
Parametrization of non-convective condensation processes

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By **M. Tiedtke**

European Centre for Medium-Range Weather Forecasts

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REFERENCES

1. THERMODYNAMICS OF MOIST AIR

A brief review of the general thermodynamics of moist air is given here.

Air can be considered as a mixture of ideal gases. The equation of state is

$$p = \rho R T \quad (1)$$

where p is pressure, ρ is density, T is absolute temperature and R is the gas constant

$$R = c_p - c_v \quad (2)$$

c_p and c_v are the heat capacities at constant pressure and at constant volume, respectively. Equation (1) is also valid for moist air, provided that the actual temperature is replaced by the virtual temperature

$$p = \rho R T_v \quad (3)$$

The virtual temperature T_v is given by

$$T_v = T(1 + \eta q) \quad (4)$$

where $\eta = \frac{1}{\varepsilon} - 1 = 0.608$ and ε is the ratio of the gas constants for dry air, and for water vapour $\varepsilon = R/R_v = 0.622$. q is the specific humidity $q = \rho_{\text{vap}}/\rho$ (ρ_{vap} is the density of water vapour and ρ is the density of moist air). The equation of state for moist air is valid since it is assumed that water vapour also behaves



as an ideal gas

$$p_{\text{vap}} = \rho_{\text{vap}} R_{\text{vap}} T \quad (5)$$

where the subscript ‘vap’ refers to water vapour.

The first law of thermodynamics is

$$\begin{aligned} \frac{ds}{dt} &= \frac{Q}{T} = \frac{1}{T} \frac{de}{dt} + \frac{p}{T} \frac{dv}{dt} = \frac{c_v}{T} \frac{dT}{dt} + \frac{R}{v} \frac{dv}{dt} \\ &= \frac{1}{T} \frac{dh}{dt} - \frac{v}{T} \frac{dp}{dt} = \frac{c_p}{T} \frac{dT}{dt} - \frac{R}{p} \frac{dp}{dt} \end{aligned} \quad (6)$$

s is the specific entropy, e is the specific internal energy ($e = c_v T$), h is the specific enthalpy ($h = c_p T$), Q is the heating rate by external sources and v is the specific volume ($v = \frac{1}{\rho}$)

From (6) we get

$$\begin{aligned} s &= c_v \ln T + R \ln v + \text{const} \\ s &= c_p \ln T - R \ln p + \text{const} \end{aligned} \quad (7)$$

For dry adiabatic processes, i.e. at $\frac{ds}{dt} = 0$, the potential temperature θ is conserved

$$\frac{d\theta}{dt} = 0, \quad \theta = T \left(\frac{p_0}{p} \right)^{R/c_p} \quad (8)$$

The specific entropy can be expressed in terms of potential temperature as

$$s = c_p \ln \theta + \text{const} \quad (9)$$

The dry adiabatic lapse rate is

$$\frac{dT}{dz} = -\frac{g}{c_p} \quad (10)$$

where g is the gravity of the earth

If the exchange of heat through condensation processes is considered, the thermodynamic equation (6) becomes

$$c_p \frac{dT}{dt} + \frac{d(r_{\text{sat}} L)}{dt} - \frac{RT}{p} \frac{dp}{dt} = Q \quad (11)$$

where L is the latent heat and r_{sat} is the saturation mixing ratio.

The Clausius–Clapeyron equation gives the slopes of the curves for the saturation water vapour pressure e_{sat} (Fig. 1).

$$\frac{de_{\text{sat}}}{dT} = \frac{L}{T(v_{\text{vap}} - v_{\text{water}})} \quad (12)$$

where v_{vap} and v_{water} are the specific volumes for water vapour and for water, respectively. Integration of equation (12) gives (assuming that $L = \text{const}$, $v_{\text{water}} \ll v_{\text{vap}}$ and $v_{\text{vap}} = R_v T / e_{\text{sat}}$) the Tetens formula (Murray (1967))

$$e_{\text{sat}} = e_{0\text{sat}} \exp \left[a \frac{T - T_0}{T - b} \right] \quad (13)$$

where

$$e_{0\text{sat}} = 6.1078 \text{ mb}$$

$$a = 17.27$$

$$T_0 = 273.16 \text{ K}$$

$$b = 35.86 \text{ K}$$

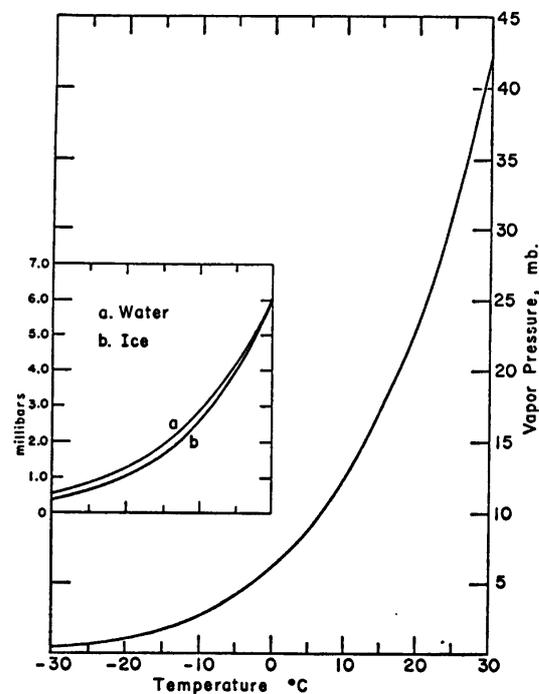


Figure 1. Phase diagram for water is the (p, T) plane. The inset shows the vapour–ice equilibrium (lower curve) and the vapour–liquid equilibrium (upper curve) at subfreezing temperatures.

2. CLOUD PHYSICAL PROCESSES

As was done for the thermodynamics, a short summary of the principles of cloud physics shall be given here.

There are essentially two kinds of process associated with clouds that must be considered in numerical models: the formation of clouds and the release of precipitation. The formation of clouds is due to condensation processes, i.e. a change of phase from water vapour to water droplets or to ice crystals and vice versa. Precipitation, on the other hand, is released by transformation of small cloud droplets into larger rain drops [Fig. 2](#).

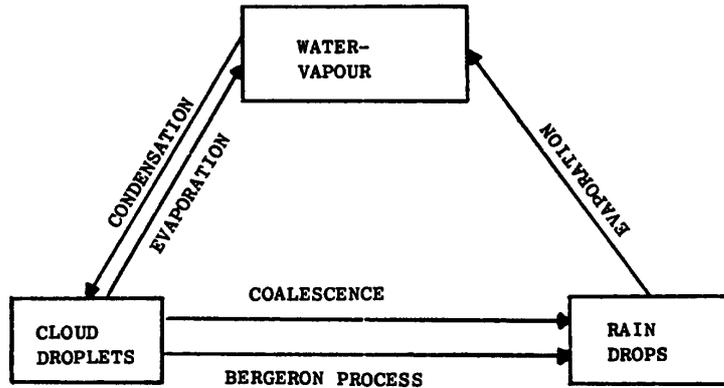


Figure 2. Schematic diagram on condensation and precipitation processes

2.1 Condensation of water vapour

We consider here the chemical equilibrium of liquid water and of water vapour. Measurements show that, for the case of pure liquid water and of pure water vapour, the equilibrium saturation vapour pressure over convex surfaces is higher than over plane surfaces according to

$$e_r = e_\infty \exp\left[\frac{\alpha}{r}\right] \quad (14)$$

where e_r is the saturation vapour pressure of a droplet of radius r , and e_∞ is the saturation vapour pressure over a plane surface. The fraction e_r/e_∞ increases with decreasing droplet radius, as shown in Table 1.

 TABLE 1. FRACTION e_r/e_∞ FOR DIFFERENT SIZES OF DROPLETS AT 0° C

r (μm)	10^{-2}	10^{-1}	1	10
e_r/e_∞	1.128	1.012	1.0012	1.0001

Thus a pure droplet of radius 10^{-2} μm needs 12.8% supersaturation in order to be in equilibrium with the surrounding water vapour. Since newly formed droplets are necessarily very small, large amounts of supersaturation are needed to start the condensation of water vapour in pure air.

In the atmosphere, however, relative humidities above 101% are rarely observed. The reason is that various kinds of small particles that serve as condensation nuclei are always present in the atmosphere. Due to the hygroscopic effect of these particles, a smaller equilibrium vapour pressure is needed for the very small droplets. We can, therefore, always assume that condensation takes place whenever the relative humidity exceeds 100%.

The droplets generated by condensation processes are very small. The size spectrum of cloud droplets in the presence of condensation nuclei ranges from 0.1 μm to 10 μm . Fig. 3 shows droplet spectra of various kinds of clouds. As can be seen, non-precipitating clouds as Sc and Cu have a rather narrow distribution. This distribution can be explained by droplet growth due to condensation, which is large for the smallest droplets and decreases for the larger droplets, thus leading to a narrow spectrum.

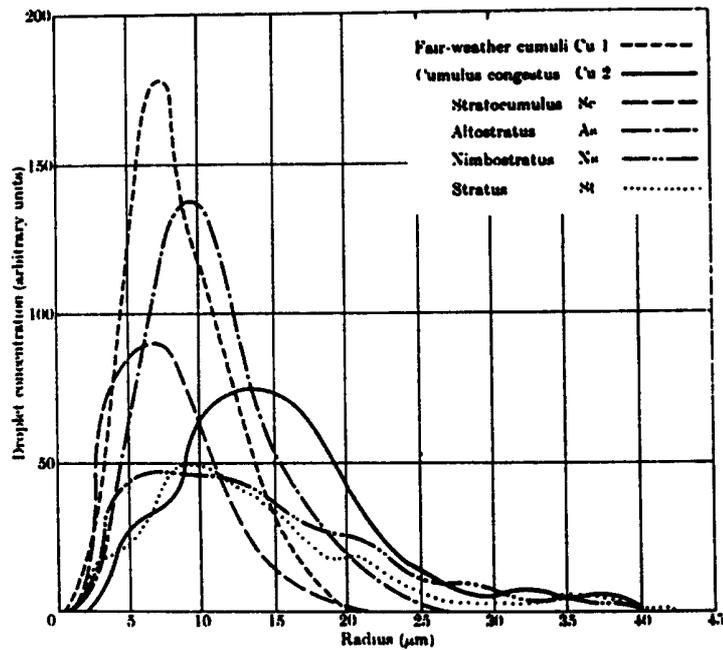


Figure 3. The mean droplet-size distributions of various cloud types. (From Diem (1948).)

2.2 Release of precipitation

The water droplets formed by condensation of water vapour are very small compared to raindrops in drizzle and in rain, which have a size of 50–100 μm and 1000 μm , respectively. Therefore, processes other than condensation must be present to form rain drops. The two processes that are believed to be the most efficient in producing rain drops are coalescence and the Bergeron process.

Coalescence occurs when water droplets are moving with different fall speeds due to their different sizes. By collision and fusion the larger droplets increase in size. The growth rate of a collector droplet may be prescribed as

$$\frac{dD}{dt} = \frac{Em}{2\rho_{\text{water}}} V \quad (15)$$

where D is the droplet diameter, V is the fall velocity of the collector relative to the small droplets, ρ_{water} is the density of the droplet and m is the cloud water content. E is the efficiency of collection. The efficiency of coalescence depends on the size spectra of the droplets and is more efficient for a broader spectrum. Coalescence becomes efficient only if droplets with radii larger than 10 μm are present. Coalescence results in a significant broadening of the spectrum of the cloud droplets. However, large rain drops are produced only in clouds of large vertical extents (depths of several km) which enables falling drops to collect enough small droplets. The coalescence process can, therefore, release precipitation in deep water clouds. This precipitation process is important in low latitudes, where deep clouds of high water content are present, but it is not very efficient at higher latitudes.

Bergeron Process. The most efficient process of precipitation at higher latitudes is associated with the formation of ice particles at temperatures below about -15°C . Generally, ice particles form by freezing of supercooled water droplets on various freezing nuclei. Freezing nuclei play a similar role to that of condensation nuclei for the condensation process. They initiate freezing at temperatures of around -15°C . Ice particles that are already formed grow very fast by collecting supercooled droplets because the equilibrium vapour pressure over ice is lower than

over water (see Fig. 1). The possibility that precipitation is initiated by the formation of ice crystals at low temperatures was first mentioned by Bergeron (1935).

Spectra of rain drops. Spectra of rain-drop sizes observed in the atmosphere are often described analytically. An analytical distribution that is often referred to is that derived by Marshall and Palmer (1948) from observational data

$$N = N_0 \exp[-\lambda D]$$

where N is the number of raindrops per unit volume of air, and N_0 and λ are empirical parameters.

3. PARAMETRIZATION OF NON-CONVECTIVE CONDENSATION AND PRECIPITATION PROCESSES IN NUMERICAL MODELS

In this paragraph we consider clouds associated with the large-scale flow as resolved in numerical models. Typical examples of this cloud type are the clouds associated with fronts. As for other processes, condensation processes and precipitation processes can only be parametrized by means of the large-scale variables given at the model's grid points, which are available at very coarse resolution of, say, about 100 km in the horizontal and 1 km in the vertical, and in time intervals of 10 min. The most important parameters are humidity, temperature and vertical motion. The vertical velocity determines the condensation rate and, therefore, the supply of liquid water content. Temperature also controls the liquid water content, because the maximum amount of vapour is a function of temperature. The temperature distribution in a cloud is also important for the type of precipitation—rain or snow—as mentioned before.

The parametrization of cloud processes must be kept very simple, since the numerical integrations must be carried out in a reasonable time. Two simple parametrization schemes that are used in numerical models are described here. In the first scheme, only condensation of water vapour is considered while clouds are not parametrized. This scheme is frequently used in numerical models. In the second scheme, the cloud water is predicted and the release of precipitation is parametrized in a simple way.

3.1 Scheme without cloud stage

In this scheme the cloud stage is skipped and it is, therefore, assumed that all the condensed water vapour is immediately converted into precipitation. This scheme is easily implemented in a numerical model. We consider the model equations for the large-scale values of temperature and moisture which, in the case of a grid-point model, prescribe the time changes of the temperature T and specific humidity r at the grid points.

$$\begin{aligned}\frac{\partial T}{\partial t} &= A(T) + \frac{L}{c_p} C \\ \frac{\partial r}{\partial t} &= A(r) - C\end{aligned}\tag{16}$$

The terms $A(T)$ and $A(r)$ include all the adiabatic and diabatic processes, except for the condensation of water vapour. The condensation rate C is the rate at which the saturation mixing ratio r_{sat} changes in saturated air.

$$C = \frac{dr_{\text{sat}}}{dt}$$



In numerical models the condensation rate is generally determined diagnostically. Therefore, preliminary values T^* and r^* are first predicted by neglecting the effects of condensation

$$\begin{aligned} T^* &= T^{n-1} + 2\Delta t A(T) \\ r^* &= r^{n-1} + 2\Delta t A(r) \end{aligned} \quad (17)$$

If the air is found to be supersaturated

$$r^* > r_{\text{sat}}(T^*)$$

T^* and r^* are adjusted to their saturation values, which are then the final values T^{n+1} and r^{n+1} at time step $n+1$

$$\begin{aligned} T^{n+1} &= T^* + \frac{L}{c_p} \frac{r^* - r_{\text{sat}}(T^*)}{1 + \frac{L}{c_p} \frac{dr_{\text{sat}}(T^*)}{dT}} \\ r^{n+1} &= r^* + \frac{r^* - r_{\text{sat}}(T^*)}{1 + \frac{L}{c_p} \frac{dr_{\text{sat}}(T^*)}{dT}} \end{aligned} \quad (18)$$

?These corrections follow from

$$\Delta T = -\frac{L}{c_p} \Delta r, \quad \Delta r = r_{\text{sat}}(T^{n+1}) - r^*$$

where the saturation value r_{sat} is that for the corrected temperature which is approximated as

$$r_{\text{sat}}(T^{n+1}) \cong r_{\text{sat}}(T^*) + \frac{dr_{\text{sat}}}{dT}(T^*) \Delta T$$

The condensation rate C is therefore given as

$$C = \frac{1}{2\Delta t} \frac{r^* - r_{\text{sat}}(T^*)}{1 + \frac{L}{c_p} \frac{dr_{\text{sat}}}{dT}} \quad (19)$$

As the assumption is made that all the condensed water vapour immediately falls out as rain, the precipitation rate is (units : $\text{m}_{\text{H}_2\text{O}}\text{sec}^{-1}$)

$$P = \frac{1}{\rho_{\text{water}_0}} \int_0^{\infty} C \rho \, dz$$

where ρ_{water} is the density of water.

3.2 A cloud parameterization scheme

Most numerical models consider the condensation processes as described before. The cloud phase is skipped by assuming that the condensed water vapour falls immediately out as precipitation. Although the release of latent heat seems the most important process for the large-scale flow, the formation of clouds and the release of precipitation may also be important as they are a part of the hydrological cycle. Consequently there have been several attempts to parametrize clouds and precipitation processes, notable schemes are those by Kessler (1969) and Ogura and Takahashi (1971). Kessler's scheme is designed for warm clouds, since it considers the release of precipitation to be mainly due to coalescence, whereas Ogura and Takahashi designed their scheme for cumulus clouds. The scheme described here takes into account the two main processes important for the release of precipitation—coalescence and the Bergeron process. This scheme has recently been proposed by Okland while staying at the ECMWF. The advantage of this scheme is that it is rather inexpensive for computer calculations and that it takes the main precipitation processes into account. As the scheme considers cloud water as a further variable, the prognostic equations are, therefore

$$\begin{aligned}\frac{\partial T}{\partial t} &= A(T) + \frac{L}{c_p}(C - E_{\text{cld}} - E_{\text{rain}}) \\ \frac{\partial r}{\partial t} &= A(r) - (C - E_{\text{cld}} - E_{\text{rain}}) \\ \frac{\partial r_{\text{cld}}}{\partial t} &= A(r_{\text{cld}}) + C - E_{\text{cld}} - E_{\text{rain}} \\ \frac{\partial P}{\partial t} &= \frac{1}{\rho_{\text{water}_z}} \int_0^{\infty} (G_{\text{rain}} - E_{\text{rain}})\end{aligned}$$

r and r_{cld} are the mixing ratios for water vapour and cloud water, respectively. E_{cld} and E_{rain} are the evaporation rates of cloud water and rain water and G_{rain} is the rate of release of precipitation due to conversion from cloud water to rain water.

The parametrization of condensation C is done in the same way as described before, and the evaporation of cloud water E_{cld} due to increase of the saturation mixing ratio is similarly calculated.

The release of precipitation is parametrized as follows. Two types of precipitating clouds are considered.

- (a) Clouds with low temperatures at cloud top: $T_{\text{top}} < T_{\text{crit}}$ with $T_{\text{crit}} \sim -15^\circ \text{K}$. For this cloud type it is assumed that the total liquid cloud water is immediately released as precipitation.
- (b) Clouds with temperatures at top above T_{crit} : $T_{\text{top}} > T_{\text{crit}}$ but with a high cloud water content $W_{\text{cld}} > W_{\text{crit}}$, with $W_{\text{crit}} \sim 2 \text{ mm}_{\text{H}_2\text{O}}$, where

$$W_{\text{cld}} = \frac{1}{\rho_{\text{water}_{\text{base}}}} \int_{\text{base}}^{\text{top}} r_{\text{cld}} \rho \, dz$$

It is further assumed that cloud water in excess of this critical value is instantaneously removed as precipitation.

Besides the condensation processes and precipitation processes, the following processes are also considered:

- (a) Evaporation of rain E_{rain}
- (b) Collection of cloud water in non precipitating cloud layers by precipitation from above $(G_{\text{rain}})_{\text{coll}}$

These processes are parameterized follows Kessler (1969).



$$E_{\text{rain}} = C_1(r_{\text{sat}} - r)p^a$$

$$(G_{\text{rain}})_{\text{coll}} = C_2 r_{\text{cld}} P^b$$

where C_1 , C_2 , a and b are constants.

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