Variational Data Assimilation Theory and Overview

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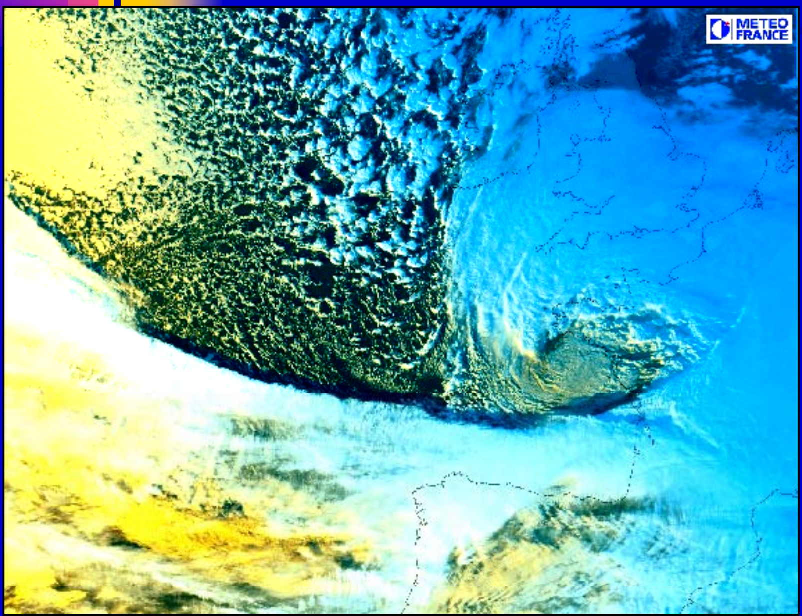
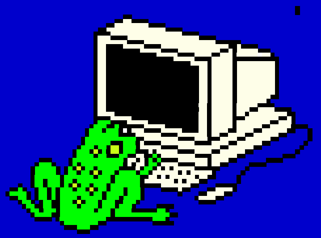
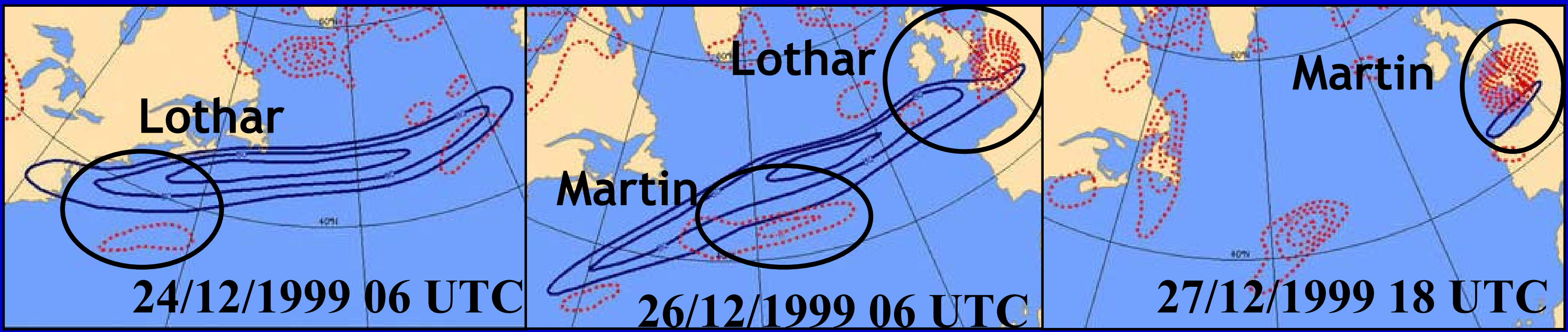
Météo-France

ECMWF Seminar 2003

NWP: from observations and models to weather maps



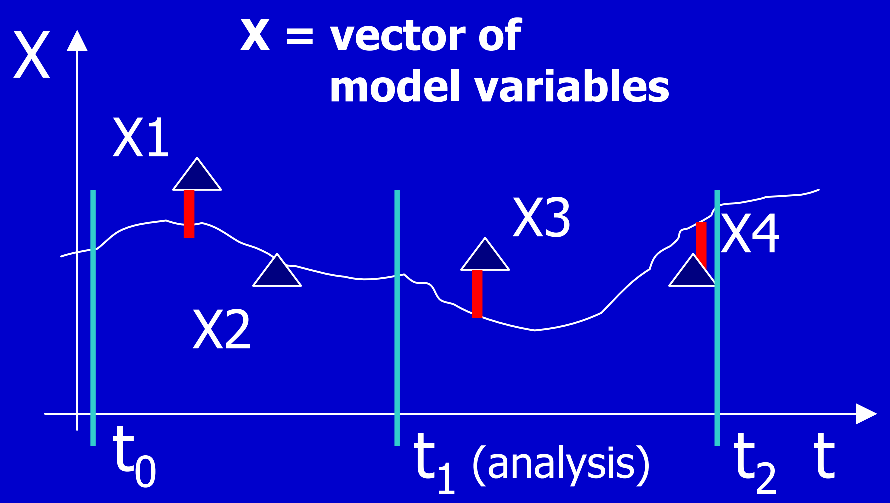




Upper-level wind Low-level vorticity

Data assimilation

Use the available observations together with the model trajectory to provide IC for the forecast or (re-)analyses



Sequential: use observations in small batches, as they become available

Continuous: over a time window, use all observations. Obs at t_2 are used for the analysis between t_0 and t_2 .

Optimisation problem

Need for a statistical approach:

Find the best compromise between various sources of information: observations, background, dynamics/physics of the system

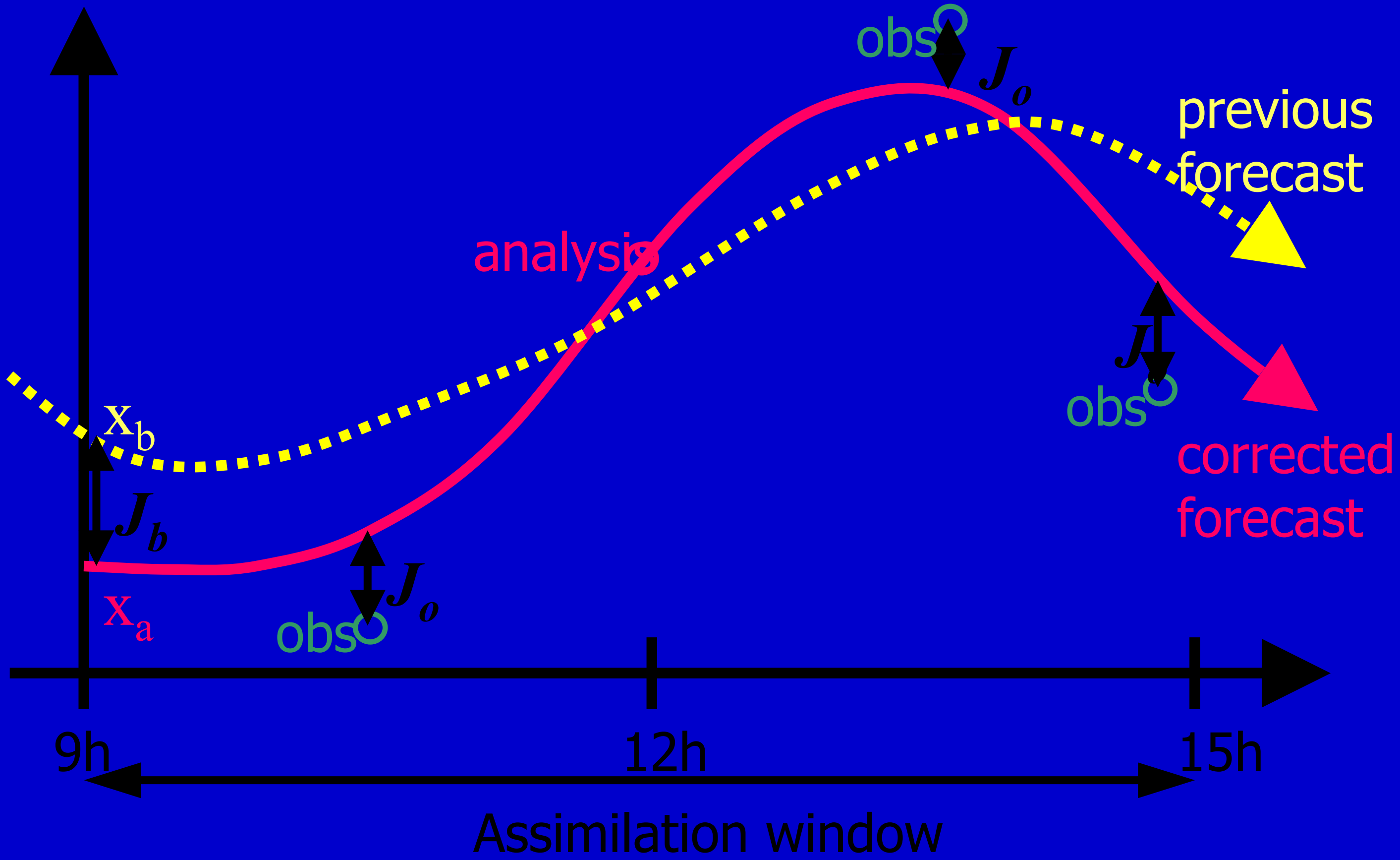
Trust them according to their error statistics

Introduction to 4D-Var

Four-dimensional variational assimilation

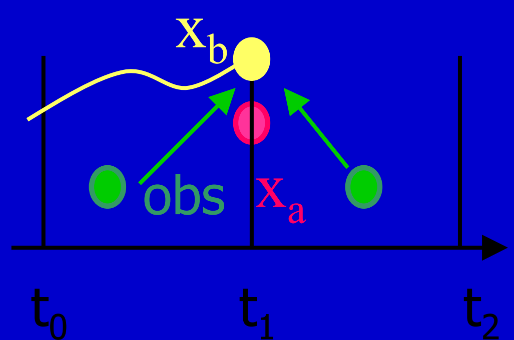
(Le Dimet and Talagrand; Lewis and Derber, 1985)

Principles of 4D-VAR assimilation

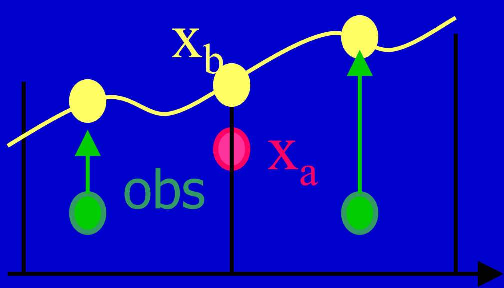


Assumption of perfect model: one looks for the starting point of the trajectory

Approximations to 4D-Var: 3D-Var and 3D-FGAT



- 3D-Var: One looks for the best compromise between the background field and all available observations as if they were at the analysis time



•3D-FGAT: one compares the model to the observations with no approximation, but performs a 3D analysis

Resolution of the optimisation problem

Minimize the cost function

$$J(\mathbf{x}) = \tfrac{1}{2}(\mathbf{x}-\mathbf{x}^b)^{\mathrm{T}}\mathbf{B}^{-1}(\mathbf{x}-\mathbf{x}^b) + \tfrac{1}{2}(\mathbf{y}-H(\mathbf{x}))^{\mathrm{T}}\mathbf{R}^{-1}(\mathbf{y}-H(\mathbf{x})),$$

where

$\mathbf{x}$ is the vector of atmospheric (model) variables, $(10^7/10^8)$

$\mathbf{x}^b$ a « background » for the analysis,

y the vector of observations, $(10^6/10^7)$

H the observation operator, including the model integration,

$\mathbf{B}$ the background error covariance matrix,

$\mathbf{R}$ the observation error covariance matrix.

Solution in the linear case

$$\mathbf{x}_a = \mathbf{x}_b + \mathbf{K}(\mathbf{y} - \mathbf{H}\mathbf{x}_b)$$

With the gain matrix

$$\mathbf{K} = \mathbf{B}\mathbf{H}^T(\mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{R})^{-1}$$

And Analysis error Covariance

$$\mathbf{A} = (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{B}$$

- This is the Optimal least-squares estimator minimum variance for the analysis error
- Or BLUE= Best Linear Unbiased Estimator
- If all errors are Gaussian, then it is also the maximum likelihood estimate

(Lorenc, 1986)

Equivalence with Kalman filter

Optimal sequential assimilation: loop over observation times t_i

Analysis

$$\mathbf{x}_a(t_i) = \mathbf{x}_b(t_i) + \mathbf{K}_i(\mathbf{y}(t_i) - \mathbf{H}_i\mathbf{x}_b(t_i))$$

$$\mathbf{K}_i = \mathbf{P}_i\mathbf{H}_i^T(\mathbf{H}_i\mathbf{P}_i\mathbf{H}_i^T + \mathbf{R}_i)^{-1}$$

Forecast

$$\mathbf{x}_b(t_{i+1}) = M(\mathbf{x}_a(t_i))$$

$$\mathbf{P}_{i+1} = \mathbf{M}_i\mathbf{P}^a_i\mathbf{M}_i^T$$

Equivalence with Kalman filter

At the end of the assimilation window,

same Optimal analysis

obtained by 4D-Var and Kalman filter

(equivalent to the Kalman smoother over the whole assimilation window)

Comparison of the model trajectory to observations performed at « the appropriate time »

Full use of the dynamics over the window to update implicit forecast error matrix

Properties of Variational methods

Can be extended to non-Gaussian errors: Var-QC (Lorenc, Andersson)

Can use a wide range of observations including those with a complex link to atmospheric variables (eg radiances, Andersson et al, 1994)

Efficient use of asynoptic data (Järvinen, Andersson)

Practical Implementation

Tangent linear hypothesis

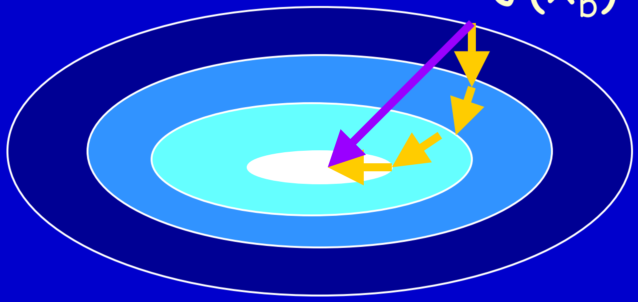
Method can be extended to weakly non-linear problems

One needs y-$\mathcal{H}(\mathcal{M}(x))$ to be able to be linearized around x_b

$$Y - \mathcal{H}(\mathcal{M}(x)) = y - \mathbf{HM}(x - x_b) - \mathcal{H}(\mathcal{M}(x_b)) + \text{second-order terms}$$

Computing technique: Minimisation

minimisation of the cost-function



J = distance to obs and x_b

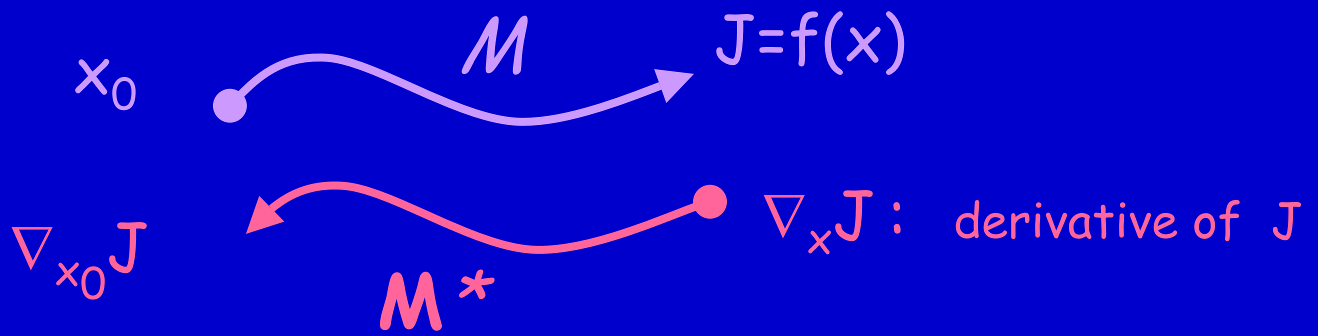
- If operators are linear, cost-function is quadratic and minimisation can be carried out efficiently

- One needs several computations of J and its gradient (first derivative)

⇒ Result = analysis

Computing technique: use of the adjoint

- Variational assimilation needs the adjoint of the operators to compute the gradient of J with respect to the IC
- The adjoint of the forecast model allows to link the gradient at any time to the gradient at the beginning of the assimilation window



« Incremental » formulation

Minimize the cost function

$$J(\delta\mathbf{x}) = \frac{1}{2}\delta\mathbf{x}^{\mathrm{T}}\mathbf{B}^{-1}\delta\mathbf{x} +$$

$$\frac{1}{2}(\mathbf{y} - H(\mathbf{x}^b) - \mathbf{H}\delta\mathbf{x})^{\mathrm{T}}\mathbf{R}^{-1}(\mathbf{y} - H(\mathbf{x}^b) - \mathbf{H}\delta\mathbf{x})$$

where

$\delta\mathbf{x}$ is the vector of « increment » to the background,

$\mathbf{H}$ is the linearized and simplified observation operator.

$\Rightarrow \delta\mathbf{x}$ can have a lower resolution than $\mathbf{x}$

- (Courtier, Thépaut, Hollingsworth, 1994)

Applications of the incremental technique in an operational context

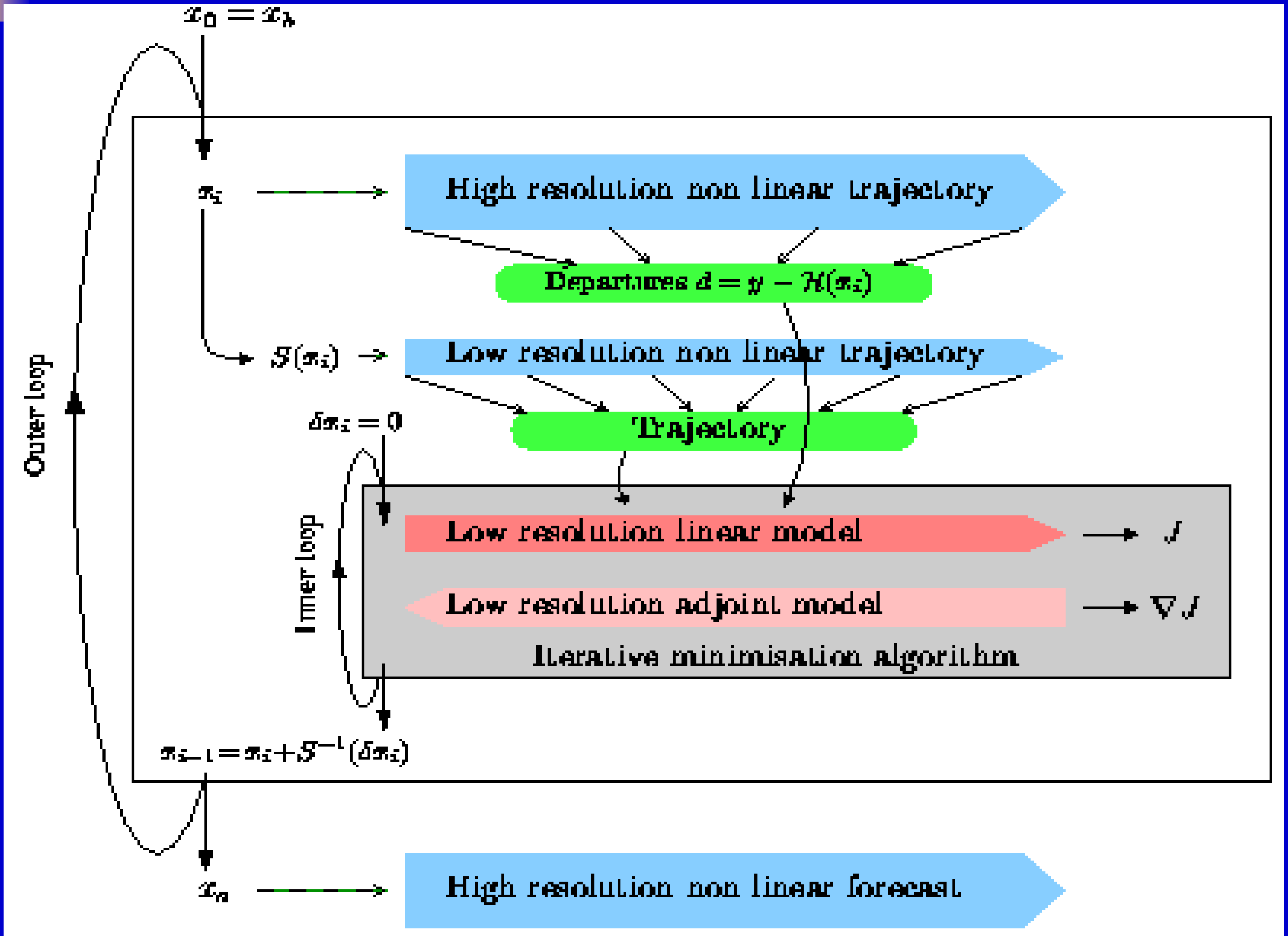


Illustration: Burgers' model

(Liu, 2002)

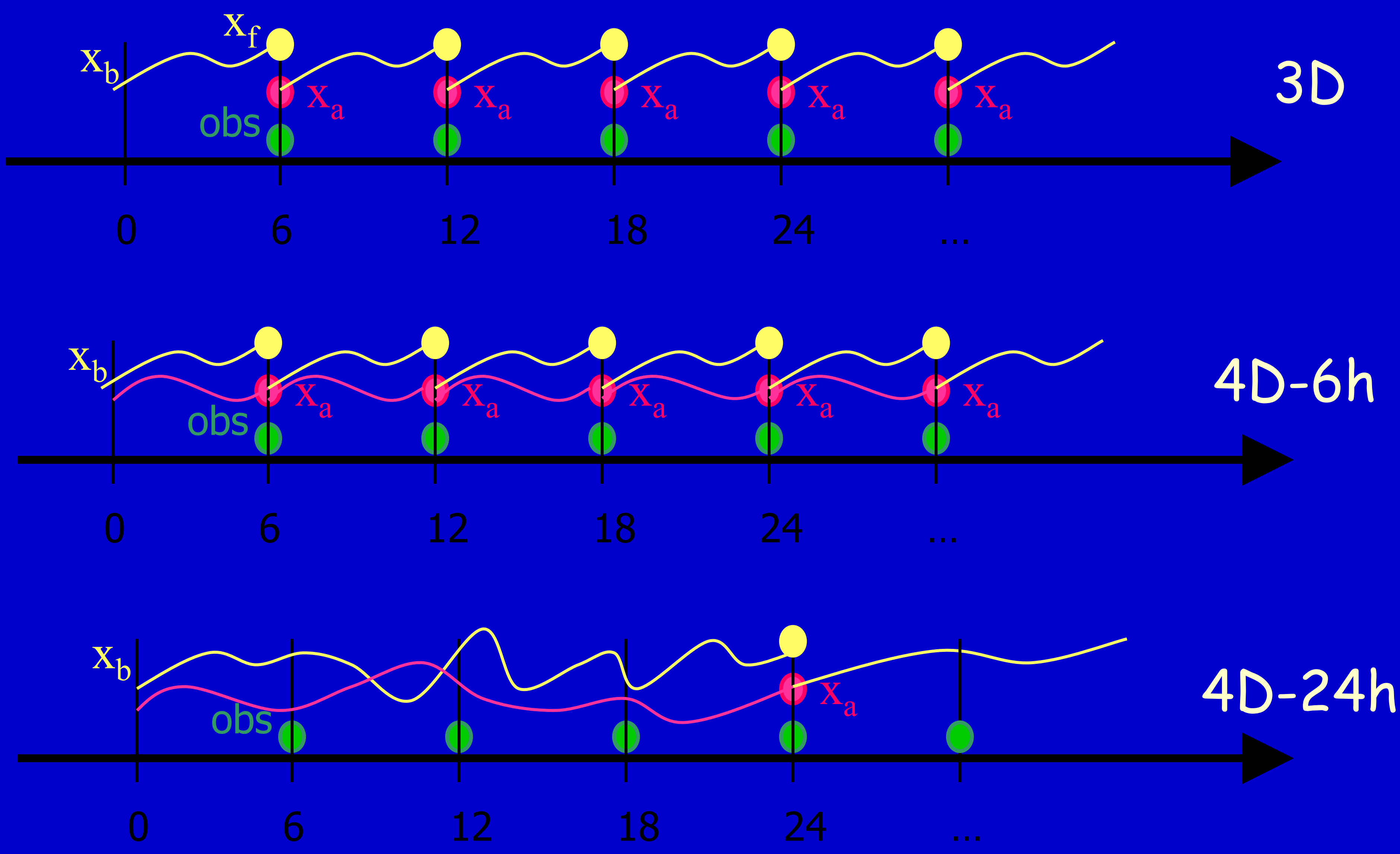
Experimental framework

- 1D circle: Length=8000km. Δx=100km.
- Burgers equation: non-linear advection-diffusion

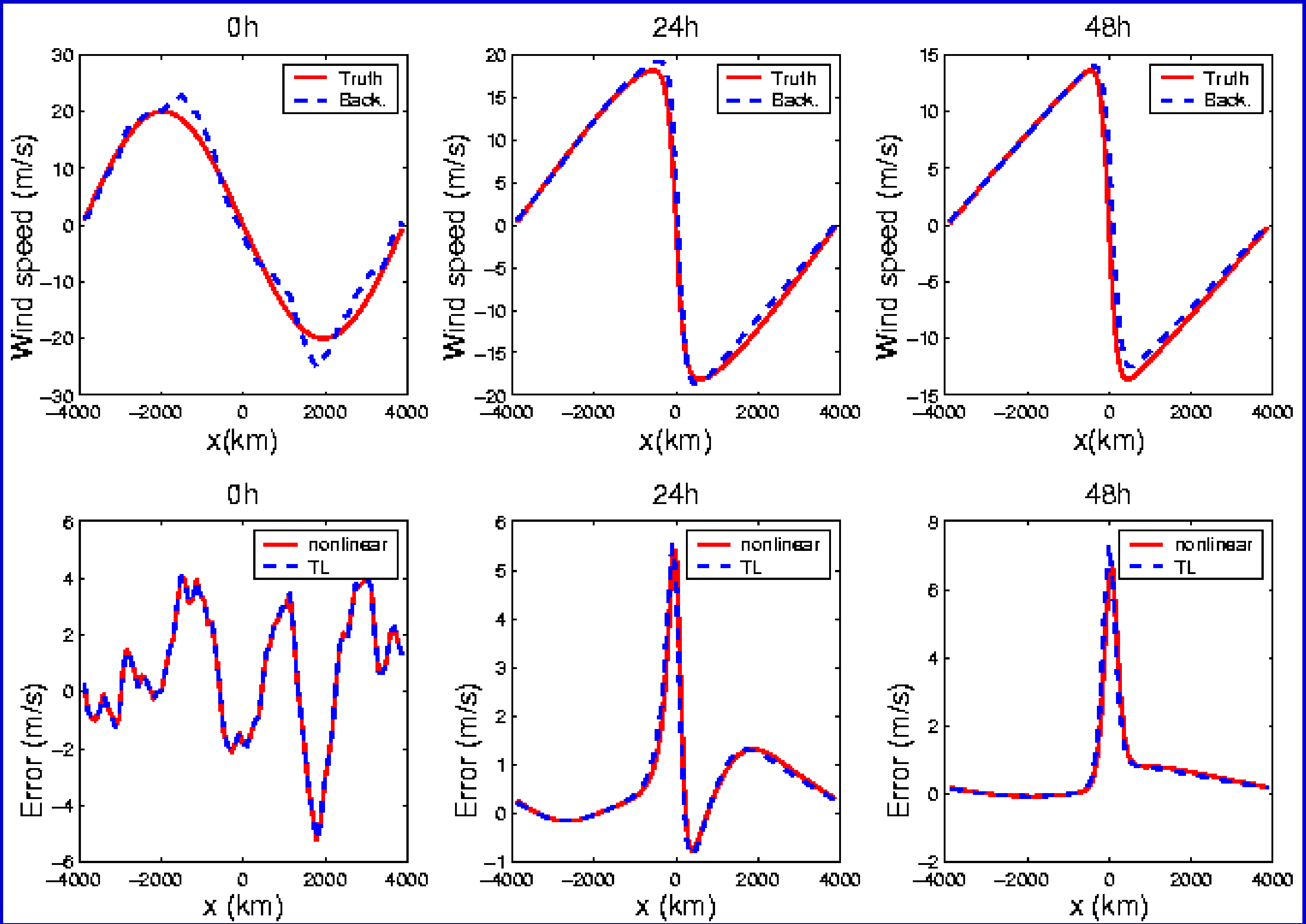
$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \upsilon\frac{\partial^2 u}{\partial x^2}$$

- Background error correlated with $L_b = 208\text{km}$
- Observations available every grid-point, every 6h.
- Errors $\sigma_o = \sigma_{\text{b}} = 2$

Various VAR experiments



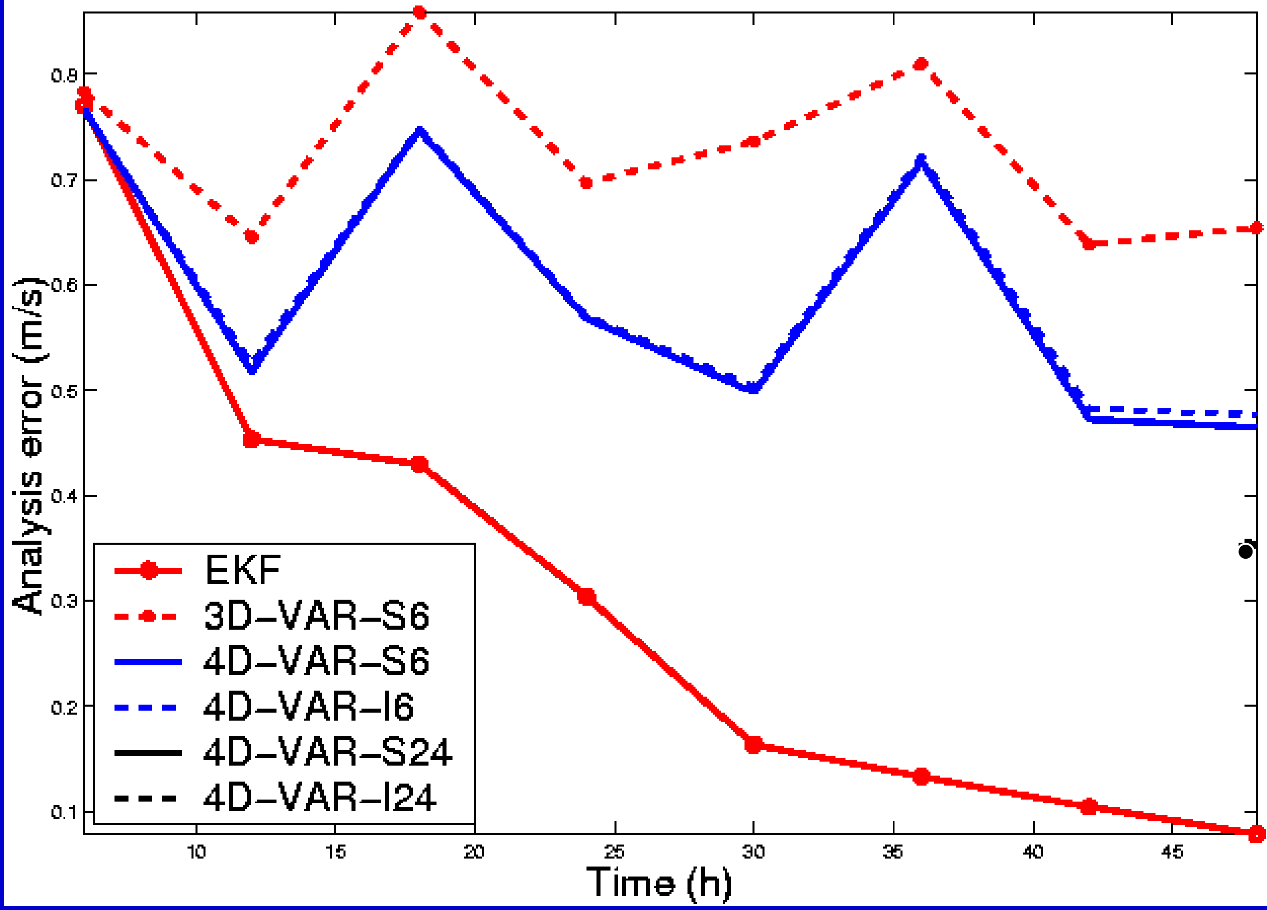
Burgers



0h
24h
48h
Truth
Back.
Wind speed (m/s)
x(km)
nonlinear
TL
Error (m/s)
x (km)

Burgers

- Illustrates the benefit of using 4D-Var in a simple context



Operational results 4D-Var vs 3D-Var

Prior studies in a quasi-operational context

- Zupanski et al, Zou, Huang and Gustafsson: promising results in a limited area model
- Thépaut, Rabier et al:
  - Very simplified version of linearized physics for TL and AD models
  - 4D-Var over 6 and 12h windows better than 3D-Var

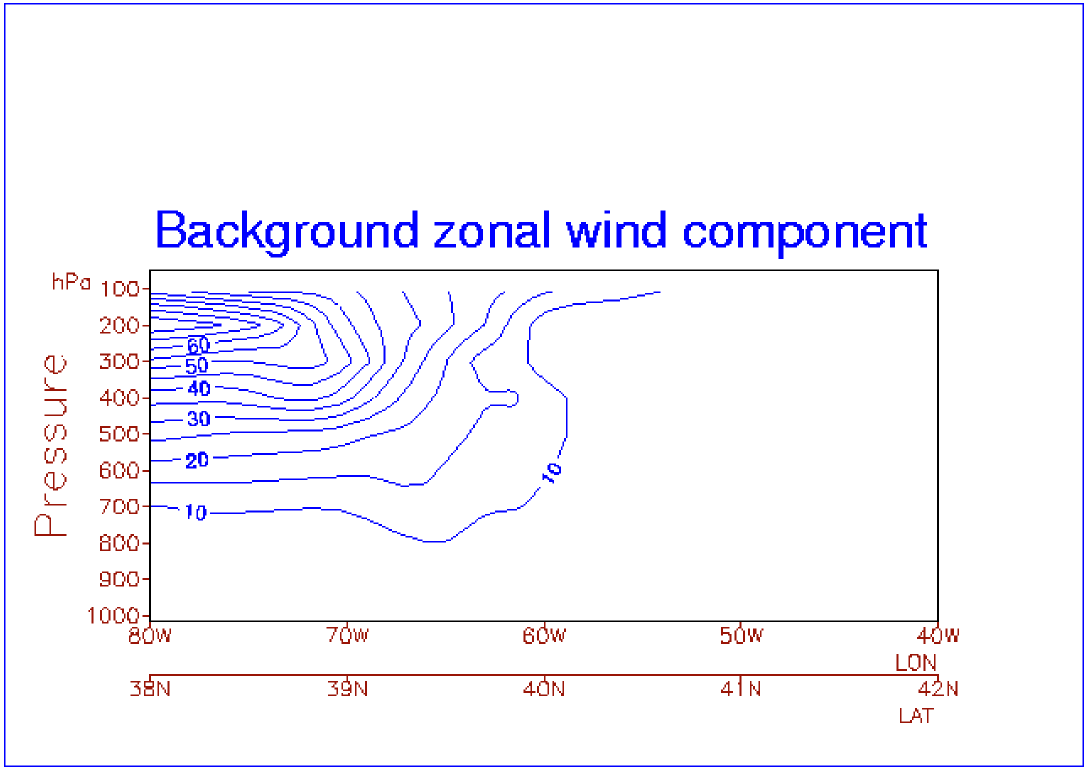
Optimisation of 4D-Var on a 6-hr window: implementation at ECMWF

(Rabier et al, 2000; Mahfouf and Rabier, 2000; Klinker et al,2000)

- Influence of the dynamics
- Impact of linearized physics
- Diagnostics on pre-operational 4D-var

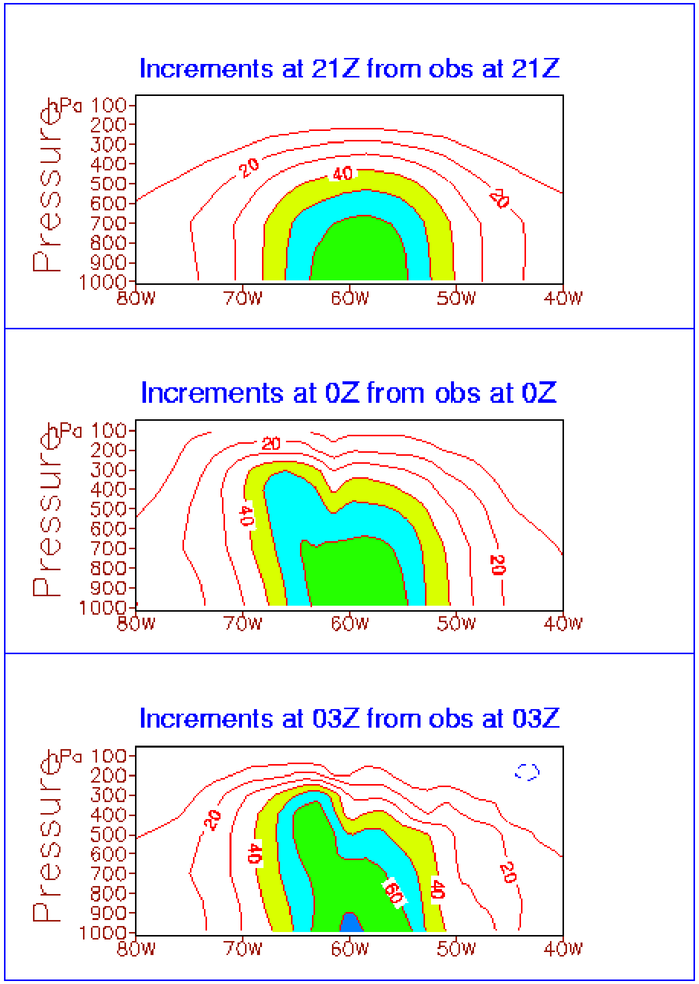
Influence of the dynamics

- Standard baroclinic area in the Atlantic



Influence of the dynamics

- Increments for a single obs
- Dynamics change the increments

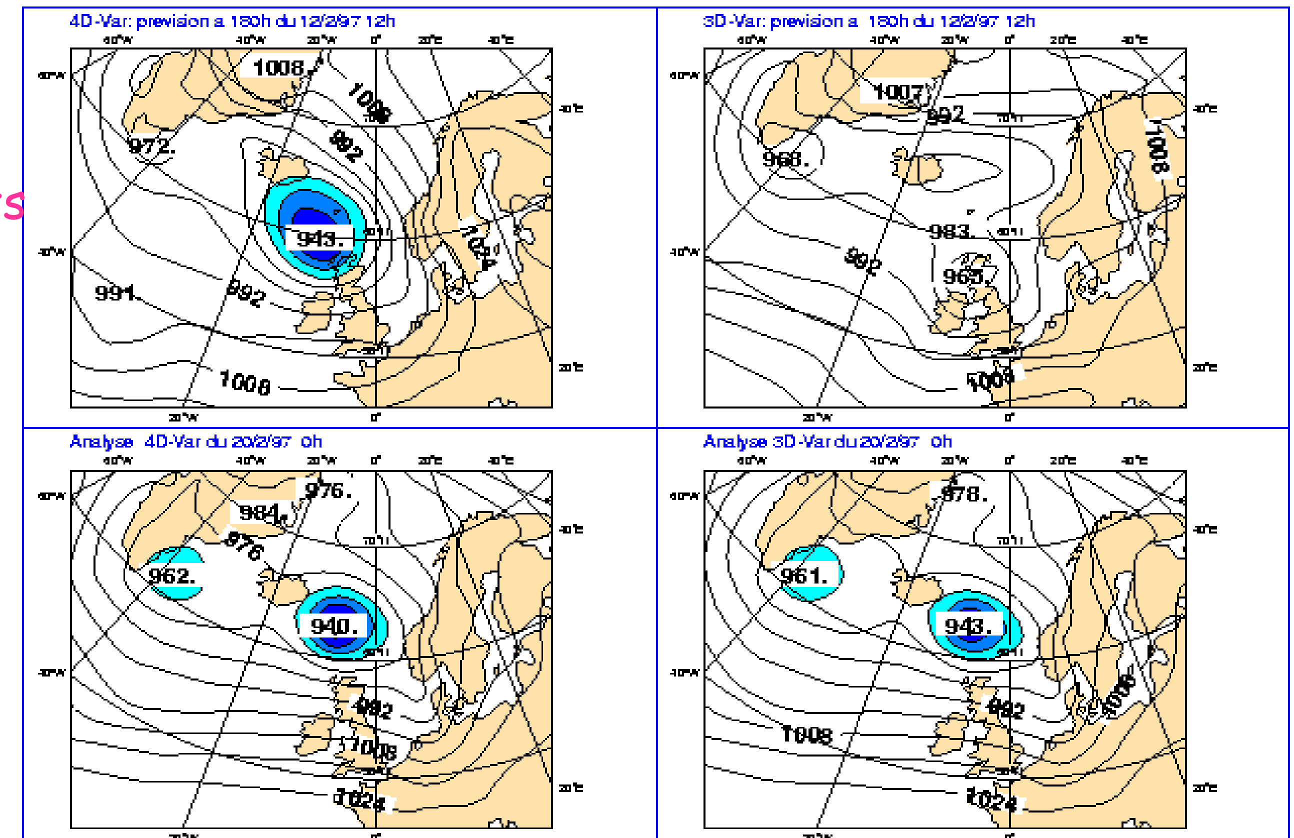


Influence of the dynamics

4D-Var 3D-Var

Forecasts

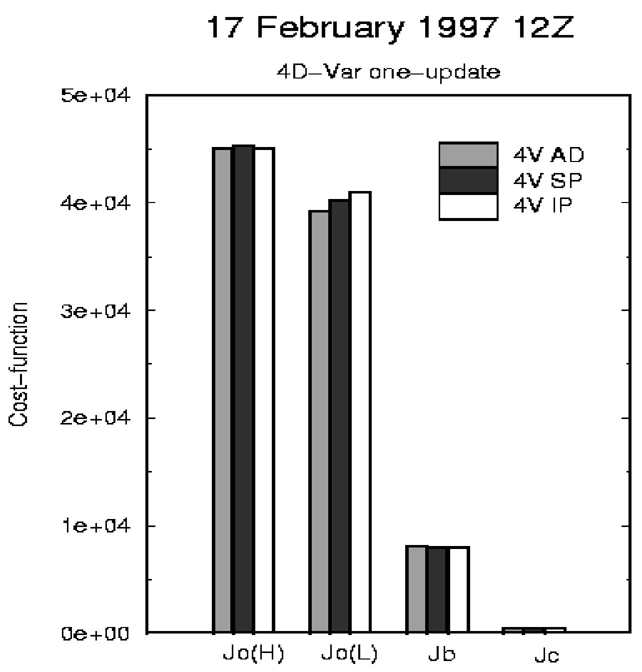
Analyses



- Better forecasts during FASTEX

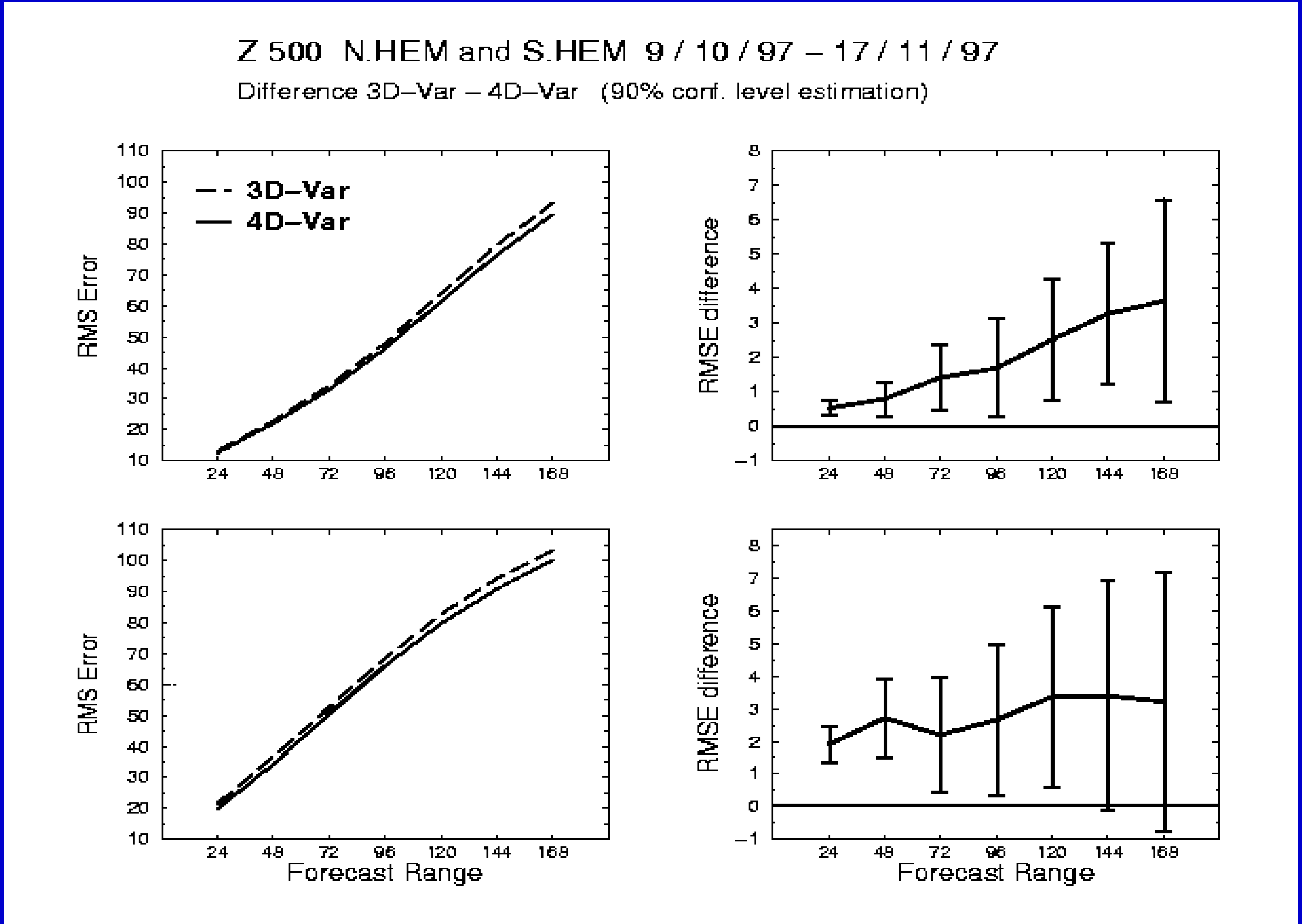
Impact of linearized physics in the minimisation

- Better agreement between minimisation and model integrations at full resolution with full physics
- Less spin-down
- Slightly better forecasts



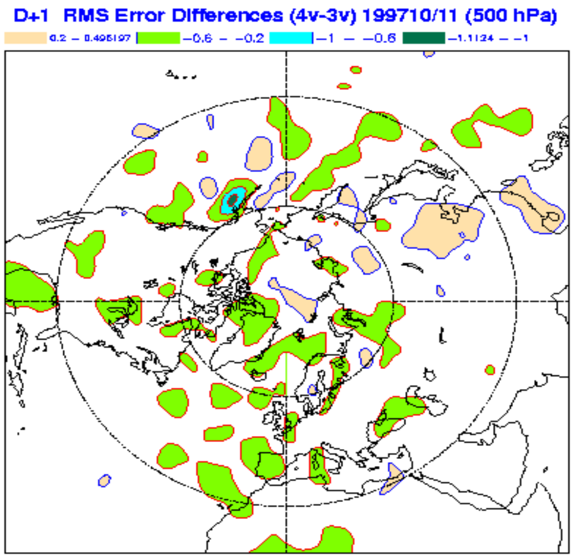
Diagnostics on pre-operational results

- Significantly better scores in both hemispheres



Diagnostics on pre-operational results

- Widespread improvement: differences between rms of 24h errors.



Further Operational developments and results

Operational Developments

- Extensions of 4D-Var

- Extract temporal information through the use of frequent data (Järvinen, Andersson)
- Developments in the Jb formulation (Derber, Fisher)
- 12-hour 4D-Var (Bouttier)
- Better handling of the trajectory (Trémolet)
- Multi-incremental (Veersé and Thépaut)
- Initialising with Digital Filter techniques (Gustafsson, Gauthier and Thépaut)
- Continuous improvement of linearized physics (Janiskova…)

Results at Météo-France

- Re-analysis during FASTEX:
  - Gain of 3m, 0.6 m/s at 300hPa for the RMS error in Geopotential and wind

    over Europe at the 72h range

- Operational implementation in 2000

(Desroziers, Hello, Thépaut, 2003)

Limitations and Perspectives

- Limitations
  - Heavy developments (coding of adjoint)
  - Limits of the Incremental technique
- Perspectives
  - Model error to be included
  - Combination with more probabilistic techniques (eg Ens Kalman filter)
  - Cloud and rain assimilation
  - Challenging issues for a higher-resolution analysis using high density satellite data

The End