APPLICATIONS OF COST-LOSS MODELS

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We examine the potential economic benefit of weather forecasts using a simple cost-loss model of economic decision making. Consider a decision maker who is sensitive to a specific adverse weather event X. If X occurs and no action has been taken, the decision maker incurs a loss L. However, the decision maker has the option of taking protective action to prevent this loss; taking action incurs a cost C irrespective of the outcome. For example, the weather event might be the occurrence of ice on roads and the action "to grit the roads"; C would be the cost of the gritting procedure and L would be the economic loss as a result of traffic delays and accidents on icy roads. The situation is summarised in Table 1.

TABLE 1. COST AND LOSS FOR DIFFERENT OUTCOMES

		Occurs	
		Yes	No
Action	Yes	C	C
	No	L	0

Over a large number of cases, let $\bar{o}$ be the fraction of occasions when X occurs. If the decision maker always protects, the cost will be C on every occasion, so the average expense is $E_{\text{always}} = C$, while if action is never taken, L will be incurred only when X occurs, so the average expense will be $E_{\text{never}} = \bar{o}L$. If $\bar{o}$ is known, but the decision maker has no additional forecast information then the optimal strategy is always to protect if $E_{always} < E_{\text{never}}$, so the mean expense is:

$$E_{\text{climate}} = min\{C, \bar{o}L\}. \tag{1}$$

Given perfect knowledge of the future weather, the decision maker would need to take action only when the event was going to occur. The mean expense would then be

$$E_{\text{perfect}} = \bar{o}C. \tag{2}$$

A deterministic forecast system gives a simple yes or no simply prediction for X to occur. The performance of the system can be summarised in a contingency table which shows the fraction of correct and incorrect forecasts of X (Table 2). The mean expense of using the forecasts is obtained by multiplying the corresponding cells of Tables 1 and 2:

$$E_{\text{forecast}} = aC + bC + cL \tag{3}$$

The difference in expense between E_{forecast} and E_{climate} is a measure of the value of the forecasts to the decision

TABLE 2. CONTINGENCY TABLE FOR FORECAST AND OCCURRENCE OF BINARY EVENT

		observed		
		Yes	No	
forecast	Yes	a	b	a+b
	No	c	d	c+d
		$a+c = \bar{o}$	$b+d = 1-\bar{o}$	a+b+c+d=1

maker. We define the relative value V of a forecast system as the reduction in expense as a proportion of that which would be achieved by a perfect forecast:

$$V = \frac{E_{\text{climate}} - E_{\text{forecast}}}{E_{\text{climate}} - E_{\text{perfect}}}. \tag{4}$$

Maximum relative value V=1 will be obtained from a perfect forecast system, while V=0 for a climate forecast. If V>0 then the user will benefit from the system.

Substituting from Eqs. (1), (2) and (3) into Eq. (4) and using hit rate $H = a/(a+c)$ and false alarm rate $F = b/(b+d)$, V can be written as

$$V = \frac{min(C/L, \bar{o}) - F(C/L)(1-\bar{o}) + H\bar{o}(1-C/L) - \bar{o}}{min(C/L, \bar{o}) - \bar{o}(C/L)} \tag{5}$$

This expression for V shows that the economic value of a forecasting system depends not only on the performance of the system (H and F), but also on the event ($\bar{o}$) and on the user (C/L).

The use of deterministic forecasts is straightforward: take action whenever X is forecast. In contrast, probability forecasts require an additional choice to be made: the decision maker must decide how high the forecast probability should be for him to take action. The choice of a threshold probability, p_t, converts the probability forecast to a deterministic one: forecasts with probability higher than p_t become "yes" forecasts, the remainder are "no" forecasts. By varying p_t from 0 to 1, a sequence of values for H and F and hence of V can be derived; the user can then choose that value of p_t which results in the largest V. Note that since V also depends on $\bar{o}$ and C/L, the appropriate value of p_t will be different for different users and different weather events.

For a given weather event and forecast system, $\bar{o}$, H and F are fixed and V depends only on C/L. Fig. 1 shows V as a function of C/L for four precipitation events for the ECMWF Ensemble Prediction System (EPS). The relative value of the EPS probability forecasts is compared to that of the deterministic control and ensemble mean (EM) forecasts. For all four events, the EPS probability forecasts have greater value for a wider range of users than either of the deterministic forecasts. For the more extreme precipitation events, the benefit of the probability forecasts over the deterministic forecasts is substantial for almost all users. This is a result of the ability of different users being able to choose different probability thresholds appropriate to their C/L. For example, a user with very large potential losses (hence small C/L) would need to take action at a low probability, while a user with relatively high costs (large C/L) would only act if the probability of X was high.

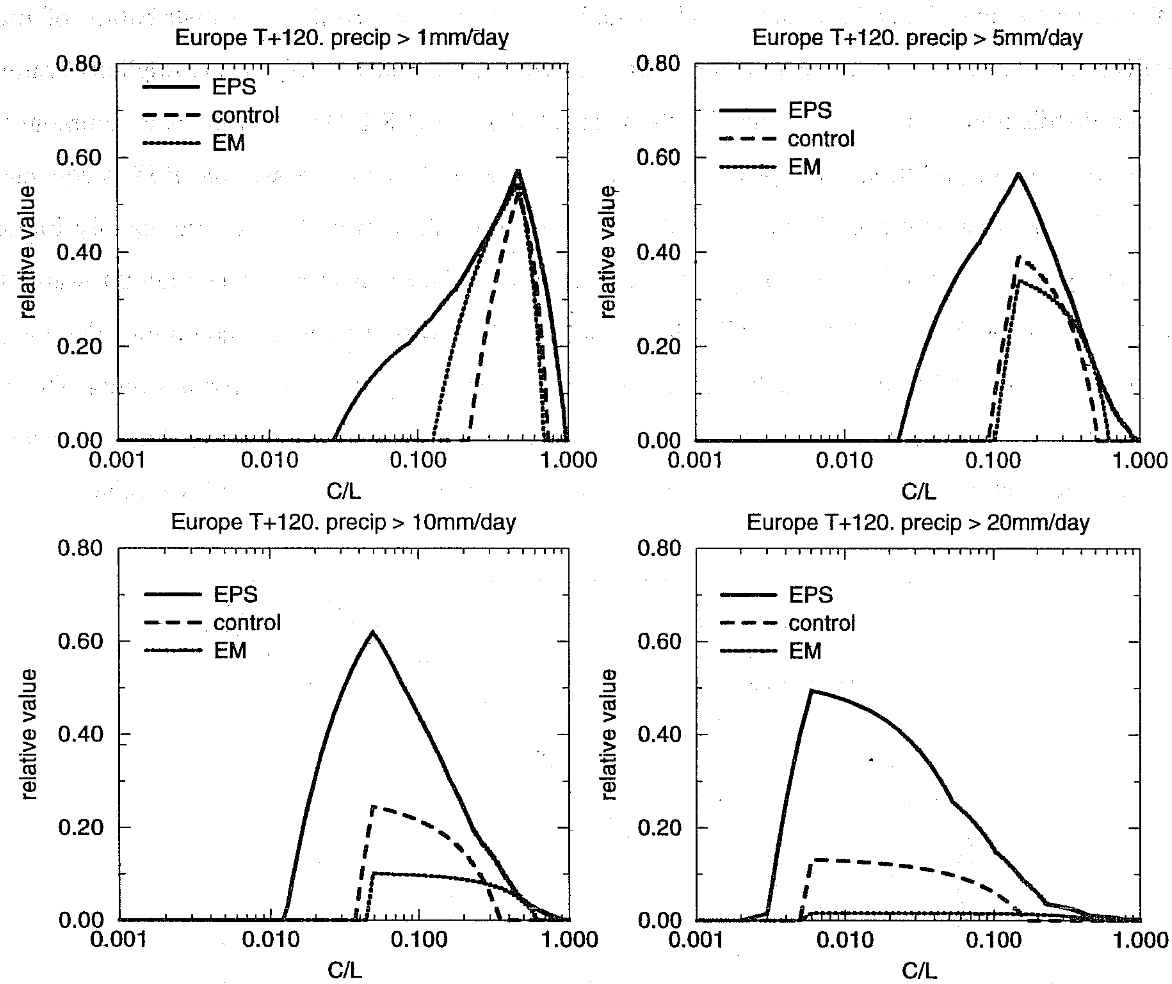

Figure 1: Relative Value for different users (C/L) for the deterministic control (dashed line) and EPS ensemble mean (dotted line) forecasts and for the EPS probability forecasts (solid line) for four precipitation events over Europe at day 5 for winter 1999-2000.

In fact, it can be shown that for reliable probability forecasts, the optimal choice of probability threshold is $p_t = C/L$

The value curves of Fig. 1 show how the benefits of a forecast system vary for different users. While some users receive no benefit from the EPS, others will gain over 50% of the benefit from a (hypothetical) perfect deterministic forecast. Maximum value V_{max} is obtained by users with $C/L = \bar{o}$. The relative value of different forecast systems can also vary between users. For instance, users with small C/L will gain more benefit from the control forecast than from the ensemble mean, whereas those users with larger C/L will prefer the ensemble mean.

If C/L is known for all users, then it is straightforward to calculate the overall value of a forecast system to that set of users. An improvement in overall value would indicate that the group of users as a whole were gaining increased benefit, although it would not guarantee that all users would benefit equally (it is possible that the value to some individual users would actually decrease). In practice, the distribution of users is generally not well known, so any overall measure of value will be based on some (possibly implicit) assumption about the distribution of users. For example, the Brier skill score (BSS, Wilks 1995) is a commonly used summary measure of performance of probability forecast systems. It can be shown that BSS is equivalent to the overall value assuming a uniform distribution of users (i.e. all values of C/L are equally important; Murphy 1966). However, it is more likely that the distribution of users is more concentrated towards lower values of C/L (Roebber and Bosart 1996). The variation of BSS with increasing maximum value (V_{max}) is shown in Fig. 2. The relationship is substantially non-linear, especially for less common events. The curves show that there is no simple relationship between BSS and V_{max}. For example, a low BSS does not necessarily mean the system has no value. In fact for BSS=0.2, some users may be achieving over 80% value.

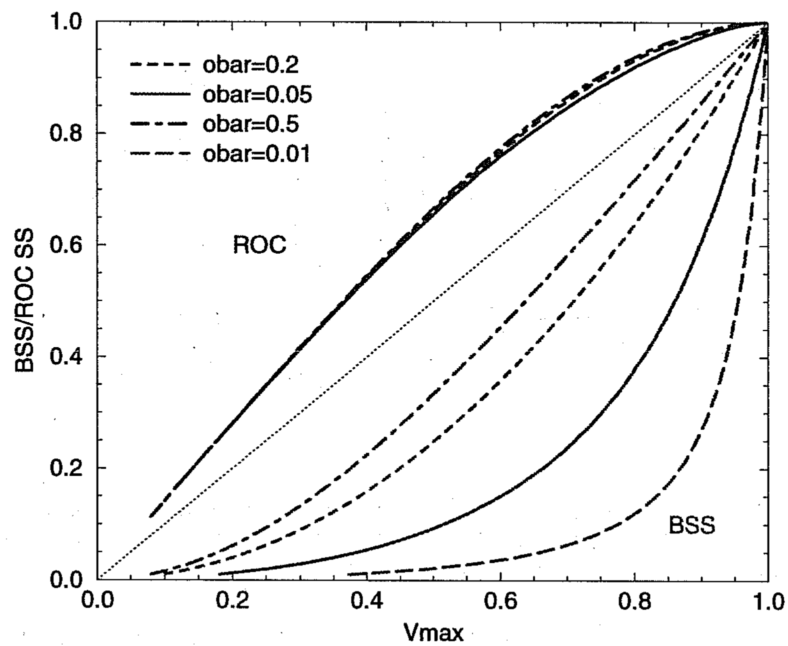

Figure 2: The variation of Brier skill score (curves below the diagonal) and ROC skill score (above the diagonal) with maximum value V_{max} for events with different observed frequencies of occurrence (obar).

Different skill measures will be related to value in different ways. The variation of a skill score based on the area under the relative operating characteristic curve (ROCSS; Richardson 2000) with V_{max} is also shown in Fig. 2. There is little dependence of the relationship between ROCS and V_{max} on $\bar{o}$; also the relationship is substantially more linear than for the BSS. Thus, in terms of economic value, the two skill scores are providing different information. The BSS provides a measure of overall value assuming all possible users are equally important, while the ROCSS provides an indication of the maximum value.

The simple cost-loss model used here provides a framework in which to investigate the potential economic benefits of weather forecasts. The economic value of forecasts depends not only on the forecasting system, but on the relevant weather event and on the user; different users will benefit to a greater or lesser extent from the same forecasting system. The decision model demonstrates the relative importance of various aspects of forecast performance to the user and the benefit of using probability forecasts rather than deterministic predictions. The model can also be used to illustrate the complexity of the relationship between economic value and forecast skill. Different skill measures have different implicit assumptions about the range of users, which should be borne in mind when using these measures to evaluate forecast systems. The cost-loss model and the potential economic value of the ECMWF EPS are discussed in more detail in Richardson (2000).

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