figures 7 and 8 show the two sides of equation (2) for radiosonde temperature and zonal wind observations, respectively, for the three EDA experiments under examination. Overall, it is apparent that the use of model error parameterizations tends to produce a more reliable EDA background, though not enough to have a well calibrated ensemble. Consistent with the results shown in Section 2, the increase in temperature spread is mainly due to the SPPT scheme, while the SKEB scheme primarily increases the EDA wind spread especially in the extra-tropics.

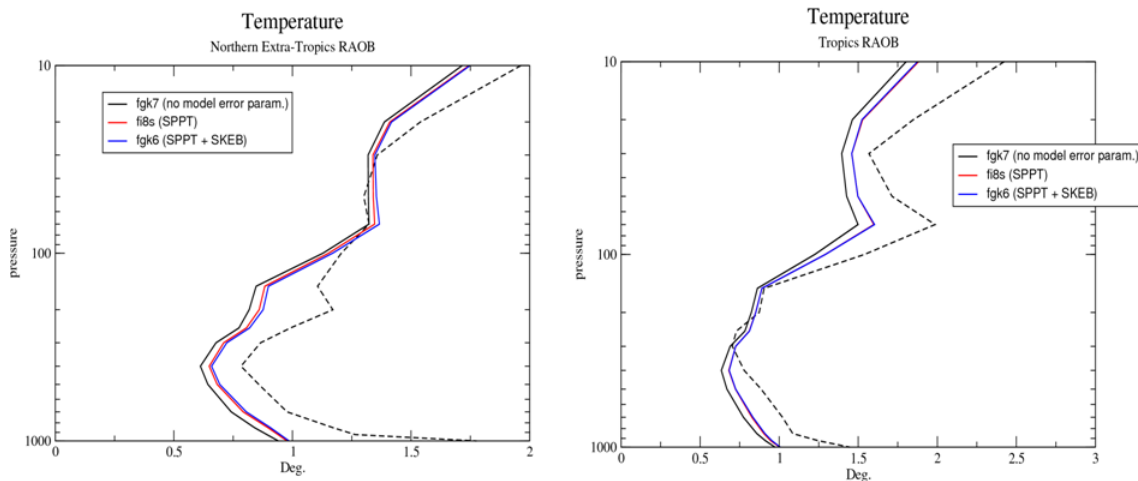


Figure 7: Time averaged (21 day period, 20100405-20100425) innovation standard deviation for radiosonde temperature observations in the Northern Hemisphere (left panel) and in the Tropics (right panel). Dashed line is the observed innovation standard deviation, continuous lines are predicted innovation standard deviations (see main text for explanation) for EDA experiments with no model error param. (fgk7, black line), with SPPT param. (fi8s, red line) and with both SPPT and SKEB param. (fgk6, red line).

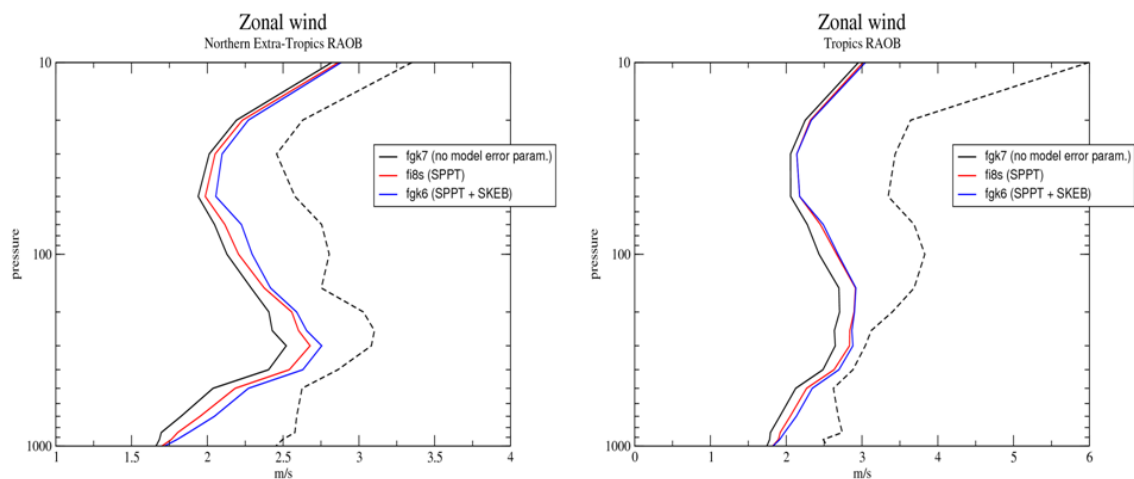


Figure 8: Time averaged (21 day period, 20100405-20100425) innovation standard deviation for radiosonde zonal wind observations in Northern Hemisphere (left panel) and in the Tropics (right panel). Dashed line is observed innovation standard deviation, continuous lines are predicted innovation standard deviations (see main text for explanation) for EDA experiments with no model error param.(black line, fgk7), SPPT param.(red line, fi8s) and SPPT + SKEB param.(blue line, fgk6).

Since the EDA spread is used as a proxy for background errors, we are interested in comparing the statistical distributions of EDA variances and background mean squared errors. A linear relationship close to the  $y=x$  line between EDA variances and background mean squared errors would show that the EDA has statistical consistency (Anderson, 1996; Talagrand et al., 1997), i.e. the EDA spread is a statistically reliable predictor of forecast *uncertainty* (Kolczynski *et al.*, 2011). To verify this hypothesis we follow Wang and Bishop, 2003, and bin Eq. (2) according to the EDA background variances. Results for radiosonde observations are presented in Fig. 9-11. It is again clear that the use of model error parameterizations improves the statistical reliability of the EDA spread as a background error variance predictor and that the additional value of the SKEB scheme is mostly in the characterization of the wind errors.

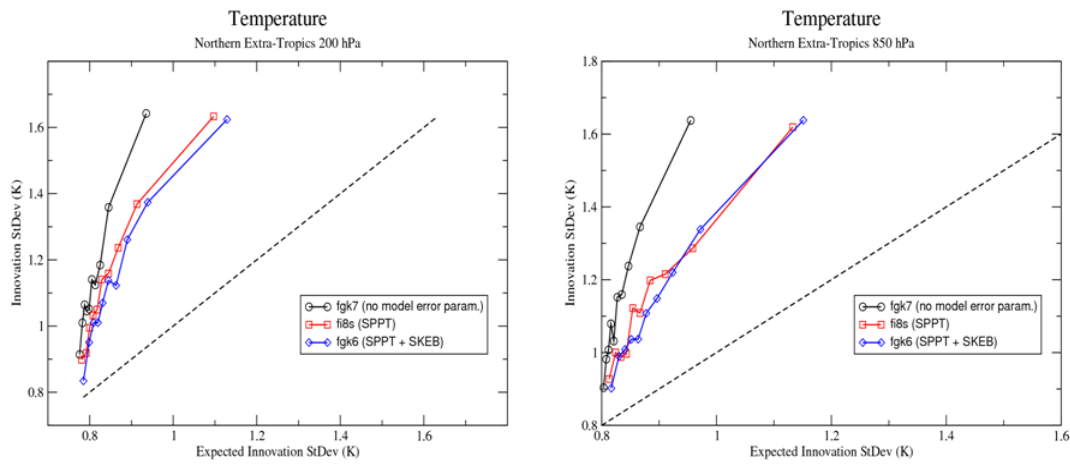


Figure 9: Time averaged (21 day period, 20100405-20100425) expected vs. observed innovation standard deviations as a function of expected innovation standard deviation (see main text for details) for EDA experiments with no model error param. (fgk7, black line), SPPT param. (fi8s, red line) and SPPT + SKEB param. (fgk6, blue line). Dashed line is  $y=x$  line. Results for radiosonde temperature observations in the North Hemisphere at 200 hPa (left panel) and 850 hPa (right panel).

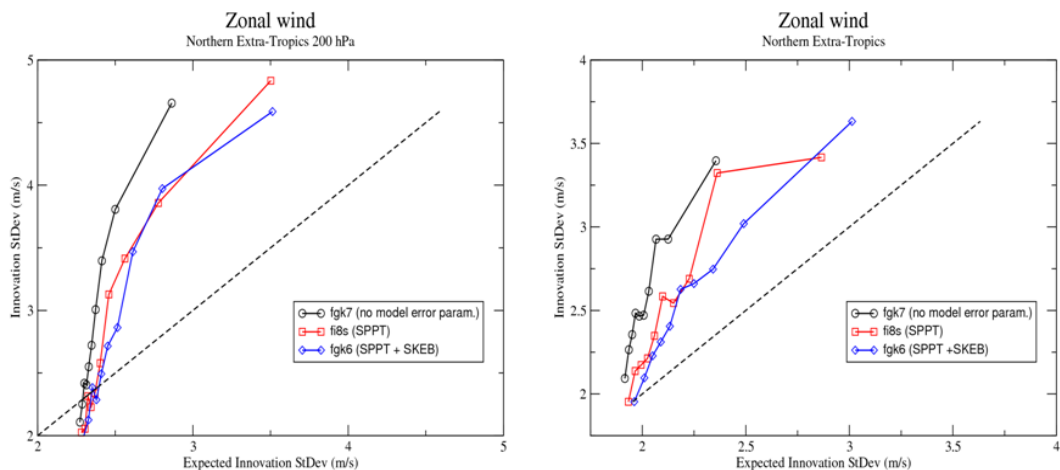


Figure 10: Time averaged (21 day period, 20100405-20100425) expected vs. observed innovation standard deviations as a function of expected innovation standard deviation (see main text for details) for EDA experiments fgk7 (no model error param.; black line), fi8s (SPPT param.; red line) and fgk6 (SPPT + SKEB param.; blue line). Dashed line is  $y=x$  line. Results for radiosonde zonal wind observations in the North Hemisphere at 200 hPa (left panel) and 850 hPa (right panel).

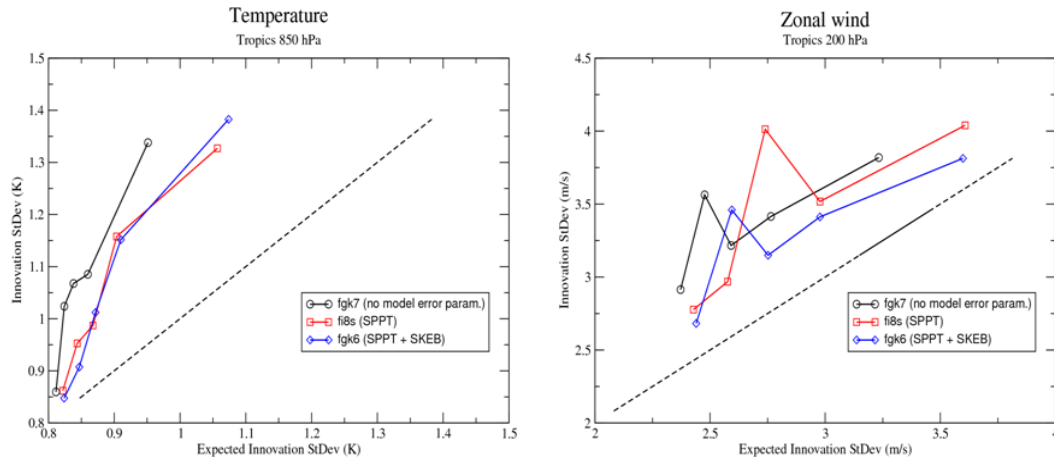


Figure 11: Time averaged (21 day period, 20100405-20100425) expected vs. observed innovation standard deviations as a function of expected innovation standard deviation (see main text for details) for EDA experiments fgk7 (no model error param.; black line), fi8s (SPPT param., red line) and fgk6 (SPPT + SKEB param.; blue line). Dashed line is  $y=x$  line. Results for radiosonde temperature observations in the Tropics at 850 hPa (left panel) and for radiosonde zonal wind observations in the Tropics at 200 hPa (right panel).

The strength of the linear relationship between the EDA spread and the background error is commonly measured in terms of the Pearson correlation coefficient. As many authors have noted (Houtekamer, 1993; Whitaker and Loughé, 1998; Mass and Grimit, 2007) this correlation can not be very large because the spread of the EDA distribution does not completely determine a single realization of the forecast error pdf. However what is of interest here is evaluating the relative strength of the linear spread-error model in the presence of the model error parameterizations. In Fig. 12 - 13 we present the temperature and vorticity spread-error correlation values as a function of model level for the three EDA experiments, both in the northern hemisphere extra-tropics and in the tropics.

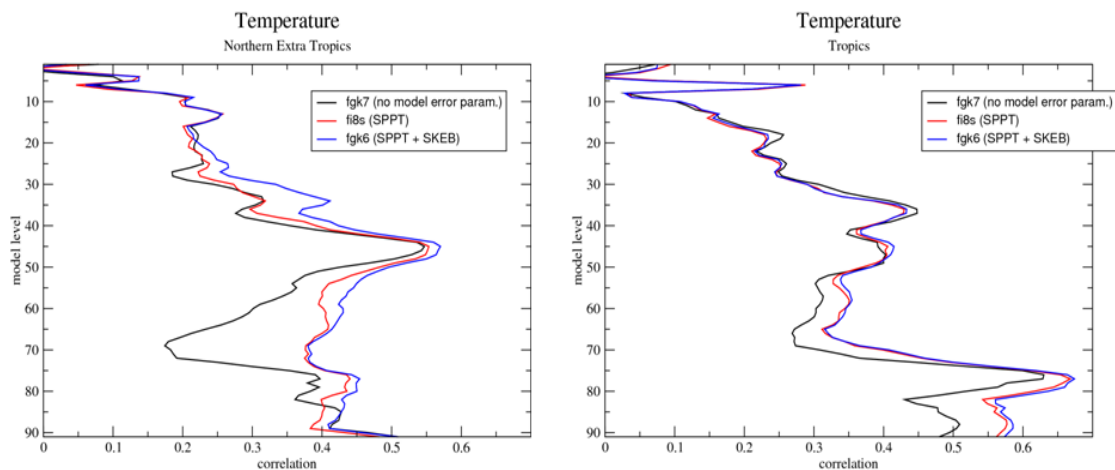


Figure 12: Time averaged (21 day period, 20100405-20100425) Pearson correlation coefficient of the EDA background standard deviation with the EDA background mean root mean squared error (computed with respect to the operational analysis) for EDA experiments with no model error param. (fgk7, black line), (SPPT param. (fi8s, red line) and SPPT + SKEB param. (fgk6, blue line). Results as a function of model level are shown for the temperature field in the northern extra-tropics (left panel) and in the tropics (right panel).

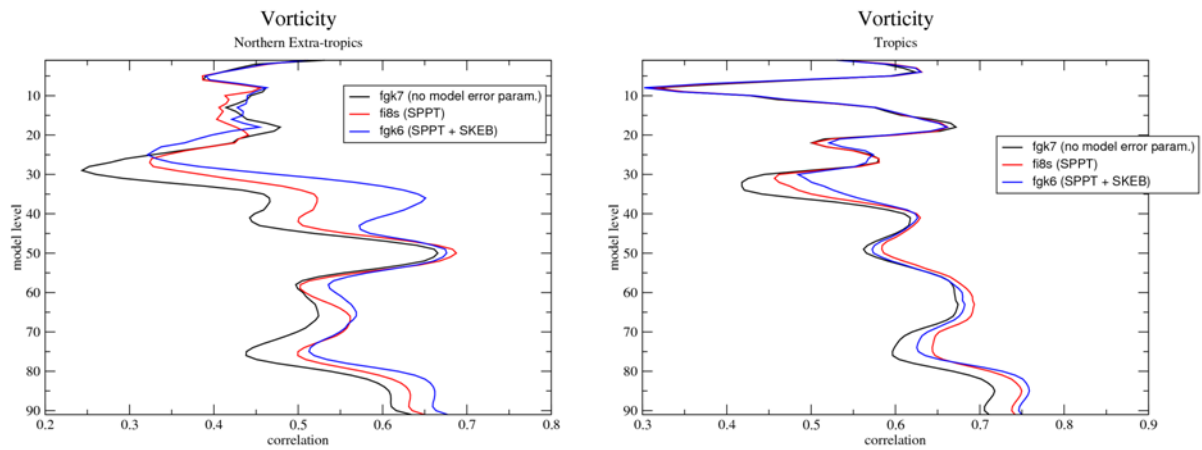


Figure 13: Time averaged (21 day period, 20100405-20100425) Pearson correlation of the EDA background standard deviation with the EDA background mean root mean squared error (computed with respect to the operational analysis) for EDA experiments with no model error param. (fgk7, black line), (SPPT param. (fi8s, red line) and SPPT + SKEB param. (fgk6, red line). Results as a function of model level are shown for the vorticity field in the northernextra-tropics (left panel) and in the tropics (right panel) .

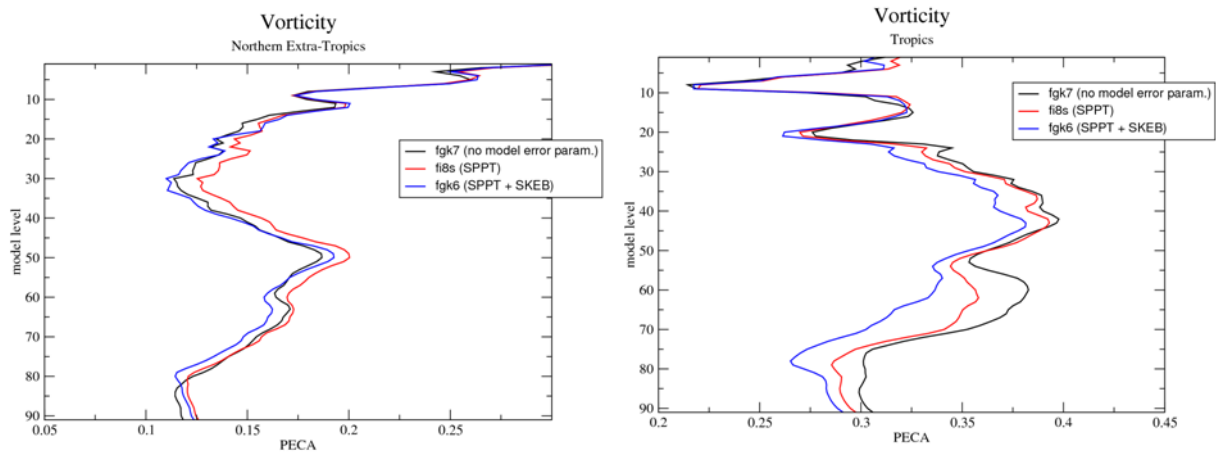


Figure 14: Time averaged (21 day period, 20100405-20100425) PECA values (computed with respect to the operational analysis) for EDA experiments with no model error param. (fgk7, black line), (SPPT param. (fi8s, red line) and SPPT + SKEB param. (fgk6, red line). Results as a function of model level are shown for the vorticity field in the northern extra-tropics (left panel) and in the tropics (right panel) .

Consistent with the results shown above, model error parameterizations increase the spread-error correlation, with the SPPT accounting for most of the increase in correlation for the temperature field while the SKEB scheme provides additional benefit in the description of the extra-tropical winds. Particularly noticeable is the increase in correlation due to SKEB for the extra-tropical vorticity between model levels 30 to 45 (approximately corresponding to the 50-150 hPa pressure layer), which has been shown to be related to the representation of model uncertainty due to the gravity wave drag parameterization.

## 5. Impact on the statistical distribution of EDA covariances

EDA perturbations are also used to diagnose the covariance structures of background errors.

At ECMWF this is currently done offline from a large sample to estimate a climatological background error matrix that is used in the 4D-Var analysis (Fisher, 2003). In the near future it is envisaged to be done on a daily basis to compute a flow-dependent component of the B matrix.

A proper analysis of the impact of the model error schemes on EDA sampled covariances would obviously require a comparison of the EDA sampled covariance with the forecast error covariance. This is not available yet.

However an indication of the effect of model error parameterizations on EDA covariances can also be inferred from the ‘‘Perturbation versus Error Correlation Analysis’’ (PECA, Wei and Toth, 2003).

This diagnostic, originally developed in the ensemble forecasting context, is defined as:

$$PECA \equiv \frac{1}{N_{ens}} \sum_{j=1}^{N_{ens}} cor(\varepsilon_b, pert_j) \quad (3)$$

i.e., as the average correlation between  $\varepsilon_b$ , which is defined as the difference between the EDA control background forecast and the verifying analysis, and  $pert_j$ , which are the EDA member’s background perturbations with respect to the control. PECA values are a measure of how much the EDA perturbations project onto the perceived background error spatial patterns and thus give an indication about the skill of the sampled covariances computed from those perturbations.

PECA values for the vorticity field as a function of model level are presented in Fig. 14, whose left panel shows results for the northern extra-tropics while the right panel refers to the tropics. The effect of the SPPT scheme appears to be neutral to positive in the northern hemisphere and negative in the tropical lower troposphere. The effect of the SKEB scheme, on the other hand, appears to be consistently negative when measured with the PECA diagnostic.

It is difficult at this stage to assess the significance of this result. But it may be the case that the spatially correlated error structures introduced by the model error schemes do not represent background error covariances well. Further investigations are required on this issue.

## 6. Summary and outlook

The EDA system has recently become an essential component of both the ECMWF deterministic 4D-Var and Ensemble Prediction System, due to its capacity to provide realistic flow-dependent estimates of analysis and background errors. Its success relies on the correct specification of the sources of uncertainty input into the assimilation cycle, among which model uncertainty plays a central role. Thus the use of the EDA sampled statistics to simulate the background errors of the reference 4D-Var provides a stringent, observation based, test for the model error parameterization schemes that originally were developed for the EPS.

The two model error schemes developed at ECMWF, SPPT and SKEB, aim to simulate different aspects of model uncertainty: random errors linked to physical parameterizations (SPPT scheme) and errors arising from energy dissipation in the dynamics and physical parameterizations of the model (SKEB). This is visible in the different impacts they have on the climatology of the EDA sample variances.

The use of both model error parameterizations has been shown to improve the statistical consistency of the EDA variances, verified against both observations (radiosondes) and reference analysis fields. Their use has also been shown to be beneficial in terms of increasing the spatial correlation of the EDA spread with the ‘perceived’ background error standard deviation. This means that the EDA spread is a more skilful predictor of background error variance when the model error parameterizations are active.

On the other hand the impact of model error parameterizations (especially SKEB) on the EDA ability to represent background error covariances is mixed in terms of the PECA diagnostic and needs further investigation. This will be performed by studying the relationship between ensemble covariance distribution and perceived background error covariances (similarly to what has been done for EDA variances); and through assimilation experiments using climatological background error statistics derived from EDA with different model error parameterizations.

## REFERENCES

- Anderson, J. L., 1996: A method for producing and evaluating probabilistic forecasts from ensemble model integrations. *J. Climate*, **9**, 1518–1530.
- Belo Pereira, M. and L. Berre, 2006: The use of an ensemble approach to study the background-error covariances in a global NWP model. *Mon. Wea. Rev.*, **134**, 2466–2489.
- Bonavita M., L. Raynaud and L. Isaksen, 2010: Estimating background-error variances with the ECMWF Ensemble of Data Assimilations system: the effect of ensemble size and day-to-day variability. *ECMWF Technical Memorandum*, No. 632.
- Buizza, R., M. Miller and T. N. Palmer, 1999: Stochastic representation of model uncertainty in the ECMWF Ensemble Prediction System, *Q. J. R. Meteorol. Soc.*, **125**, 2887–2908.
- Buizza, R., M. Leutbecher and L. Isaksen, 2008: Potential use of an ensemble of analyses in the ECMWF Ensemble Prediction System. *Quart. J. Roy. Meteor. Soc.*, **134**, 2051–2066.
- Fisher, M., 2003: Background error covariance modelling. *Proceedings of the ECMWF Seminar on recent developments in data assimilation for atmosphere and ocean*, ECMWF, pages 45–63. (Available from: <http://www.ecmwf.int/publications/>)
- Grimit E. P. and C. F. Mass, 2007: Measuring the Ensemble Spread–Error Relationship with a Probabilistic Approach: Stochastic Ensemble Results. *Mon. Wea. Rev.*, **135**, 203–221
- Houtekamer, P. L., 1993: Global and local skill forecasts. *Mon. Wea. Rev.*, **121**, 1834–1846.

- Isaksen, L., M. Bonavita, R. Buizza, M. Fisher, J. Haseler, M. Leutbecher and Laure Raynaud, 2010: Ensemble of data assimilations at ECMWF. *ECMWF Technical Memorandum* No. 636. (Available from: <http://www.ecmwf.int/publications/>)
- H. L. Mitchell and P. L. Houtekamer, 2000: An Adaptive Ensemble Kalman Filter. *Mon. Wea. Rev.*, **128**, 416-433.
- W. C. Kolczynski, Jr., D. R. Stauffer, S. E. Haupt, N. S. Altman and A. Deng, 2011: Investigation of Ensemble Variance as a Measure of True Forecast Variance. *Mon. Wea. Rev.*, Early Online Release. doi: 10.1175/MWR-D-10-05081.1
- Palmer, T.N., R. Buizza, F. Doblas-Reyes, T. Jung, M. Leutbecher, G.J. Shutts, M. Steinheimer and A. Weisheimer, 2009: Stochastic Parametrization and model uncertainty. *ECMWF Technical Memorandum*, No. 598. (Available from: <http://www.ecmwf.int/publications/>)
- Talagrand, O., R. Vautard, and B. Strauss, 1997: Evaluation of probabilistic prediction systems. *Proc. Workshop on Predictability*, Reading, United Kingdom, ECMWF, 1–25.
- Y. Trémolet, 2007: Model-error estimation in 4D-Var. *Q. J. R. Meteorol. Soc.*, **133**, 1267–1280.
- Varella H, Berre L, Desroziers G. 2011. Diagnostic and impact studies of a wavelet formulation of background-error correlations in a global model. *Q. J. R. Meteorol. Soc.* DOI:10.1002/qj.845
- Wang, X. and C. H. Bishop, 2003: A comparison of breeding and ensemble transform Kalman filter ensemble forecast schemes. *J. Atmos. Sci.*, **60**, 1140-1158
- Wei, M. and Z. Toth, 2003: A New Measure of Ensemble Performance: Perturbation versus Error Correlation Analysis (PECA). *Mon. Wea. Rev.*, **131**, 1549-1565
- Whitaker, J. S., and A. F. Lough, 1998: The relationship between ensemble spread and ensemble mean skill. *Mon. Wea. Rev.*, **126**, 3292–330.