Case Dependent Implicit Normal Mode Balance Operators

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Abstract

An implicit normal mode balance approach is used to represent the balanced part of analysis increments within advanced data assimilation systems. The methodology introduced here (1) naturally extends previous linear (Rossby) operators used in variational operational systems to include a first-order, case-dependent correction using a tangent-linear Baer-Tribbia's type of balance combined with Temperton's implicit NMI formulation; (2) represents an efficient, accurate and coherent treatment of nonlinearities compatible with tangent-linear code; (3) can naturally include tangent linear physical processes. Because of the incorporation of physical forcing terms in the balance conditions, the procedure implicitly imposes flow-dependent and physically dependent (hereafter referred to as *case-dependent*) background-error covariances.

1. Introduction

The use of Hough mode representation directly into the analysis step has previously been considered for instance by Parrish (1988) and in early 3D-Var development work at NCEP (Parrish and Derber, 1992, see end of Sec. 1 and Sec. 2), ECMWF (Heckley et al. 1992). The motivations at the time for doing this were to partition adequately the contribution of Rossby and Gravity mode contributions in the final analysis state. We recall that the analysis was performed on the total fields rather than analysis increments as done nowadays. A dual goal was achieved by doing this since controlling gravity wave imbalances in the resulting analysis could harmoniously be imposed in this representation through ideas from Nonlinear Normal Mode Initialization (NNMI) balance concepts. The shift to an incremental variational (VAR) formulation where analysis increments are analyzed at each inner-loop of the complete VAR analysis (Courtier et al. 1994, Sec 3) alleviated the need for a full nonlinear balancing procedure so that only simple linear relationships; e.g. the linear balance equation, could be used with success to link mass and rotational wind increments.

In an attempt to introduce flow-dependent background error covariances in ECMWF's 4D-Var analysis, Fisher (2003) introduced the quasi-geostrophic (QG) set of balance equations into the definition of the balanced part of the analysis increment. The r.h.s. terms of the QG Omega equation (Hoskins et al. 1978) being nonlinear in terms of streamfunction and temperature, this automatically introduces flow-dependency in the problem.

This effect also appears in the nonlinear Charney's balance equation complementary to the Omega equation. It allowed particularly more synoptically coherent divergent wind increments whereas the previous ECMWF regression approach was unable to detect such basic-state properties. Fisher's approach could be extended to

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include diabatic effects as suggested by Fillion et al. (2005) for instance. Those effects can on some occasions of intense physical forcing be modulating the so-defined balanced circulation and potentially crucial for an effective data assimilation over cloudy/rainy regions and could be a crucial ingredient to consider. This is currently lacking in current 4D-Var feasibility studies such as in Benedetti and Janiskova (2007).

There are growing evidences of limitations and problematical computer code development one faces when going in the direction of extending the use of the QG balance equations. Practical implementations performed up to now made simplifying assumptions when building tangent-linear QG balance operators such as the use of a pressure coordinate form of the equations and simple static stability for instance. In the vicinity of significant topography, the former assumption can be shown to neglect structures in the balanced part of the analysis increment for instance (see Lindskog's contribution, this proceeding, for an example of this and extra terms to consider). The concept of dynamical and moist-physical balance (related to spin-up problems) requires a careful numerical approach to enforce "balance" that stays as close as possible to the forecast model's numeric and sequence of application of physical parameterization schemes (e.g. moist-convection, large-scale condensation, cloud scheme and coupling with radiation processes). Since we deal with analysis increments here, tangent-linear (TL) processes are of concern. Implementing a QG approach requires the construction of TL/Adjoint (AD) extra code not used in the TL model (or perturbative model) and also implies additional coding if additional forcings are needed on the r.h.s. of the Omega equation. It appears highly desirable to avoid the proliferation of such parallel computer code, if possible, while at the same time maintaining or even improving the degree of dynamical/physical balance through TL forcings desired. Failing to achieve these requirements may lead to poor performance or even negative impacts of assimilating observations related to cloudy/rainy regions for instance. It is the purpose of the present study to demonstrate the existence of such an approach applicable in VAR analysis systems.

2. Theory

In the following, we present the procedure to define (1) the tangent-linear balance operators; (2) the introduction of the extended control vector; (3) the methodology on how to construct background error statistics for the new analysis variables.

2.1. Tangent-linear balance: Rossby and Gravity contributions

The idea to use nonlinear normal mode theory (Baer-Tribbia 1977) in the present data assimilation context (where QG equations are used) is obvious considering the link between QG theory and NNMI (Leith 1980, hereafter L80). We borrow Temperton's (1988) notation (hereafter T88), make the link with L80's results and will assume the reader has a previous knowledge of these and related studies. Daley's book (1991) can also be a valuable reference since we avoid discussing the basic details of normal mode representations.

Let **x** represents the vector of model variables. The forecast model may be written as:

$$\frac{\partial \mathbf{x}}{\partial t} = i\,\mathbf{A}\,\mathbf{x} + \varepsilon\, N(\mathbf{x}) \tag{1}$$

where **A** is a constant coefficient matrix representing the linear terms in the model, $N(\mathbf{x})$ represents the nonlinear terms (including dynamical as well as physical nonlinearities) and ε is a scaling parameter (typically Rossby number) which measures the relative importance of nonlinearities. We now assume we have at hand the **G** (Gravity) and **R** (Rossby) projectors. Machenhauer's (1977) balance sets

$$\mathbf{G}\frac{\partial \mathbf{x}}{\partial t} = 0 \tag{2}$$

There is a large amount of indications necessary here concerning this last statement based on practical results of implementation into NWP models in the past. We postpone a bit these restrictions and will come back on the most serious ones later. Using (1) and basic algebraic properties of **A** and **G**, the latter condition is easily shown to imply

$$\mathbf{G}\,\mathbf{x} = i\varepsilon\,(\mathbf{AG})^{-1}\,\mathbf{G}\,N(\mathbf{x}) \tag{3}$$

Starting from a linearly balanced state $\mathbf{x}_0 = \mathbf{R}\mathbf{x}$, L80 showed (within the context of Boussinesq equations) that the first order correction to $\mathbf{x}_0$ produces a state $\mathbf{x}_1$ given by

$$\mathbf{x}_1 = \mathbf{x}_0 + i\varepsilon\,(\mathbf{AG})^{-1}\,\mathbf{G}\,N(\mathbf{x}_0) \tag{4}$$

s.t. (1) it implies a QG omega field and (2) geopotential *and* streamfunction fields are corrected from their linearly balanced values (used to define $\mathbf{x}_0$) according to a nonlinear relationship related to the classical nonlinear balance equation (ref. Sec 5). We stress here, based on this last statement and known results from NNMI theory that the streamfunction field has a *balanced gravity* component. We come back later on this. The preceding results are the starting point to extend Fisher's approach. Starting from Eq.(1), the tangent-linear equations governing the evolution of the perturbation $\delta\mathbf{x}$ about the basic state $\bar{\mathbf{x}}$ are

$$\delta\left(\frac{\partial \mathbf{x}}{\partial t}\right) = i\,\mathbf{A}\,\delta\mathbf{x} + \varepsilon\,\mathbf{N}_{\bar{\mathbf{x}}}\,\delta\mathbf{x} \tag{5}$$

We now ask, rather than Machenhauer's condition applied to the full fields but to the perturbed state about the basic state, the condition:

$$\mathbf{G}\,\delta\left(\frac{\partial \mathbf{x}}{\partial t}\right) = 0 \tag{6}$$

$$\mathbf{G}\,\mathbf{A}\delta\mathbf{x} = i\varepsilon\,\mathbf{G}\mathbf{N}_{\bar{\mathbf{x}}}\,\delta\mathbf{x} \tag{7}$$

In the same way nonlinearities were treated in NNMI, we make use of Andersen's (1977) forward timestep trick but now using the TL model (see T88, for the derivation of his eqn. 2.14) to get (dropping the scaling parameter):

$$\mathbf{A}\,\Delta(\delta\mathbf{x})_{\mathrm{G}} = i\,\left(\delta\frac{\partial \mathbf{x}}{\partial t}\bigg|_{\bar{\mathbf{x}}}\right)_{\mathrm{G}} \tag{8}$$

where $\Delta(\delta\mathbf{x})_{\mathrm{G}}$ is the *change* that has to be made to the state we want to balance and $(\delta\frac{\partial \mathbf{x}}{\partial t}\big|_{\bar{\mathbf{x}}})_{\mathrm{G}}$ is the gravity mode projection of the TL time-tendency about $\bar{\mathbf{x}}$.

Remark 1: The condition Eq.(6), similarly to Wergen (1987) and Ballish's Incremental NNMI (1992) proposals, considers the inclusion of diabatic effects in the initialization procedure (e.g. maintenance of the thermal tide developed in the background fields during a data assimilation cycle). In our case here, the background time tendency is our reference for defining a balanced increment; i.e. it is in that sense that we use the word "balance" in this study.

Remark 2: Due to its relationship with QG theory, the TL-balance condition (8) has similar "flow-dependent" virtues as those exemplified by Fisher (2003). The *practical* advantage of (8) however, as stated in Sec.1 is the fact that we do not need do build TL/AD code for taking into account nonlinearities, the existing TL model is used through a simple call for one forward timestep. This option must be available with the computer code structure of the VAR analysis however as it is the case for the Environment Canada (EC) framework. In addition, including physical forcing in the balance condition (8) is straightforward within a call to the TL/AD code (again at EC), allowing for a complete compatibility of the dynamical and physical forcings. We give an example of this in the next section.

Remark 3: There is obviously no iteration involved in solving (8). Equation (8) state how, starting from a perturbation state vector $\delta\mathbf{x}_R$, a correction vector $\Delta(\delta\mathbf{x})_{\mathrm{G}}$ (along the gravity subspace) can be added to get to the TL-balance subspace about $\bar{\mathbf{x}}$ (noted $\mathfrak{B}$). The latter should not be confused with the TL Machenhauer's balance subspace.

Remark 4: It is well known that NNMI implementations in operational NWP models can produce realistic topographically induced vertical motions (Daley 1979, Temperton 1991). The normal mode framework being compatible with the geometry and numeric of the numerical model, these latter properties are more consistently taken into account than with QG equations.

It is appropriate at this point to introduce the second aspect of the approach taken here starting from the current operational practice at operational centers with VAR analysis systems (including the ECMWF one before their introduction of the QG equations). It was assumed until now (for approximation purposes) that the streamfunction analysis increment provided the information to build the balanced mass increment through a linear balance relationship. It is clear however, based on normal mode theory, that Rossby and Gravity modes have vorticity components (e.g. see T88). We introduce the new grid-point control-vector (ignoring the usual preconditioning for simplicity) as:

$$\delta\boldsymbol{\xi}=(\delta\psi_R,\delta\psi_u,\delta\chi_u,\delta T_u,\delta q,\delta p_{s_u})^{\mathrm{T}} \tag{9}$$

As compared to the previous formulation, the streamfunction increment has been separated in two parts, the "Rossby" part and an "unbalanced" part. For global analysis grids, where large-scale divergent Rossby circulation is important, a more appropriate definition would include the Rossby part of divergence so that one would have

$$\delta\boldsymbol{\xi}=(\delta\psi_R,\delta\chi_R,\delta\psi_u,\delta\chi_u,\delta T_u,\delta q,\delta p_{s_u})^{\mathrm{T}} \tag{9a}$$

For limited-area analysis such as ours (or smaller), we use the approximation (9). A consistent and elaborate method to rigorously define the Rossby part is described in Sec. 2.2 below. As a first implementation test into our LAM4D analysis scheme, and because no special re-computations of background error statistics for these new variables has been done, we compute the Rossby part of the balanced increment by the usual VAR operators and set to zero the unbalanced part of streamfunction; i.e. $\delta\psi_R = \delta\psi$;

$$\delta P_R = \mathbf{F}\,(\delta\psi_R) \text{ where } \mathbf{F} \equiv \nabla \bullet (f\,\nabla\delta\psi_R)\ ;\quad \delta\chi_R = 0$$
$$(\delta T_R\,,\delta p_{s_R}) = \mathbf{V}\,\delta P_R\ ;\ \mathbf{V} \text{ defined by linear regression}$$

We note that the Rossby modes used here are consistent with T88's approximation to be used hereafter (i.e. zero divergence modes). In the vertical however, the reconstruction of temperature and surface-pressure increments still uses the regression approach of Parrish and Derber (1992), rather than T91's procedure (see Sec. 3). The result is, obviously, still Rossby type increments however. In an analogous manner as Baer-Tribbia's (1977) approach, we refer to that initial step

$$\delta\mathbf{x}_R = \mathbf{R}\delta R \quad \text{s.t.} \quad \delta\mathbf{x}_R = (\delta\psi_R,\ \delta\chi_R = 0, \delta T_R, \delta p_{s_R})^{\mathsf{T}}$$

as the "order zero" balance step. It is interesting to note that this balancing step *is not* flow-dependent. The construction of the gravity-mode correction $\Delta(\delta\,\mathbf{x})_{\mathrm{G}}$ that goes with this Rossby mode correction satisfies:

$$\mathbf{A}\,\Delta(\delta\mathbf{x})_{\mathrm{G}} = i\,\left(\delta\frac{\partial\mathbf{x}}{\partial t}\bigg|_{\bar{\mathbf{x}}}\right)_{\mathrm{G}} = i\,\mathbf{G}\left[\mathbf{TL}(\bar{\mathbf{x}},\delta\mathbf{x}_R,\delta q)\right] \tag{10}$$

where $\delta\mathbf{x}_B = \delta\mathbf{x}_R + \Delta\delta\mathbf{x}_{\mathrm{G}}$; *s.t.* $\delta\mathbf{x}_B \in \mathfrak{B}$,

is the tangent-linear balance increment. Note that in (10), we have made explicit the tangent-linear input variables used by the TL model. Note that δq is used as input to the TL model in (10). Obviously, that part of the definition of the balanced increment is not only important to ensure better dynamical balance but has a very important property of being *case-dependent*; i.e. the basic-state dynamical and physical properties are taken into account explicitly in (10). The balanced gravity part of the analysis increment vector being (neglecting the symbol Δ due to the zeroing step of the gravity-mode part of the increment):

$$\delta\mathbf{x}_G = (\delta\psi_G, \delta\chi_G, \delta T_G, \delta p_{s_G})^{\mathsf{T}} \tag{11}$$

The full tangent-linear balanced increment being:

$$\begin{aligned}\delta\mathbf{x}_B &= (\delta\psi_B,\ \delta\chi_B, \delta T_B, \delta p_{s_B})^{\mathsf{T}} \\ &= (\delta\psi_R,\ \delta\chi_R = 0, \delta T_R, \delta p_{s_R})^{\mathsf{T}} + (\delta\psi_G, \delta\chi_G, \delta T_G, \delta p_{s_G})^{\mathsf{T}}\end{aligned} \tag{12}$$

The final computation of the analysis increment vector in VAR, taking moisture into account, is:

$$\delta\mathbf{x} = (\delta\psi_B, \delta\chi_B, \delta T_B, \delta q, \delta p_{s_B})^{\mathsf{T}} + (\delta\psi_u, \delta\chi_u, \delta T_u, 0, \delta p_{s_u})^{\mathsf{T}} \tag{13}$$

In (8), we need to find the gravity projection of the TL time-tendencies appearing on the r.h.s (the linear operator **A** is known). That aspect pertains to Temperton's T88 theory of *implicit* NNMI (INMI). It is demonstrated that (within some approximations on the beta-terms), no explicit knowledge of the horizontal normal mode structures and frequencies are required to apply NNMI. The price to pay being the need to solve a set of elliptic equations for each vertical mode. We now make a next step in the direction of T91 baroclinic implementation of INMI. We assume the reader has carefully examined the scheme in T91 and introduce directly his numerical procedure except that here the nonlinear time tendencies are replaced by TL time-tendencies (ref. 8 above). The procedure is as follows:

1. Run the TL model for one forward timestep to obtain the "observed" tendencies

$$(\frac{\partial \zeta}{\partial t})_o\,,(\frac{\partial D}{\partial t})_o\,,\left\{(\frac{\partial T}{\partial t})_o\,,\ (\frac{\partial \ln p_s}{\partial t})_o\right\} \rightarrow (\frac{\partial P}{\partial t})_o$$

2. Solve for each vertical mode "*l*" desired:

   (N.B.: T88's simplified scheme is used here for illustration purposes)

$$(\nabla^2 - \frac{f^2}{\Phi_l})\Delta\hat{P}_l \;=\; \frac{\partial \hat{D}_{o,l}}{\partial t}$$

$$(\nabla^2 - \frac{f^2}{\Phi_l})(\frac{\partial \hat{P}_l}{\partial t})_G \;=\; \nabla^2 \frac{\partial \hat{P}_{o,l}}{\partial t} - f\,\nabla^2 \frac{\partial \hat{\zeta}_{o,l}}{\partial t}$$

3. Compute $\Delta\hat{D}_l = \frac{1}{\Phi_l}(\frac{\partial \hat{P}_l}{\partial t})_G \;;\; \Delta\hat{\zeta}_l \;=\; \frac{f}{\Phi_l}\Delta\hat{P}_l$

4. Project back in gridpoint space the balancing increments in vertical mode space.

The procedure to construct the vertical normal modes is described in T91 and references together with the procedure to invert P increment to T and $\ln(p_s)$ increments. Further comments on this latter aspect are given in the next sub-section.

2.2. Background error statistics

In order to upgrade the control vector to (9), one need to define what the Rossby part of the streamfunction error (including velocity potential for global analysis grids) is from an ensemble of error samples in order to build error variances and auto-correlation spectra. In current operational practice, one simply uses the streamfunction component as the basic balancing variable, so this definition is not necessary. The solution to this new requirement can be found in T88's linear INMI (LINMI). This procedure simply sets to zero the gravity mode component of a given state vector (Williamson 1976). Using T88's simplified scheme (see his Sec. 4) for illustration purposes, assume we have an error sample of the form

$$\delta\boldsymbol{\varepsilon}_o = (\delta u, \delta v, \delta T, \delta q, \delta p_s)_o\,.$$

The projection of that state vector onto Rossy modes is given by the following steps:

1. Compute $(\delta\zeta, \delta D = 0, \delta P = \delta\phi + RT^* \ln p_s)_o$

2. Project onto vertical normal modes

3. For each vertical mode:

   a) Solve for the mass correction for each vertical modes "*l*":

$$(\nabla^2 - \frac{f^2}{\Phi_l})\Delta(\delta P)_l = (-\nabla^2 P_o + f\,\zeta_o)_l$$

   b) Compute the Rossby part of δP:

$$(\delta P_R)_l = (\delta P_o)_l + (\Delta(\delta P))_l$$

c) Compute the Rossby part of $\delta\zeta$:

$$(\delta\zeta_R)_l = (\delta\zeta_o)_l + \frac{f}{\Phi_l}(\Delta(\delta P))_l$$

4. Project back on gridpoint space in the vertical to get:

$$(\delta\zeta, \delta D = 0, \delta P)_R$$

5. Recover the mass variables (following T91, Sec 3.):

$$(\delta P)_R \rightarrow (\delta T, \delta \ln p_s)_R$$

6. Finalise to get the Rossby part of the error sample:

$$\delta\boldsymbol{\varepsilon}_R = (\delta\psi, 0, \delta T, \delta p_s)_R$$

Remark 1: It is important to stress here that the above T88's LINMI scheme is the least accurate in terms of retention of the standard structure of the *Rossby* modes; i.e. they are stationary (zero natural frequency) and non-divergent. As mentioned in Temperton (1989) (hereafter T89), for non-rotated system of coordinates on the sphere where the linearized dynamics remains separable, it is possible to design a hierarchy of INMI approximation to the standard explicit normal mode construction that better approximate the Rossby mode properties. This means that for such global analysis grids (e.g. EC's global 4D-Var, ECMWF 4D-Var, Met-Office's 4D-Var, NCEP's 3D-Var, JMA's global 4D-Var for instance), the approach suggested here to alter the control vector using (2-10), (2-11) it is highly desirable to use say scheme "B" of T89 or if possible, go to the standard representation (system "A" in T89). Note that in principle, nothing precludes using an explicit representation for the Rossby modes and then using the implicit representation for the gravity correction (INMI type) coming from (8) during the minimization.

Remark 2: The inversion in step 5 above can be subtle if, as mentioned in T91, the lowest model level is not at the ground and if geopotential (or variable P) and temperature variables are not vertically staggered. Parrish and Derber (1991) (See their sec.2) encountered that specific issue and opted for an EOF representation in the vertical. In our approach here, EOF of *total* streamfunction error cannot be used since it does not strictly represent the Rossby part of the error. At EC, a Charney-Phillips type of vertical grid has recently been developed that makes the hydrostatic operator invertible and a consistent approach can be designed so that the inversion step 5 above can be performed.

3. Results with the Canadian LAM4D

We describe in the following, some basic results of the incorporation of the new balance operators. At Environment Canada (EC), a Limited-Area 4D-Var analysis system has been developed in order to enable the analysis of synoptic and mesoscale weather. This system in its North American continent extension is referred to as *LAM4D*. The main objective of this new analysis system being the improved forecast of precipitation up to 48h (more emphasis on the first 24h), with a replacement of the Canadian Regional analysis system currently operational at CMC. Tangent-linear (TL) and adjoint (AD) versions of GEM-LAM were developed. Putted simply, LAM4D uses bi-Fourier spectral representation on a rotated limited area domain rather than spherical harmonics on the sphere. Otherwise, the two configurations of the code were designed to be mostly transparent to the user. Helmholtz's functions are being used in the two analysis systems with non-separable background error correlations in their respective spectral spaces.

The horizontal resolution of the Nonlinear GEM-LAM NA-Continental model is 15 km (the current operational regional model being at 15 km). Figure 1 shows the analysis grids for the Regional FGAT 3D-Var and LAM 4D-Var systems respectively. However, for the experiments presented here, the inner-loop resolution of the incremental LAM4D is set at 80 km rather than 35 km (6h time assimilation window for both systems). The analysis and models have 58 vertical levels.

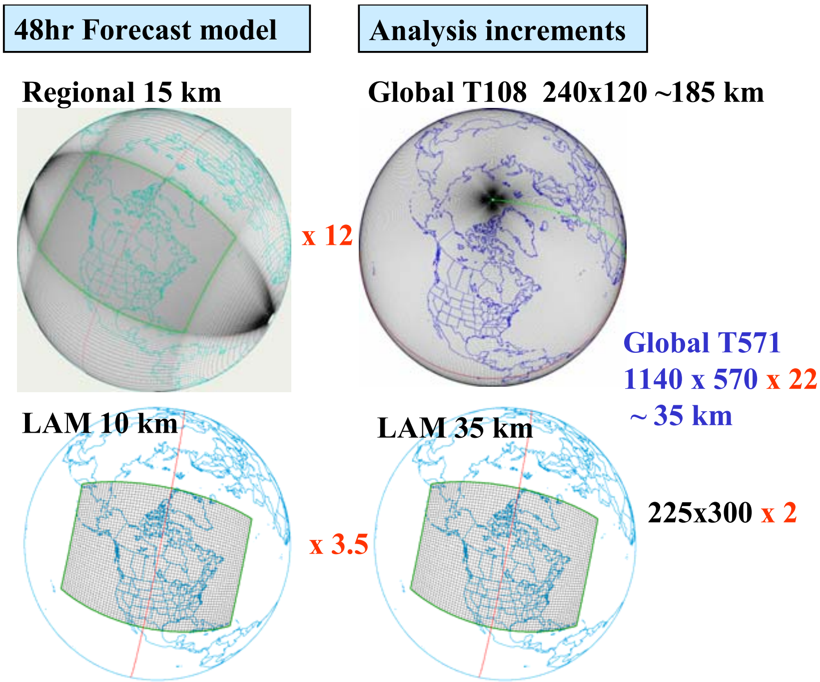


Figure 1: Regional FGAT 3D-Var anaylysis grid; i.e. regular Gaussian grid at T108. Below, LAM4D analysis grid for the target configuration.

The first experiment was designed to illustrate the flow-dependency imposed by the new balance operators in a similar way as illustrated by Fisher (2003) using QG balance operators. Since the former are defined from a one forward time step of the TL model, this requires technically (at EC) that we proceed in 4D-Var mode. It is possible in our context to run the TL model in various modes on demand during the minimization process. This is performed using so called "events" that have to be defined in the 4D-Var code. This means we can run the TL model in the forward observation operator with TL physics and run another TL for one time step adiabatically or whatever process requested when building the balanced part of the analysis increment during the minimization. This is a powerful feature that enables flexibility that was particularly in need for our definition of balance operators.

Of particular illustrative and testing importance is also the possibility for us to run the TL model in what we call "identity" mode; i.e. during the time stepping of the TL model, the regular scheme is replaced by a simple copy of the fields for one time step to the other. In other words, this call to the TL model goes through entry and exit programs but the fields are left untouched for the time stepping part of the code. This has been a feature useful for debugging many technical aspects that we of course will skip here. Using a single temperature observation at 500 hPa, the latter "identity" mode is useful since it allows using radiosonde observations centered at time 0h of the 4dvar analysis window and exclude the flow-dependent structure of the analysis increment coming simply from the TL dynamics rather than showing evidence of the flow-dependent properties of the new balance operators. The case examined here is March 2nd 2007 where a significant snow storm was approaching the southern part Quebec, Canada. The 6h background specific humidity at 500 hPa valid at 9 UTC is shown in figure 3.

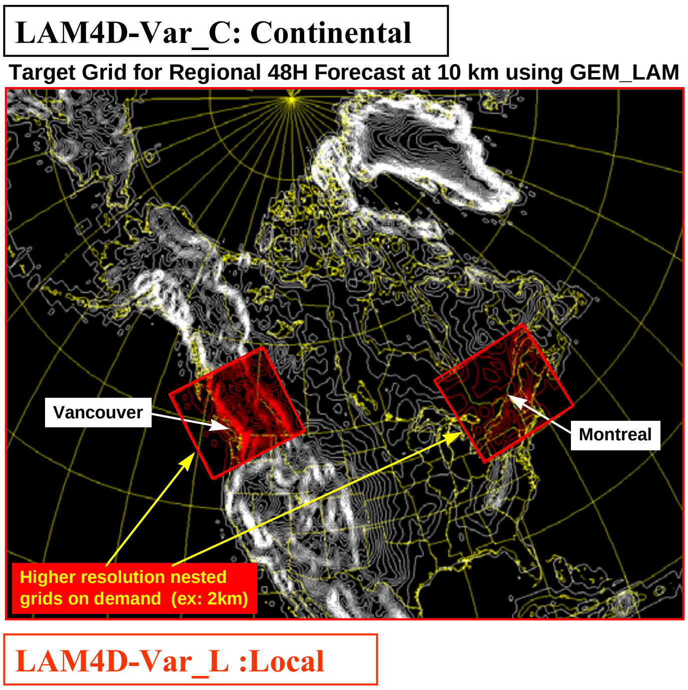


Figure 2: GEM-LAM grid extension corresponding to the lat-lon grid of LAM4D in Fig. 1 (black domain) together with local grids for 24 h forecasts.

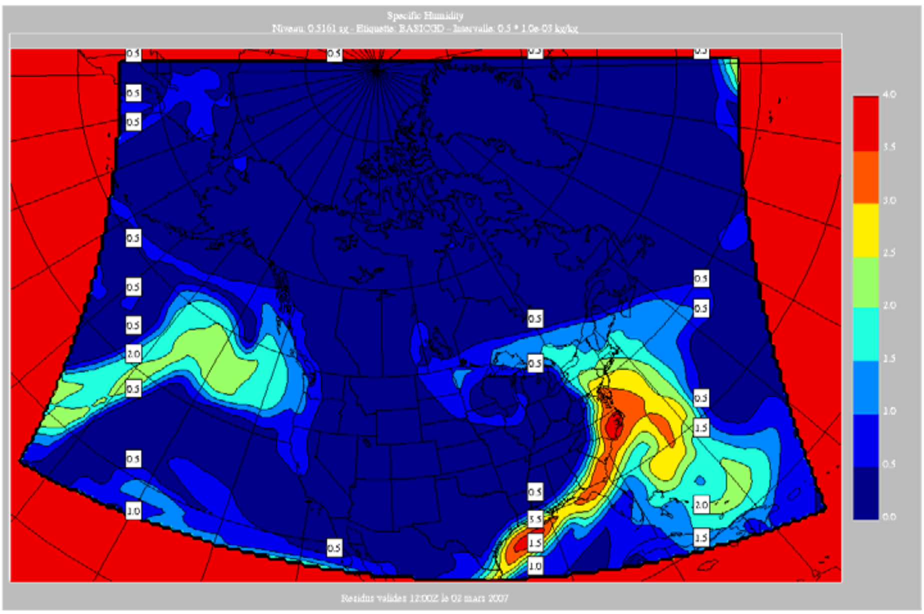


Figure 3: 500 hPa specific humidity for 2nd March 2007, 9 UTC.

We now turn to the possibility of imposing a consistent divergent wind (then vertical velocity) at the analysis time that is in agreement with moist physical processes. This is particularly important for data assimilation over cloudy/rainy regions and also for the improved quantitative precipitation forecasts. As mentioned at the beginning of Sec. 2, we may include diabatic processes into the nonlinear term when enforcing a balance between linear and tangent-linear gravity mode terms. This imposes a dynamical and physical constraint so as to stay close to the background time-tendency.

It is relevant to stress here that the QG Omega equation is normally solved on all degrees of freedom in the vertical. Considering L80's results, this suggests solving our TL balance condition for all vertical modes. This point was argued originally by Phillips (1981) (p.3) but gives important restrictions on this concerning strong diabatic and topographic effects where a first zeroing of gravity modes would be detrimental. That issue became fundamental in practical implementations of NNMI and the zeroing step was discarded. Nowadays, especially in the incremental context here, where we try to stay close to the background time-tendencies, this remains in the lines of thought of the original work of Wergen (1987) but where he could impose more restrictions than here based on the explicit natural frequencies of the normal modes treated and vertical modes initialized.

As stressed originally by L80 (Sec 8.), diabatic processes do depend on vertical velocity so that a simple TL approach as we intend to use here may be too crude and would require higher order approximation. It turns out that this does not appear to be the case with the strong winter storm examined here. Before discussing analysis results, we give in Fig. 5, results supporting our choice of incorporating a diabatic balance only in the 2 inner loop of LAM4D-Var. The first loop is using adiabatic TL balance operators. The TL error appears because of large-scale condensation and necessitates that we include it only in the second loop.

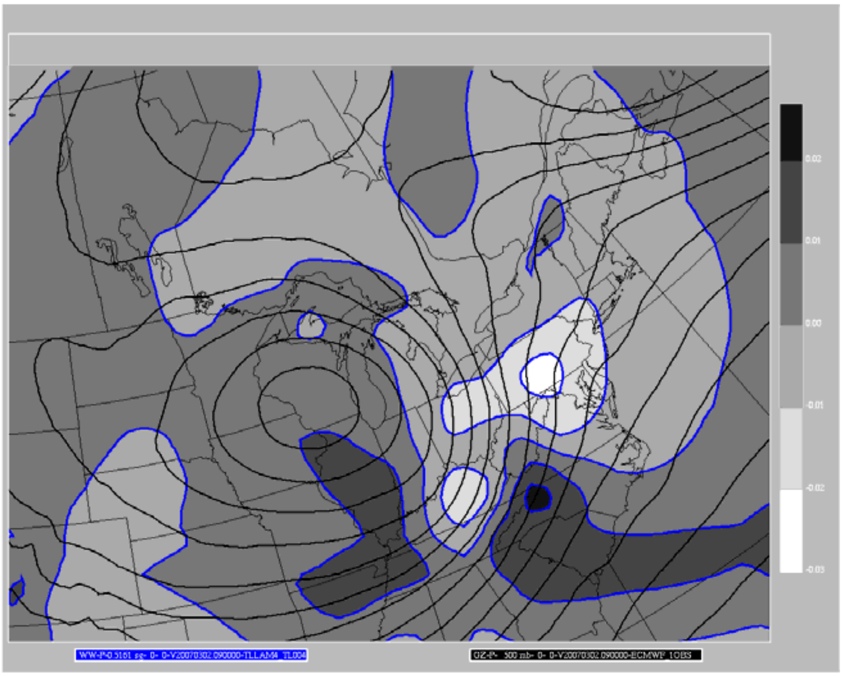
Figure 6 shows that the old balance operators do not produce any flow-dependent analysis increment of the vertical velocity whereas the use of the new balance operators does, see Fig. 7. Figure 8 shows that the use of the new balance operators, diabatic or not, produces a reduction rate of the functional comparable to the old balance methodology (results of the latter not show; Total number of iterations involved differ by 4). Finally, Fig. 9 shows that the use of a diabatic balance operator can produce a model-consistent balanced vertical velocity in agreement with diabatic effects (large-scale condensation here)

4. Summary and conclusions

We have introduced an extension of balance operators to tangent-linear ones based on implicit normal mode theory (Temperton 1988, 1989, 1991). Theoretical results on the connection between quasi-geostrophic theory and normal mode initialization (Baer-Tribbia 1977) such as in Leith (1980) allowed us to formulate a simple procedure to define balance operators. The latter allows also for a consistent incorporation of physical forcings. The tangent-linear model is used adequately for one forward time step, thus avoiding the unnecessary proliferation of extra TL coding not present in the TL model to account for dynamical and physical forcings.

A slightly new control vector and associated characterization of background error statistics was presented and based on Linear INMI (LINMI) from Temperton (1988). Preliminary results in the context of the Canadian LAM4D-Var analysis system exhibited evidence of flow-dependent analysis increments and coherent treatment of physical processes. One area of application of the new method where potential gain may me achieved is through the combination with current challenging attempts to assimilate cloudy/rainy data for instance. Obviously, moist-physical processes and direct coupling with dynamics through diabatic balance operators may play a dominant role.

Using previous balance operators



Using new balance operators

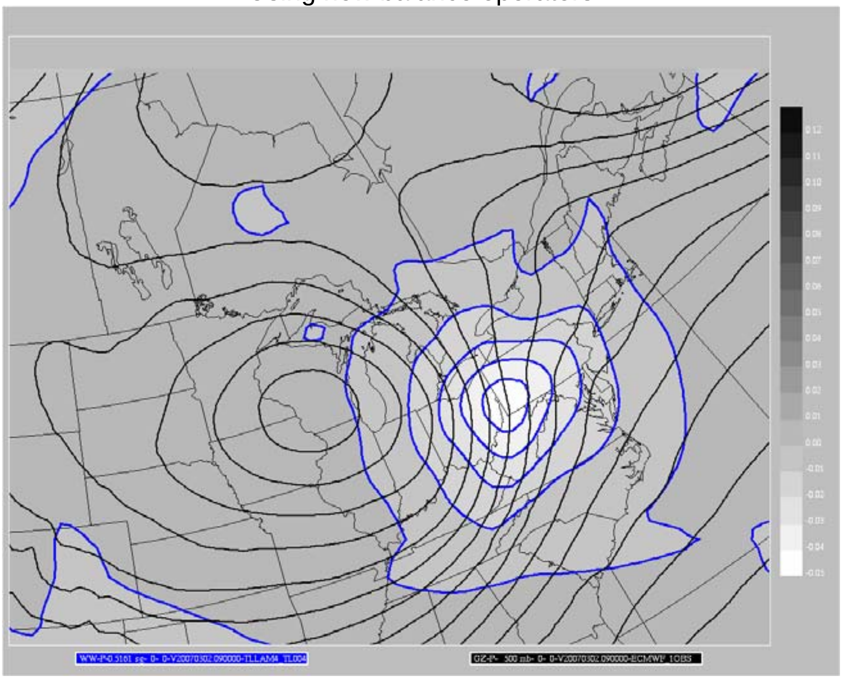
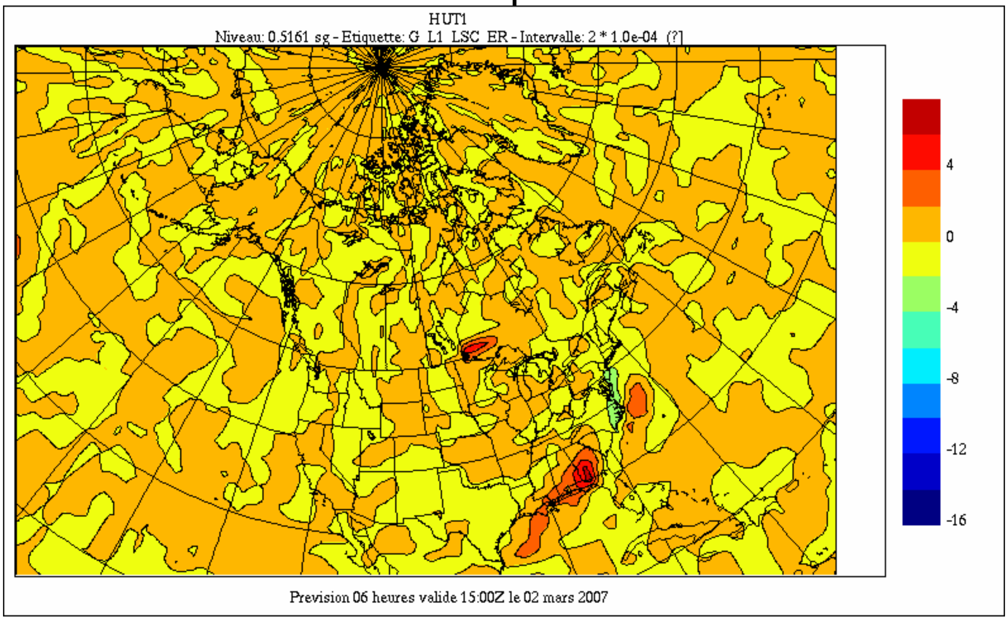


Figure 4: Vertical velocity increment (m/s) at 500 hPa valid at analysis time; i.e. for 2nd March 2007, 9 UTC when assimilating a temperature observation at 500 hPa valid at the analysis time (blue contours). Upper panel, using the old definition of balance operators; Lower panel: using new definition of balance operators. Experiments performed in "Identity mode" using US radiosonde 72520. Contour interval 1 cm/s. Maximum values: Upper panel: + 2.3 cm/s; Lower panel: - 4.6 cm/s. Superimposed, 500 hPa geopotential.

Loop 1



Loop 2

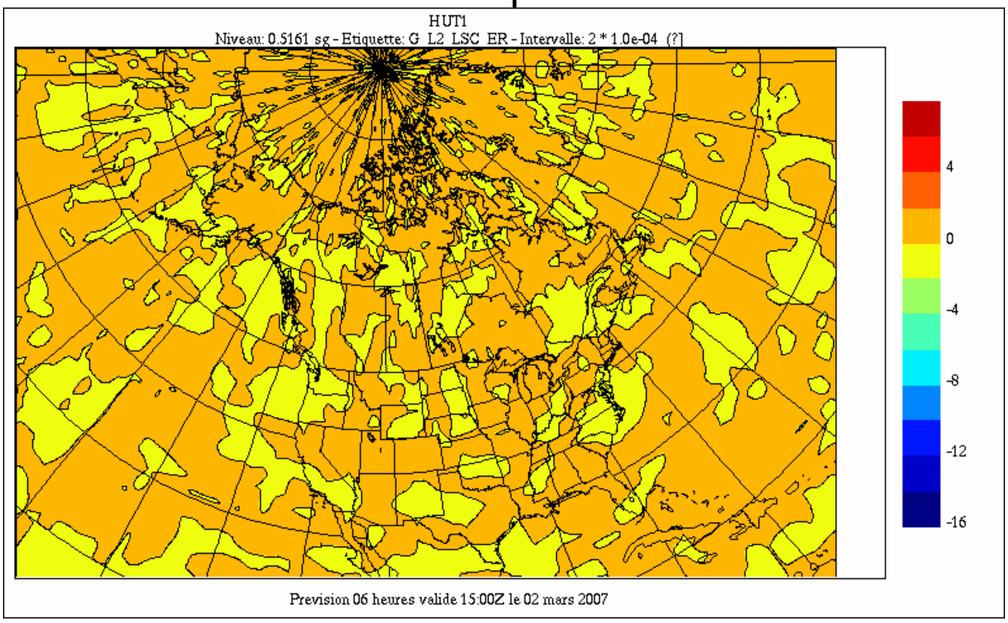


Figure 5: Specific-humidity error of the tangent-linear model w.r.t. two nonlinear runs. Large-scale condensation included. Moist-convection is absent and vertical diffusion is present. Upper panel: Using perturbation fields obtained from a 1st loop minimization. Lower panel: Using perturbation fields obtained from a 2ndloop minimization. In the first case, the error appears dominantly over regions where large-scale condensation is active and associated with the winter storm on the east coast (dominant over the cold front, see also Fig. 3).

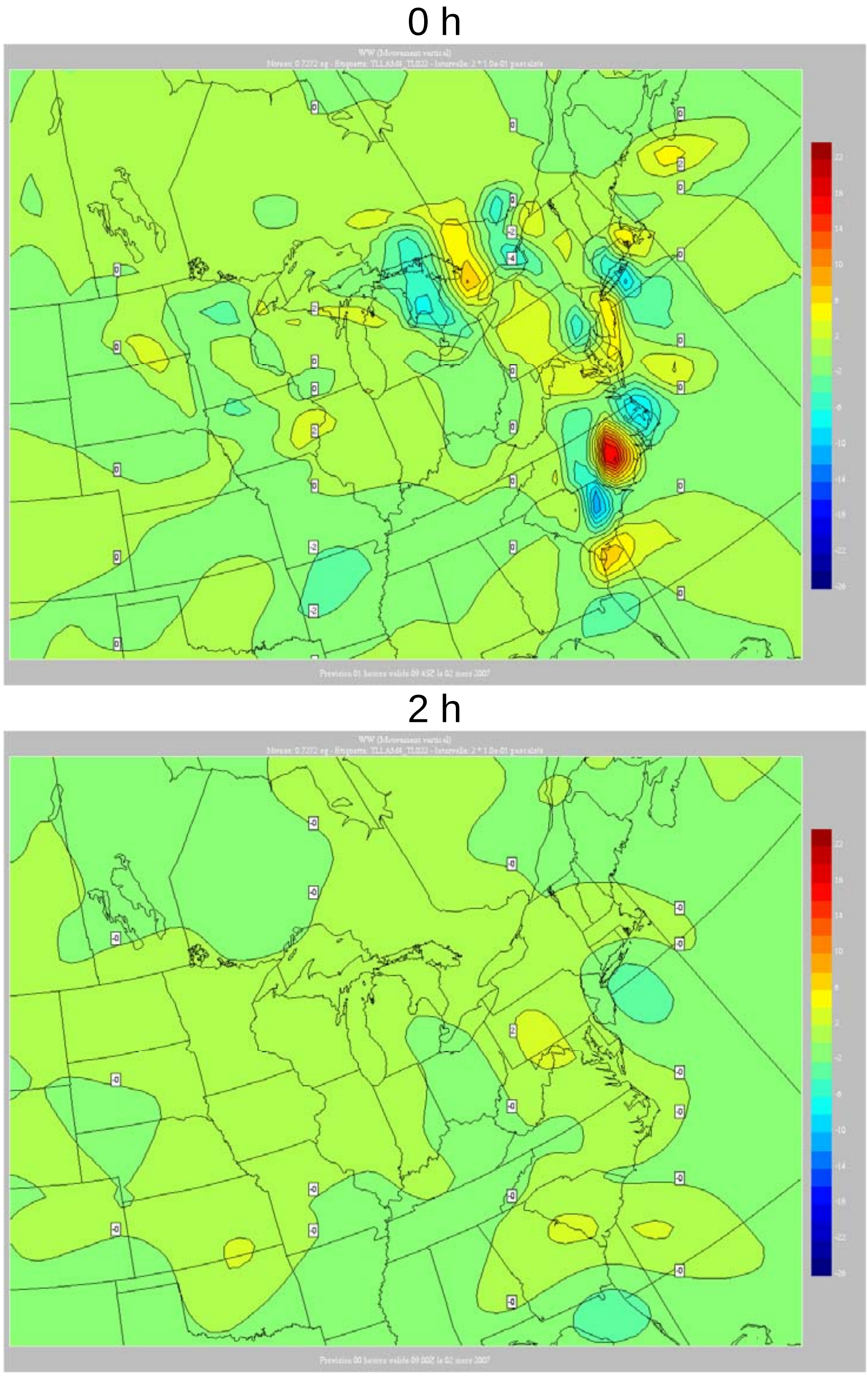


Figure 6: Vertical velocity increments at 700 hPa (Pa/s) (over the winter storm) at the end of the 2 loop LAM4D-Var minimization using old balance. Upper panel: 2^nd March 2007, at 9 UTC.; Lower panel: after 2h integration with the TL model where large scale condensation is active. The latter is also active in the TL during the minimization. The comma-shape signature of the snow-storm (see Fig.3) is apparent in the vertical velocity structure and is dominated by moist-physical processes.

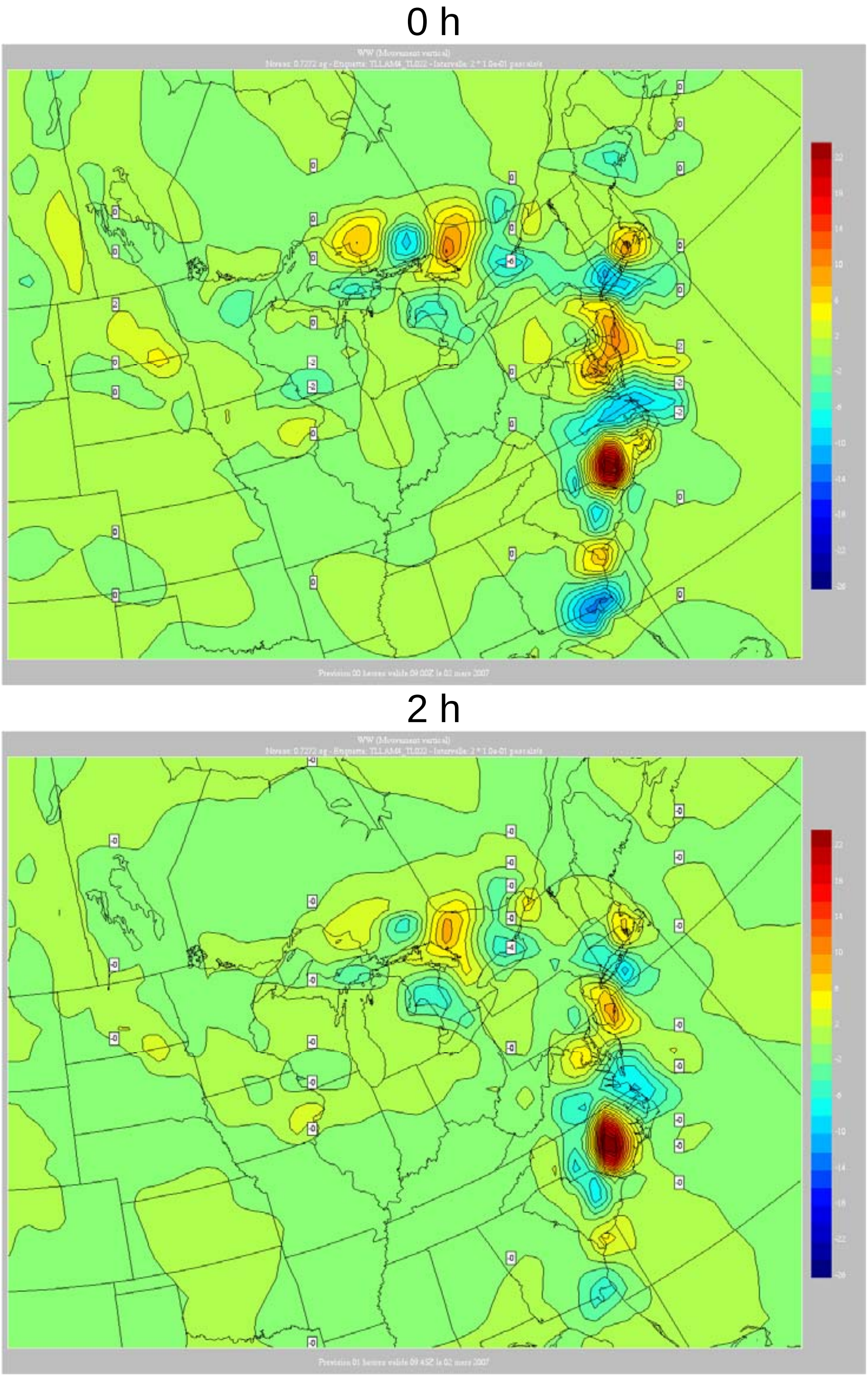


Figure 7: As in Fig. 6 but with the use of the new case-dependent balance operators. The structure of the vertical velocity increment at the initial time is well organized synoptically and appears to be closely matched with the dynamical and especially large-scale diabatic forcing present in the TL model. There is a close correspondence at 0h and 2h showing that the model (using a 22 min time step) has not developed a different kind of vertical velocity than presented as initial conditions.

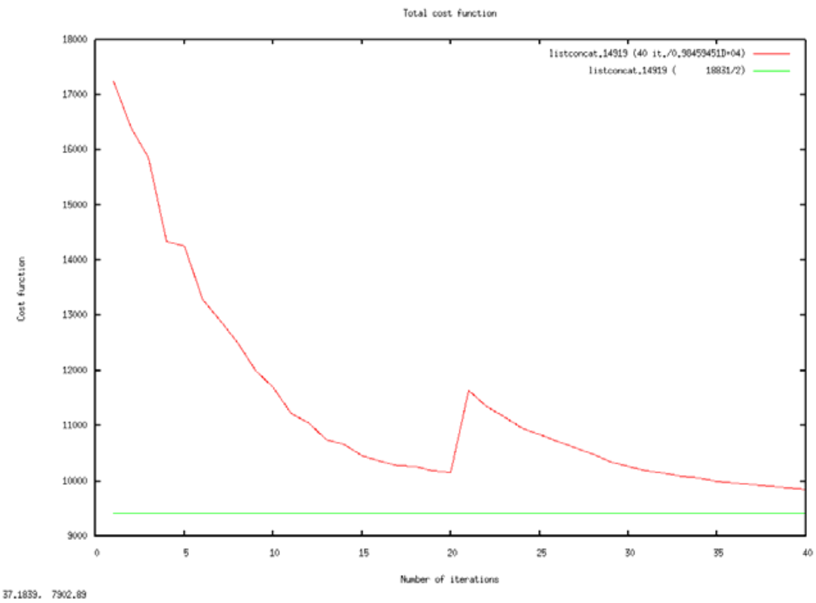


Figure 8: Value of the functional for 20 iterations (both loops). No large-scale condensation in TL and TL balance operators for 1st loop. Vertical diffusion is present. The 2nd loop uses large-scale condensation in both TL and TL balance operators. The horizontal resolution of the analysis is at 80 km for both inner loops and at 15 km for the NL model integration for the outer loop. All data normally used in operational regional analysis used.

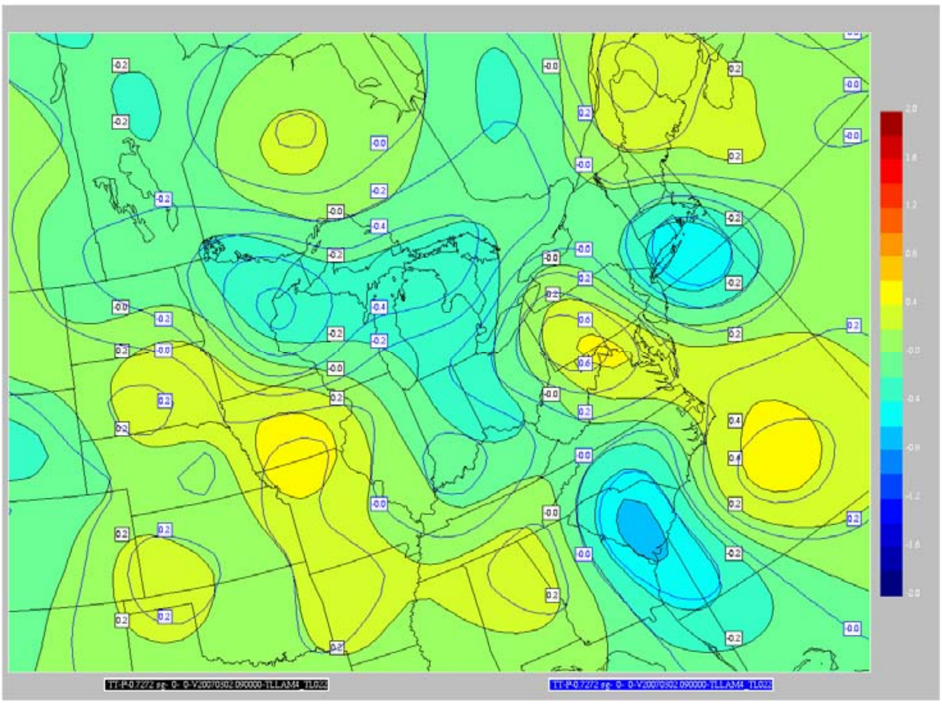


Figure 9: Temperature difference at 700 hPa between two minimizations. Both are LAM4D with 2 inner loops. One is using old balance operators and the other uses the new case dependent balance operators. Temperature differences reaching 0.5 deg near the low-pressure center.

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