CONVECTIVELY GENERATED GRAVITY WAVES AND THEIR PARAMETRIZATION

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## 1. INTRODUCTION

That convection can generate gravity waves has been shown by direct observation (eg Bradbury, 1963) and by use of numerical simulations (eg Mason and Sykes, 1982). A key result of both observational and modelling studies is that the efficiency of wave generation depends on the strength of the vertical shear. In the presence of such shear, the convective clouds have a blocking effect on the flow and generate vertically propagating waves analogous to those generated by orography. Those waves transport momentum. Parametrization of that effect is the subject of this paper. A more detailed account and a more extensive bibliography may be found in Kershaw (1995).

## 2. THEORY

There have been few attempts to parametrize convective sources of gravity waves explicitly. Indeed, apart from the work reported here, the author is aware of only one, by Rind et al (1988). They suggest that the magnitude of the momentum flux due to convection waves is given by:

$$C\varrho k M^2 N \tag{1}$$

where **C** is a constant, $\varrho$ is the density at the top of the convective layer, **k** is the horizontal wavenumber, **M** is the vertically integrated convective mass flux, and **N** is the Brunt-Väisälä frequency at the top of the convective layer. This formula is unsatisfactory in several respects. It is not dimensionally consistent (unless **C** has rather strange dimensions). It does not include the observed dependence on vertical shear. It has the wrong dependence on **N**.

Theoretical analysis suggests a different formula:

$$\varrho\overline{uw} = -\frac{\alpha}{2}\varrho_c\sqrt{1-F^2}F\overline{w_c^2} \tag{2}$$

where $\alpha$ is a non-dimensional efficiency factor, representing the proportion of convective kinetic energy which is converted to wave energy, the subscript **c** labels quantities in the middle of the convective layer, and the positive **x** direction is that of the wind at the top of the convective layer, in the frame of reference moving with the cloud. The overbar represents a horizontal average.

**F** is a non-dimensional Froude number, defined by

$$F=\frac{(U-c_x)k}{N} \tag{3}$$

where **U** is the wind speed at cloud top and $c_x$ is the phase velocity of the waves, assumed equal to the in-cloud velocity at cloud top. Clearly, the magnitude of the Froude number must be less than 1, otherwise vertically propagating waves cannot occur. This means that only waves which are longer than a critical horizontal wavelength can transport momentum.

To first order in **F**, we can approximate equation (2) by:

$$\varrho\overline{uw}=-\frac{\alpha}{2}\varrho_c F\overline{w_c^2} \tag{4}$$

Thus the momentum flux is directly proportional to the vertical shear and inversely proportional to **N**. It depends only on the efficiency factor, α, the energy of convection and the Froude number.

## 3. NUMERICAL VALIDATION

Observational validation of equation (4) would be difficult, but numerical validation is feasible. A cloud resolving model (CRM) can be used to simulate convection in a range of conditions, and the terms of the equation can be diagnosed or estimated. This has been done for an idealised cold-air outbreak case, with varying vertical shear, varying intensity of convection and varying stability (N) above the convective layer. Vertically propagating waves which were quasi-stationary with respect to the clouds, and which had horizontal wavelengths ranging from 8 to 15km, did develop in these simulations. The results show that, in that regime at least, the wave momentum flux does vary in the expected way with shear, convective energy, and stability.

The right hand side of equation (4), divided by α, might be termed the convective forcing of the waves. This convective forcing can be plotted against the wave momentum flux as in Fig 1. Each point on the scatter plot gives us an estimate of α. The line of best fit gives a value of about **20%**. However, simulations made with higher horizontal resolution suggest that **15%±5%** is probably a better estimate, *for this regime.* Different regimes, eg organised tropical clusters or squall lines, might well have different efficiencies for gravity wave generation. They might also excite waves of a different (probably larger) horizontal wavelength.

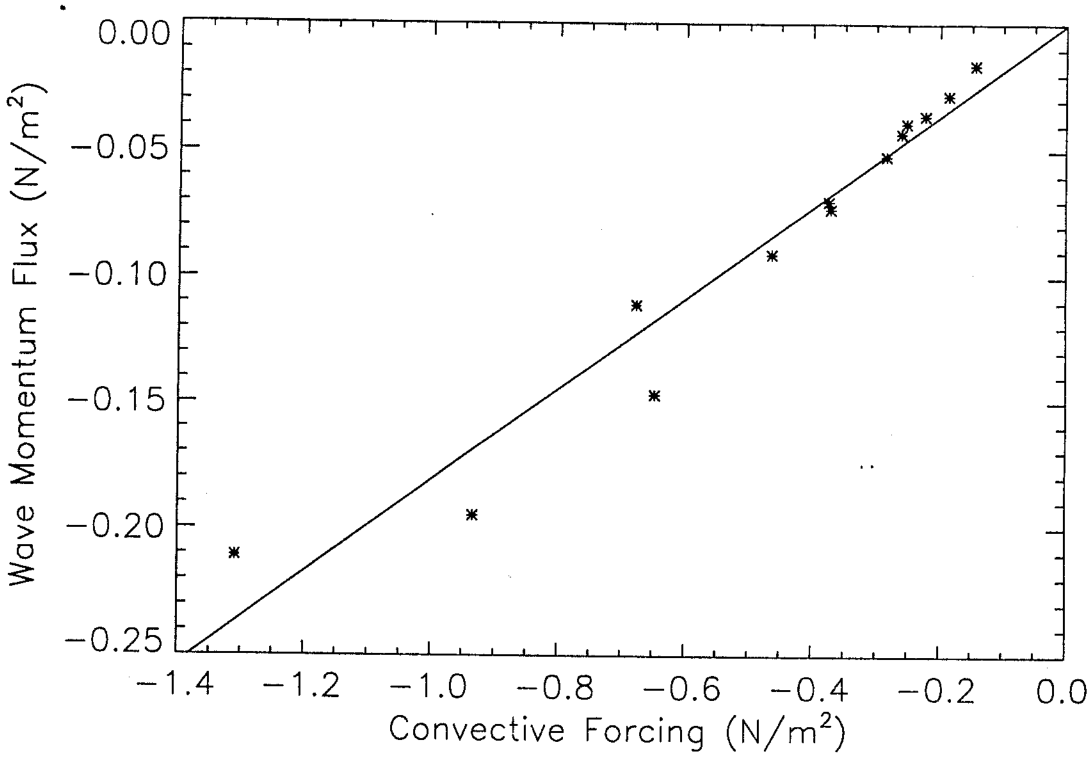


**Fig. 1 Scatter plot of momentum flux against convective forcing from CRM experiments.**

## 4. DISCUSSION

Equation (4), or its more accurate form equation (2), provides a possible parametrization of momentum transport by convection waves. It probably only makes sense to include such a parametrization, in a climate or NWP model, in conjunction with a parametrization of the transport of momentum by convection itself, such as that described in Kershaw and Gregory (1997) and Gregory et al (1997). Fig 2 shows the reason why: the momentum flux due to the convection itself is typically an order of magnitude larger than the flux due to the gravity waves. In this example, in which the wind speed is increasing linearly with height, the convection is exerting a drag on the flow in the upper part of the convective layer. The waves act to reduce the flux divergence, and hence the drag, near cloud top, and transfer some of the drag to higher levels. Both effects, the acceleration near cloud top and the drag aloft, must be included. The calculation of this drag can be done in the same way as for orographic waves, save that the non-zero phase speed must be taken into account.

Estimation of the phase velocity of the waves is easier in the context of a convective momentum transport scheme because the in-cloud velocity is a by-product of the calculations. CAPE is often calculated within convection schemes, so an estimate of the energy of the convective activity can easily be obtained too. **U** and **N** are explicitly resolved and predicted by the model, so the only remaining difficulty is the estimation of horizontal wavelength. One possible approach is to start with a typical observed wavelength, say 10km,

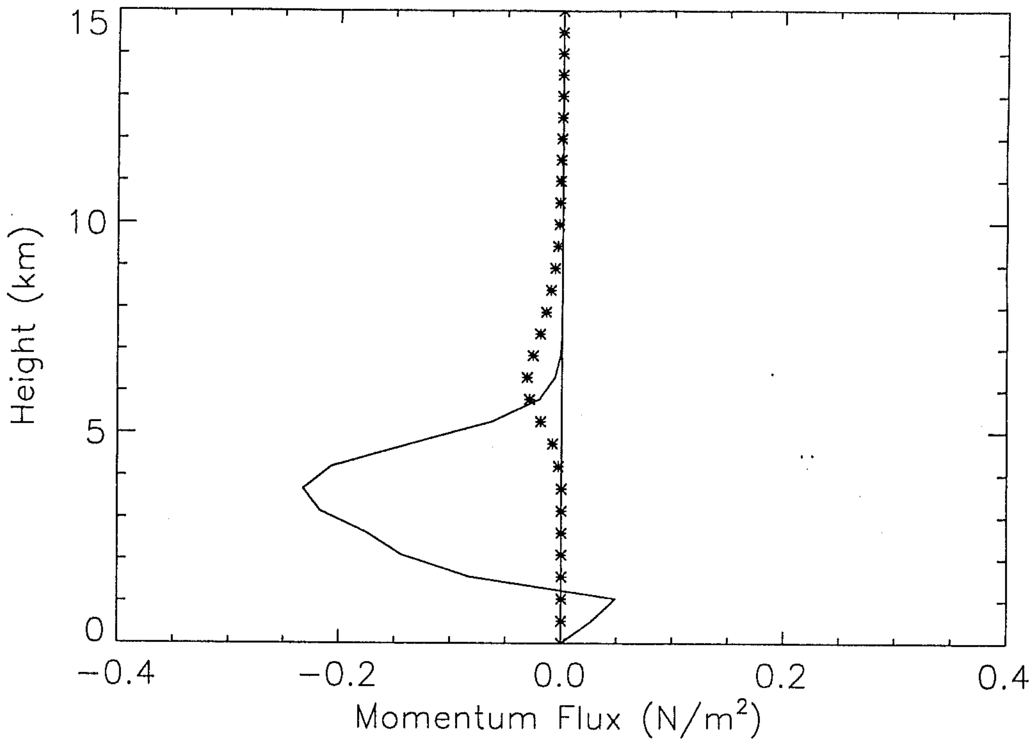


**Fig. 2 Vertical profile of momentum flux in CRM experiment (solid: convective, stars: wave).**

calculate **F**, and increase the wavelength if necessary to ensure that the magnitude of **F** is less than **1**. Alternatively, some means of estimating the spectrum of waves excited by convection must be found. This is one area which could benefit from further work using CRMs, by using them to test the parametrization equations in a wider range of convective regimes.

## 5. REFERENCES

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