The Representation of Moist Convection in Atmospheric Models.

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1. Introduction

Convection plays an important role in the maintenance of the general circulation of the atmosphere, both through latent heat release, the vertical transport of heat, moisture and momentum and the interaction of clouds with radiation. Circulation systems affected by convection range from those on a planetary scale (Hadley and Walker cells), through regional scale systems (for example monsoon circulations) to synoptic scale features. The recent paper by Emanuel et al (1994) provides an overview of the role convection plays in many of these circulation systems. Given the importance of convection it is vital that this process must be included in models of the large-scale atmospheric circulation.

The purpose of this paper is to provide a broad overview of the techniques used to represent convective processes in models of the atmospheric circulation. The aim is to introduce the reader to concepts involved, current techniques and schemes and uncertainties which remain to be resolved. As an overview it is necessarily brief at points and the reader is referred to the reference list for more detailed descriptions.

1.1 The need for parametrization.

Within large-scale models of the atmosphere the horizontal length scale on which convection exists is below the resolution used and so the effects of such clouds must be parametrized in terms of the large-scale flow. Expressing the temperature structure of the atmosphere in terms of a large-scale mean and an unresolved eddy contribution,

$$\phi=\overline{\phi}+\phi' \tag{1}$$

the thermodynamic equation of the large-scale flow may be written as,

$$\frac{\partial\theta}{\partial t}+\nabla.\underline{v}\theta+\frac{\partial\overline{\omega}\,\overline{\theta}}{\partial p}=\frac{LQ}{c_p\Pi}+\frac{\partial\overline{\omega'\theta'}}{\partial p}=Q1 \qquad (2)$$

where Q1 is termed the apparent convective heat source (Yanai et al, 1973). The left hand side contains large-scale terms only while the right hand side of eqn (2) represents the impact of small-scale motions upon the large-scale flow. Convection is seen to affect the large-scale flow through condensational heating and the vertical transport of heat. A similar treatment is applicable to the moisture equation leading to the definition of Q2, the apparent moisture sink due to convection and also the horizontal momentum equation (see section 3).

1.2 The aim of convective parametrization schemes

Convective parametrization schemes aim to represent Q1 and Q2 in terms of large-scale atmospheric variables predicted by a numerical model. In doing so they must first predict the vertical distribution of latent heating and transport properties due to convection, through the use of simple cloud models. Secondly, they must predict the overall magnitude of the energy release from the convection. This is usually termed the closure problem and involves a relationship between convective activity and the large-scale structure and flow.

A caveat to these aims is that they are achieved whilst maintaining a realistic thermodynamic vertical structure. In general convection schemes will come into balance with the large-scale forcing (moisture convergence and evaporation) regardless of the closure assumption used (see section 2.4.3c for further discussion) through a modification of the vertical thermodynamic structure. For example if a convection scheme underestimates the amount of convective heating, then the profile will cool under the action of radiation and large-scale ascent. This will increase the amount of instability within the profile and so cause convective activity to increase. If the resultant thermodynamic structure is unrealistic then the convection scheme cannot be said to be representing atmospheric convection accurately even if the net surface rainfall is well simulated.

Two other points must also be made regarding the performance of convection schemes. They must be able to represent the mean distribution of rainfall accurately and also maintain a realistic level of atmospheric variability. Recent work (Slingo et al, 1994) has shown that the simulation of tropical variability from synoptic to intra-seasonal timescale by general circulation models (GCMs) is sensitive to the convection scheme used, and especially the formulation of the closure (see section 2.4.3d).

1.3 An historical perspective.

In recent years there has been a shift of views regarding how convection should be represented in large-scale models. This is partly due to increased computational resources available to the field of atmospheric sciences but also the desire to include parametrizations of greater physical realism into GCMs. This shift is illustrated by the convection schemes described in the ECMWF 1985 seminar report into the parametrization of physical processes in large-scale models. There three types of convection scheme were discussed in detail; moist convective adjustment schemes, Kuo type schemes and the then relatively new Betts-Miller adjustment scheme. There was little discussion of a fourth type of scheme, the so-called mass flux scheme which, because of its greater complexity, was only in use in a few models (for example the forecast and climate models of the UK Meteorological Office) at that time.

Since the mid-1980s this latter type of scheme has become more widely used. A survey of the convection schemes used by the 30 different modelling groups involved in the "Atmospheric Model Intercomparison Project" (AMIP) (Gates, 1992) shows that 5 models use moist convective adjustment schemes, 8 Kuo type schemes of varying complexity, 1 the Betts- Miller scheme and 16 models various mass flux schemes.

In light of these developments this paper concentrates upon the mass flux formulation with only a brief outline of other types of scheme being given. The discussion will also be concentrated mainly upon the representation of deep convection. Shallow convection plays a large role in determining the nature of the global circulation, with simulations of the tropical flow being extremely sensitive to the parametrization used, as illustrated by the work of Tiedtke et al (1988). However since that paper little progress has been made to reduce this uncertainty. It should be noted that the mass flux formulation described below is also applicable for shallow convective processes as illustrated by the schemes of Tiedtke (1989) and Gregory and Rowntree (1990).

2. A review of current parametrization schemes.

2.1 Moist Convective Adjustment schemes

This was the earliest type of scheme developed (Smagorinsky, 1963). Essentially mass is exchanged between layers of the atmosphere which are conditionally unstable to vertical ascent, i.e. the saturated equivalent potential temperature of a lower layer is greater than that of the adjacent layer above. Once this criterion is established through the action of dynamics and other physical processes, then mass is exchanged between the layers to achieve neutrality and any moisture condensed in the process is removed as precipitation.

The advantage of these schemes is that they are simple to implement in numerical models and computationally inexpensive to use. However they lack physical basis and ignore the penetrative nature of deep convective clouds. They are still often used in numerical models alongside more complicated convection schemes to remove any residual instability remaining after the action of convection and so maintain numerical stability of the dynamics scheme.

2.2 Kuo Type Schemes

These schemes are based upon the CISK hypothesis, linking the presence of deep convection with moisture convergence in a region of the atmosphere (Kuo 1965, 1974). Convection is assumed to occur when there is moisture convergence present, the rainfall (R) being given by,

$$R=(1-b)\langle F_q \rangle \tag{3}$$

where F_q is the vertically integrated moisture supply into the column of the atmosphere (including the surface evaporation) over the depth of the cloud. The coefficient 'b' was initially assumed to be a constant but in later versions of the scheme was usually taken to be a function of relative humidity (for example, Anthes 1977). The heating due to convection is distributed in the vertical through the use of a cloud model and it is assumed that the cloud affects the large-scale atmosphere through the instantaneous mixing of the cloudy air into the clear air.

The scheme was initially designed for use in hurricane models but has found wide use in both climate and NWP applications. Again the scheme is relatively simple and inexpensive to implement in a numerical model. However one disadvantage is that no convection occurs until moisture convergence is present. Although it is commonly observed that moisture convergence accompanies convection it is not the mechanism by which convection exists, although it may maintain the large-scale atmosphere in a state in which convection may occur by countering the drying tendency of convection. Convection is essentially a buoyant process. Hence if the equivalent potential temperature of the boundary layer is greater than that of the free atmosphere convection may develop if boundary layer air is lifted by some mechanism (turbulence, orographic lifting) even if there is no moisture forcing. Indeed the presence of convection may lead to low level moisture convergence and so the maintenance and possible growth of convective activity. The Kuo scheme with its reliance upon the presence of moisture convergence is unable to represent such a mechanism. Slingo et al (1994) suggests that this may be the reason that the UGAMP climate model fails to simulate realistic tropical synoptic variability when a Kuo scheme is used (see section 2.4.3(d)). Emanuel et al (1994) discuss the nature of buoyant convection and its link to moisture convergence in greater detail.

2.3 Betts-Miller Adjustment scheme

Although classed an adjustment scheme this scheme has a better physical basis than moist convective adjustment schemes. Based upon the observations that deep convective atmospheres tend to have characteristic thermodynamic structures (Betts, 1986), typically following a virtual moist adiabat (θ_{ESV}) from the boundary layer to the freezing level with a return to the equivalent potential temperature through cloud base at the top of the convective layer (figure 1), in the presence of deep convection the atmosphere is relaxed back to this structure over a time scale τ. The convective heating and drying are given by,

$$Q1=\frac{(\theta_R-\overline{\theta})}{\tau} \qquad Q2=\frac{(q_R-\overline{q})}{\tau} \tag{4}$$

where Φ_R is the reference profile of Φ and $\overline{\Phi}$ is the large-scale value of Φ.

The reference profile for moisture is defined from that for temperature through specification of sub-saturation at three points; cloud base, freezing level and cloud top. The adjustment timescale is on the order of one or two hours depending upon the resolution of the model in which the scheme is implemented. Recent modifications include the inclusion of moist convective downdraughts . The scheme also provides a description of shallow convection in which the boundary layer is adjusted back to a characteristic structure based upon a mixing line approach. The reader is referred to Betts (1986) for further details.

2.4 Mass flux convection schemes.

Although the theory around which mass flux scheme are based has been known for some time, their greater expense has meant that until the late 1980s they have not been extensively used in models. However with increased computing power these schemes have become more popular and as discussed previously are used in many climate and NWP models. A further reason for this is that they provide a physical understanding of how local convection affects the larger-scale atmosphere.

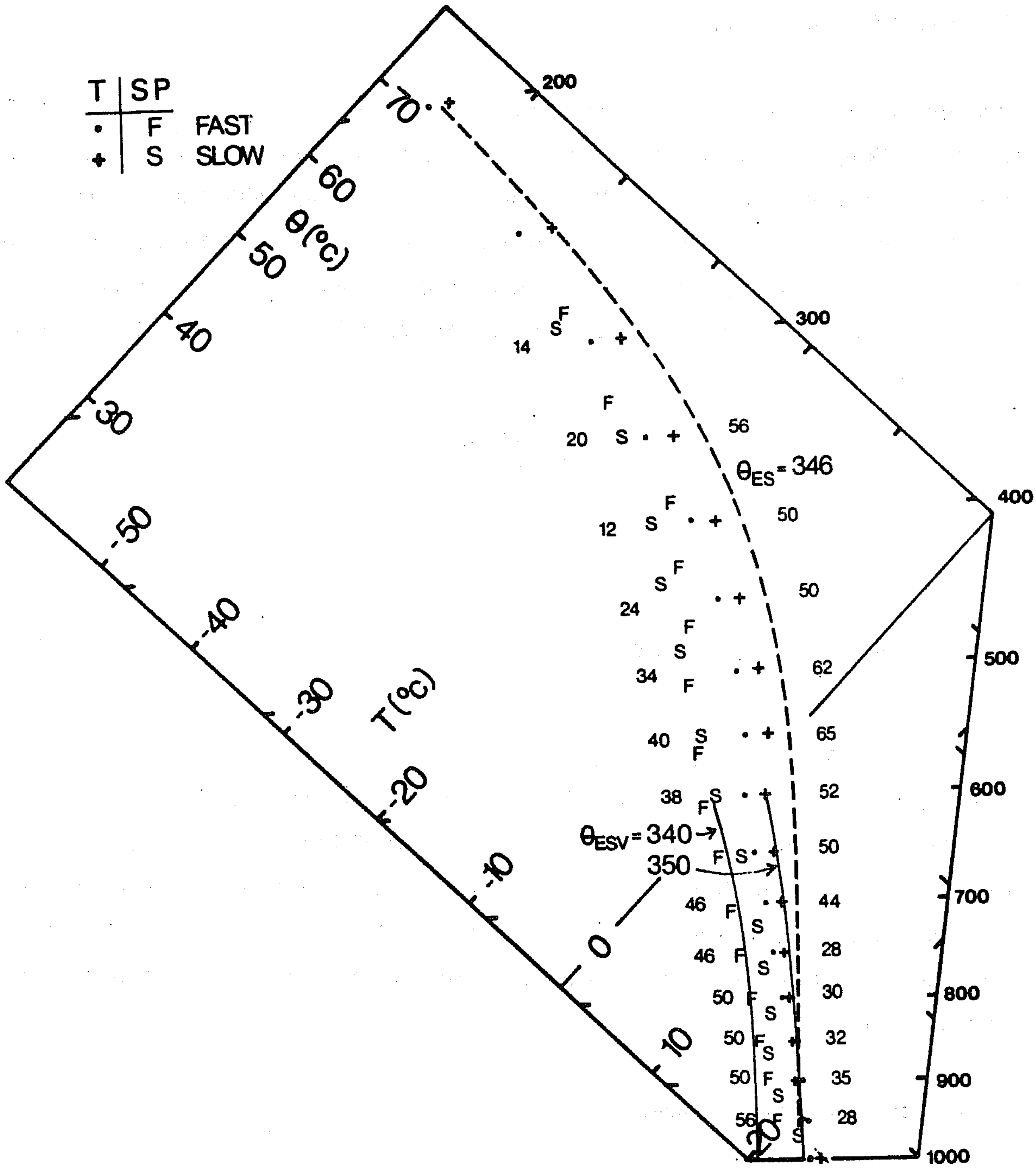


Figure 1 Composite wake soundings for GATE slow and fast moving lines showing temperature (T) and saturation points. (From Betts and Miller, 1986)

2.4.1 Formulation

The mass flux approach was first formulated by Ooyama (1971). It is assumed within some area A, taken to be the grid point of a numerical model, that a fraction σ is covered by cloud. Hence the area average of ϕ is given by

$$\overline{\phi}=\sigma\overline{\phi}^{c}+(1-\sigma)\overline{\phi}^{e} \tag{5}$$

where ϕ^{c} is the value of ϕ within cloudy air, ϕ^{e} that within the environment of the cloud.

It is normally assumed that σ<<1 and so,

$$\overline{\phi}=\overline{\phi}^{e} \tag{6}$$

Consider now the eddy flux divergent contribution to Q1. From eqn(1) it can be written that

$$\overline{\omega'\theta'}=\overline{\omega\theta}-\overline{\omega}\,\overline{\theta} \tag{7}$$

Using equation (5) to expand the first term of the r.h.s. of equation (7) and $\overline{\omega}$ in the second term, after rearrangement equation (7) can be written as,

$$\overline{\omega'\theta'}=\sigma[\overline{\omega\theta}^{c}-\overline{\omega}^{c}\overline{\theta}]+(1-\sigma)[\overline{\omega\theta}^{e}-\overline{\omega}^{e}\overline{\theta}] \tag{8}$$

Using equation (6) and assuming the vertical velocity in the environment is much smaller than that within cloudy air and so motions are only weakly correlated with thermodynamic variables, the second term on the r.h.s. of equation (8) can be neglected and the vertical eddy flux due to convective motions written approximately as,

$$\overline{\omega'\theta'}=\sigma[\overline{\omega\theta}^{c}-\overline{\omega}^{c}\overline{\theta}] \tag{9}$$

Hence from equation (2) Q1 can be expressed as,

$$Q1=(1-\sigma)\frac{L\overline{Q}^{e}}{c_{p}\Pi}+\sigma\frac{L\overline{Q}^{c}}{c_{p}\Pi}-\frac{\partial\sigma(\overline{\omega\theta}^{c}-\overline{\omega}^{c}\overline{\theta})}{\partial p} \tag{10}$$

The latent heating term associated with the cloud environment is usually interpreted as the evaporation of cloud condensate and precipitation detrained from the cloud into clear air leading to a cooling of the cloud environment.

From equation (10) it is seen that Q1 has been expressed in terms of cloud and large-scale variables. The latter are available from the grid point fields of a numerical model while the former are usually obtained from the use of a one dimensional steady state entraining plume model of the cloud.

Using such a model, for cloud i the thermodynamic and mass continuity equations are,

$$E_i\overline{\theta}-D_i\overline{\theta}^c+\frac{\partial\sigma_i\overline{\omega\theta}_i^c}{\partial p}=\sigma_i\frac{LQ_i^c}{c_p\Pi} \tag{11}$$

$$E_i-D_i+\frac{\partial\sigma_i\overline{\omega}^c{}_i}{\partial p}=0 \tag{12}$$

where E is the entrainment rate, i.e the rate at which air is included into the cloud through its sides to meet the increase of vertical mass flux in the cloud with height, and D the detrainment rate (the rate at which air leaves the cloud as the cloud vertical mass flux decreases with height) defined as

$$\begin{aligned} E_i&=\frac{\partial\sigma_i\overline{\omega}^c{}_i}{\partial p} \quad if \quad -\frac{\partial\sigma_i\overline{\omega}^c{}_i}{\partial p}<0 \\ D_i&=\frac{\partial\sigma_i\overline{\omega}^c{}_i}{\partial p} \quad if \quad -\frac{\partial\sigma_i\overline{\omega}^c{}_i}{\partial p}>0 \end{aligned} \tag{13}$$

These equations are for one cloud only whereas in the area under consideration it is assumed that there are many clouds. Summing over all cloud within the ensemble, equations (11) and (12) can be written as,

$$E\overline{\theta}-\Sigma_{\mathrm{det}}D_i\overline{\theta}^c+\frac{\partial\sigma\overline{\omega\theta}^c}{\partial p}=\sigma\frac{LQ^c}{c_p\Pi} \tag{14}$$

$$E-D+\frac{\partial\sigma\overline{\omega}^c}{\partial p}=0 \tag{15}$$

where $E=\Sigma_iE_i$ and $D=\Sigma_iD_i$, the summation being carried out over all clouds. The summation on the lhs of equation (14) is over all clouds which are undergoing terminal detrainment.

Substitution of equations (14) and (15) into equation (10) gives after rearrangement,

$$Q1=\sigma\overline{\omega}^{c}\frac{\partial\overline{\theta}}{\partial p}+\Sigma_{\mathrm{det}}D_{i}(\overline{\theta}_{i}^{c}-\overline{\theta})+(1-\sigma)\frac{L\overline{Q}^{e}}{c_{p}\Pi} \tag{16}$$

and similarly for Q2.

Convection modifies the large-scale atmosphere through compensating subsidence, motion in the clear environment surrounding the cloud compensating motion within the cloud (a consequence of mass continuity), detrainment of cloudy air into the environment, usually near cloud top, and the evaporation of cloud condensate and precipitation within the environment, whether at cloud top through detrainment or below cloud base as precipitation falls to the surface. Several authors have attempted to diagnose which of these terms is dominant using observations (e.g Yanai et al, 1973) or, more recently, through the use of cloud resolving models (e.g. Gregory and Miller, 1989). Figure 2 from the latter compares the vertical structure of Q1 with that of compensating subsidence and the evaporation of condensate within the cloud environment. Compensating subsidence is seen to follow Q1 closely in the vertical, exceeding it at most levels. This excess of heating is compensated by the evaporation term. The detrainment of cloudy air plays only a secondary role for deep convection, although it is of greater importance in shallow convection.

A similar analysis can be carried out for Q2. However the study of Gregory and Miller (1989) suggests that compensating subsidence is not a good approximation for Q2. The reader is referred there for further discussion. In practice, in the absence of a better understanding of how convection affects the moisture structure of the atmosphere, the above theory is used.

Application of the above theory in a numerical model is a two stage process. Firstly a cloud model must be used to estimate the vertical distribution of the cloudy quantities, and secondly the magnitude of the mass flux at the base of the cloud must be determined, usually by some reference to the large-scale structure and forcing (the closure problem).

2.4.2 Use of cloud models

Two approaches have been suggested for using cloud models to estimate the quantities required to implement the mass flux theory as a parametrization scheme. These are the use of a spectral cloud ensemble, in which several different cloud models are used to represent differing cloud types within a grid box of a GCM, and the bulk cloud model approach, where a single cloud model is used. The strengths and weaknesses of each approach are discussed below.

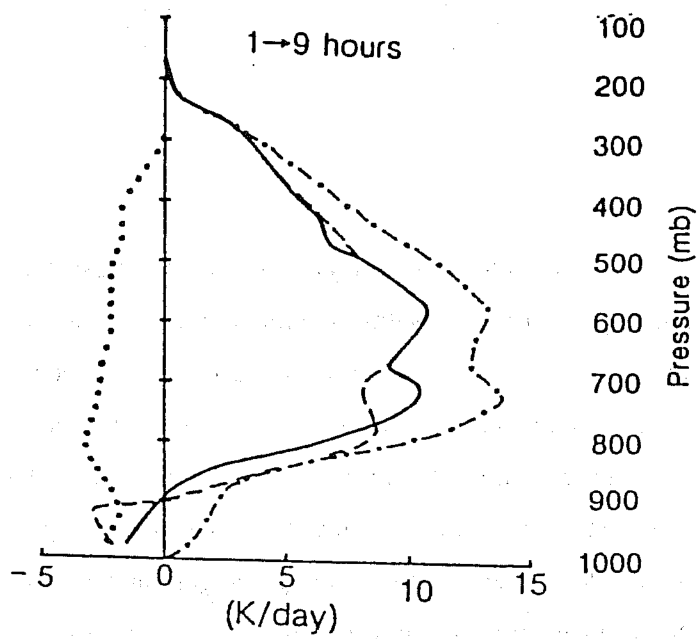


Figure 2 Contribution of compensating subsidence (—·—·—), evaporation of cloud and precipitation (·····) to Q1 (————————) for convection in a GATE easterly wave. (From Gregory and Miller, 1989)

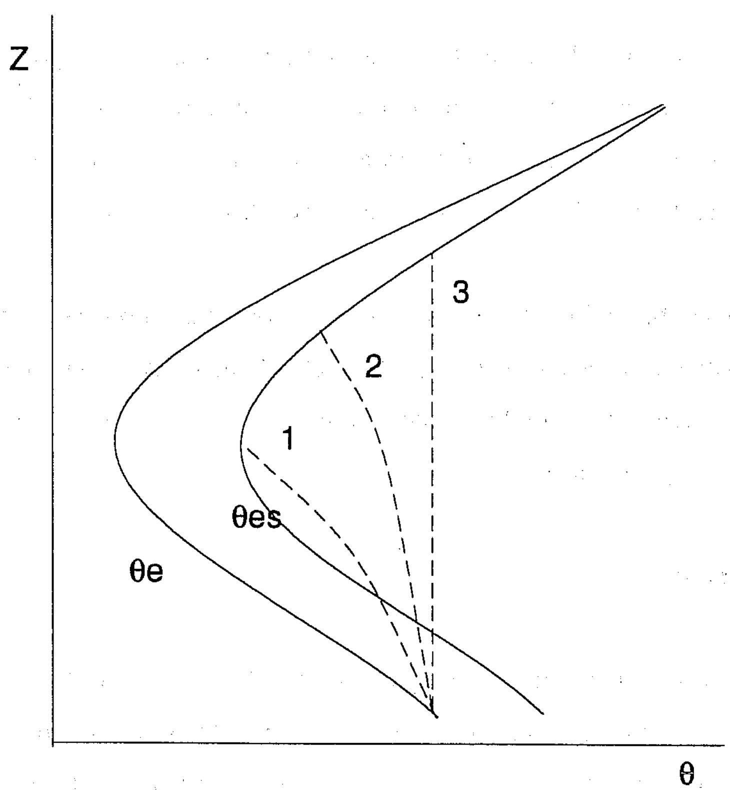


Figure 3 Schematic of spectral cloud ensemble approach. Clouds 1,2 and 3 each have different entrainment rates and so different thermodynamic properties, reaching zero buoyancy at different heights.

(a) Spectral cloud ensemble approach

This technique was first introduced by Arakawa and Schubert (1974). They assumed that within a grid box of a GCM a spectrum of different height clouds existed. Each cloud is characterised by a different entrainment rate and so different cloud top height (figure 3). Clouds with large entrainment rates terminate lower in the atmosphere (cloud 1), while those with lower entrainment rates follow a more undilute ascent and so terminate in the upper troposphere (cloud 3). In theory the number of different cloud types within the distribution is arbitrary with clouds allowed to terminate at any level in the atmosphere, but in practice is limited by the number of vertical levels in the model, with one cloud type detraining at each model layer. As the pressure of these layers varies from time step to time step due to changes in surface pressure, entrainment rates for each cloud type have to be continually re-evaluated each time the scheme is used.

In estimating their vertical structure each cloud type is allowed to affect the others, the ascent of air through cloud type 1 being affected by changes to the cloud environment from cloud types 2 and 3. Cloud base mass flux must be estimated for each cloud type, determining how much that cloud type contributes to the ensemble mean. An iterative solution is required to solve the problem which is not necessarily stable. In recent years the Arakawa - Schubert scheme has undergone some modification (e.g. Moorthi and Suarez, 1992) in order to reduce its complexity and increase its robustness, while still retaining the spectral cloud ensemble approach.

(b) Bulk cloud model approach

Because of the complexity and hence computational expense of the spectral cloud ensemble approach several schemes have been developed using the simpler bulk cloud model approach developed by Yanai et al (1973). In this method only one cloud model is used to represent an ensemble of clouds within a grid box, the cloud properties predicted being assumed to be averages over the cloud ensemble (figure 4). Both the mass flux scheme of Tiedtke (1989) used in the ECMWF model and the UK Meteorological Office scheme (Gregory and Rowntree, 1990) use this approach. The method is cheaper and simpler to implement. Both the above schemes have the additional advantage over the Arakawa-Schubert scheme that they are able to represent convection which is not rooted in the planetary boundary layer (for example as seen in mid-latitude warm fronts). Although the approach is simpler than the full Arakawa-Schubert scheme it performs well in both climate and NWP applications.

One problem with the use of a bulk cloud model is that clouds may only detrain at one level and this

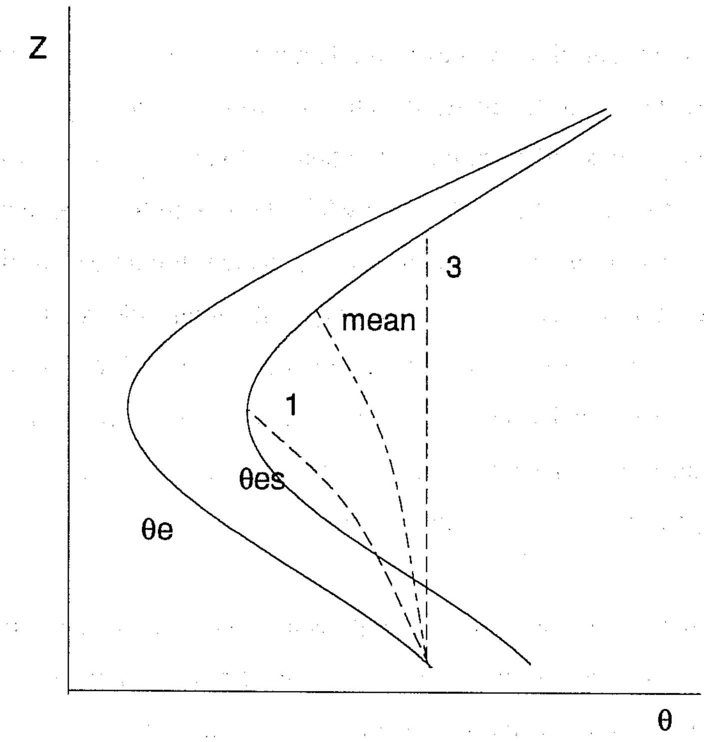


Figure 4 Schematic of bulk cloud approach. Mean cloud properties represent a variety of clouds with properties ranging from ascent 1 through 3.

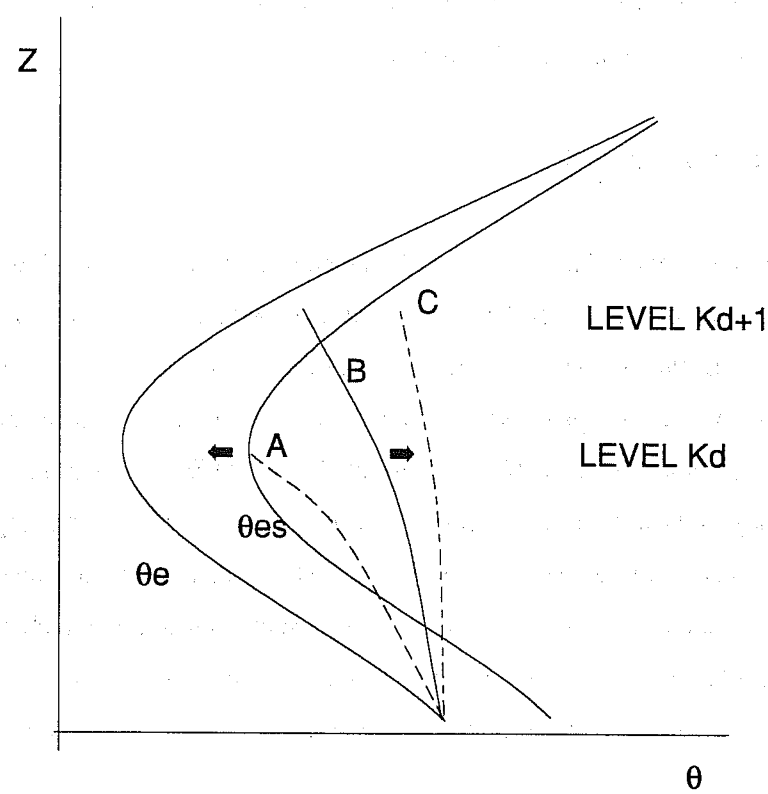


Figure 5 Schematic of forced detrainment.

may lead to an underestimation of the cloud top height of the deepest clouds within the assumed ensemble. The UK Meteorological Office scheme overcomes this deficiency through its use of "forced detrainment", illustrated in figure 5. Essentially if on ascent to model level k_D+1 the parcel becomes negatively buoyant (ascent B) it is assumed that part of the ensemble detrains at the lower level k_D with neutral buoyancy (equivalent to the detrained cloud having a higher entrainment rate, ascent A). Because the detraining air has a lower θ_e than that of the ensemble mean, the ensemble mean temperature is raised at level k_D. The amount of neutrally buoyant air detrained at the lower level is just sufficient to allow the parcel to be positively buoyant in level k_D+1, hence allowing the parcel ascent to continue. The θ_e of the buoyant parcel at level k_D+1 implies that this air has undergone a less dilute ascent from the convective initiating layer (ascent C). This process is limited by the condition that a parcel may not ascend higher than the neutral buoyancy level of an undilute parcel ascending from the cloud's originating level.

It should be noted that although the mean θ_e of the parcel increases, for individual parcel ascents it always decreases. In the example in figure 5, no detrainment occurs until level k_D is reached and so the detrainment level of shallow convection within the ensemble may be over estimated. This can be compensated by the inclusion of a background detrainment which occurs as the parcel ascends, even when the parcel is positively buoyant.

(c) Importance of downdraughts.

The above discussion has related to the ascent of air though clouds. However downdraughts, driven by the evaporation of falling precipitation and also the drag of precipitation on the air through which it falls, are also an important component of many deep convective systems. As well as acting as a heat sink (through the evaporation of precipitation) they bring cold dry air from the mid-troposphere into the boundary layer, so potentially enhancing surface fluxes. The study of Gregory and Miller (1989) showed that downdraughts make a large contribution to the net convective heating. Diagnosing heating rates from a cloud resolving model simulating deep convection over the GATE area, they found that the net heating due to updraughts was twice Q1, the excess heat being compensated by the cooling due to evaporation of precipitation within downdraughts (figure 6).

In recent years several mass flux schemes have been updated to incorporate the effects of convective scale downdraughts (e.g. Gregory and Allen, 1991). They are represented by inverted entraining plumes, adiabatic warming on descent of air being compensated by cooling due to the evaporation of precipitation, maintaining negative buoyancy. Typically the starting level of the downdraught is taken to be the highest level of free sinking of an equal mixture of updraught and environment air brought

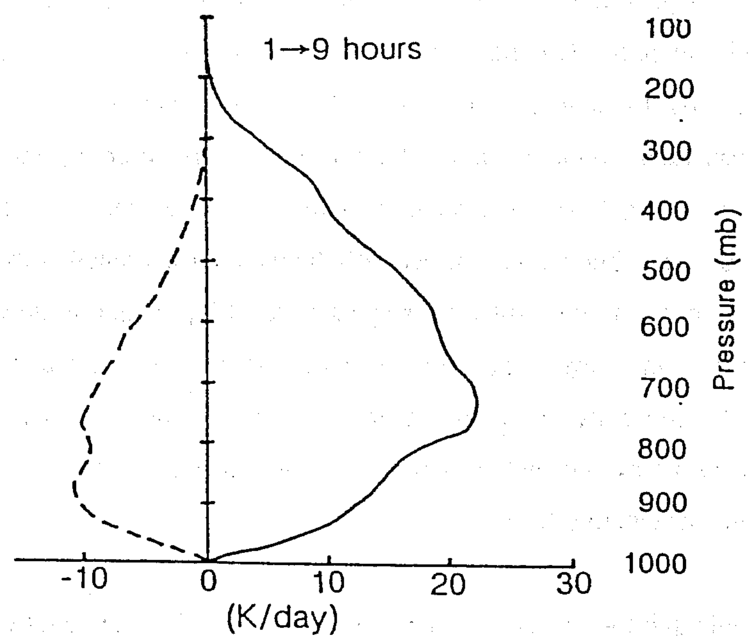


Figure 6 Contribution of updraughts (————) and downdraughts (— — —) to Q1 for convection in a GATE easterly wave. (From Gregory and Miller, 1989)

to saturation. The initial downdraught mass flux is linearly related to that of the updraught at some reference level, typically cloud base (as in the ECMWF mass flux scheme), but not necessarily so. For example the mass flux scheme used in the UK Met. Office Unified Model (Gregory and Rowntree, 1990; Gregory and Allen, 1991) uses the mass flux in the upper part of the updraught to determine the initial downdraught mass flux as this is found to be a better representation of the activity of the deepest clouds within the ensemble.

Downdraughts affect the large-scale environment through compensating "up-sidence" and detrainment of downdraught air within the boundary layer. Figure 7 illustrates the impact of including convective downdraughts into the UK Met. Office Unified Model at climate resolution (for northern winter, December, January and February). Greater rainfall is seen over the tropical continents (fig 7a,b) while mixing ratios in the lowest model layer are reduced (fig 7c) and surface evaporation enhanced (fig 7d) over large regions of the tropical oceans where convection occurs.

(d) Microphysical processes.

The production of precipitation within clouds is governed by complex microphysical processes. These are poorly represented in most convection schemes. Although the latent heat of freezing is included, little distinction is made between the formation of rain and ice precipitation. The mass flux scheme of Gregory and Rowntree (1990) illustrates the simplicity of the schemes currently used. In this scheme clouds are allowed to precipitate once a critical depth is reached which varies whether the cloud forms over land or sea (4km over land, 1.5km over sea) or whether the cloud is glaciated (1km). These values are based upon observations made by Ludlam (1980). On precipitation clouds retain 1g/kg of condensate (or an amount equal to the saturation mixing ratio if it is smaller) which is then transported upwards to be precipitated out at a higher level or detrained into the cloud environment.

The radiation budget of GCMs is very sensitive to the microphysical formulation used. For example, the simulation of top of atmosphere outgoing longwave radiation in the tropics is very sensitive to the amount of condensed water detrained from the top of convective systems. Figure 8 shows the variation in zonal mean outgoing longwave radiation from simulations of the Met. Office Unified Model produced by changing the amount of condensed water transported into the upper levels of the cloud from an amount equal to the saturation mixing ratio, to twice and 5 times that value. Changes on the equator of up to $40 Wm^{-2}$ are seen as the amount of condensed water detrained is increased. There is little data on the amount of cloud water in the upper tropical troposphere and so it is difficult to validate a convection scheme's performance in this regard. This is a key area where further observations must be obtained and where more realistic microphysics schemes must be introduced into

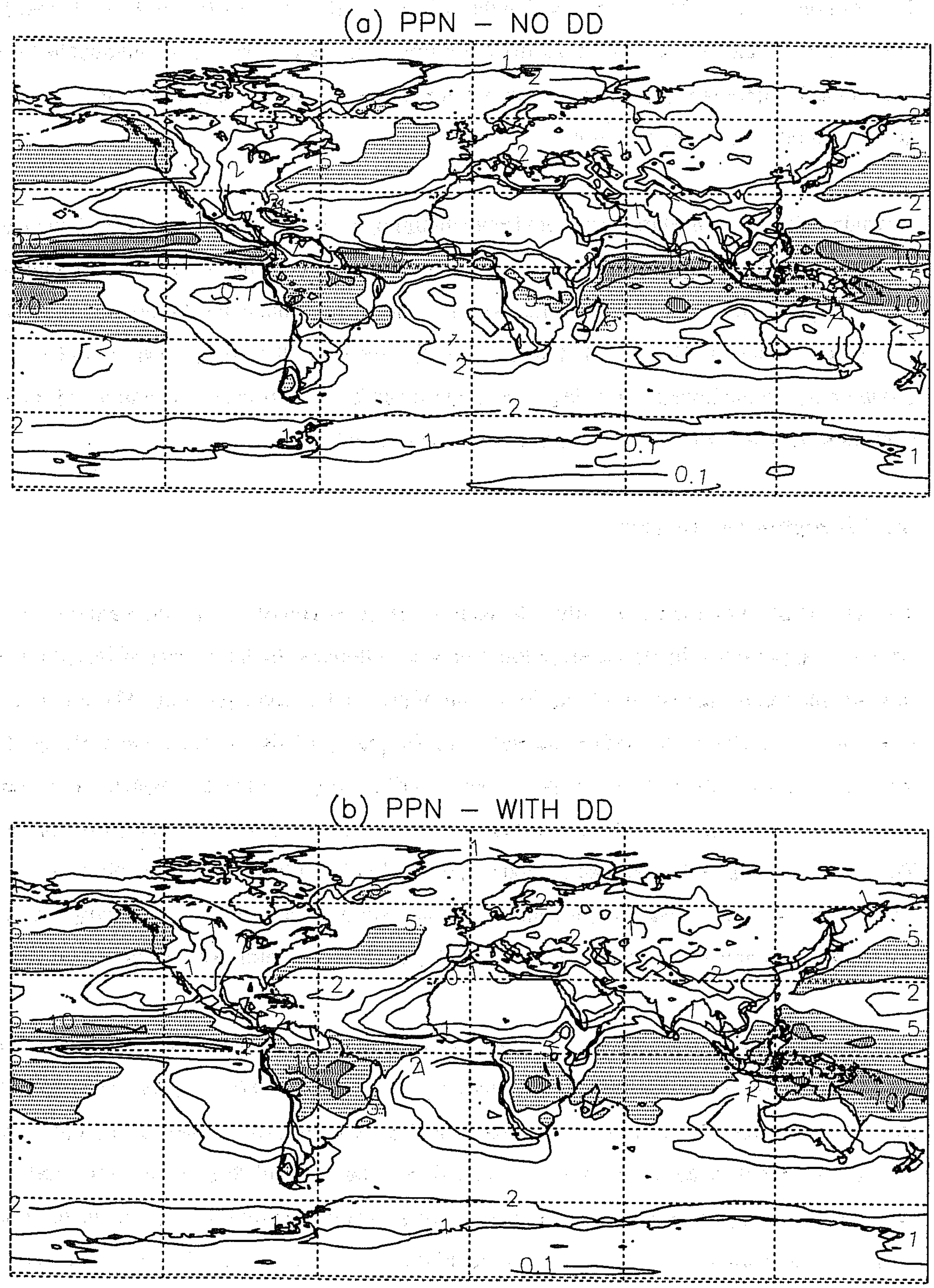
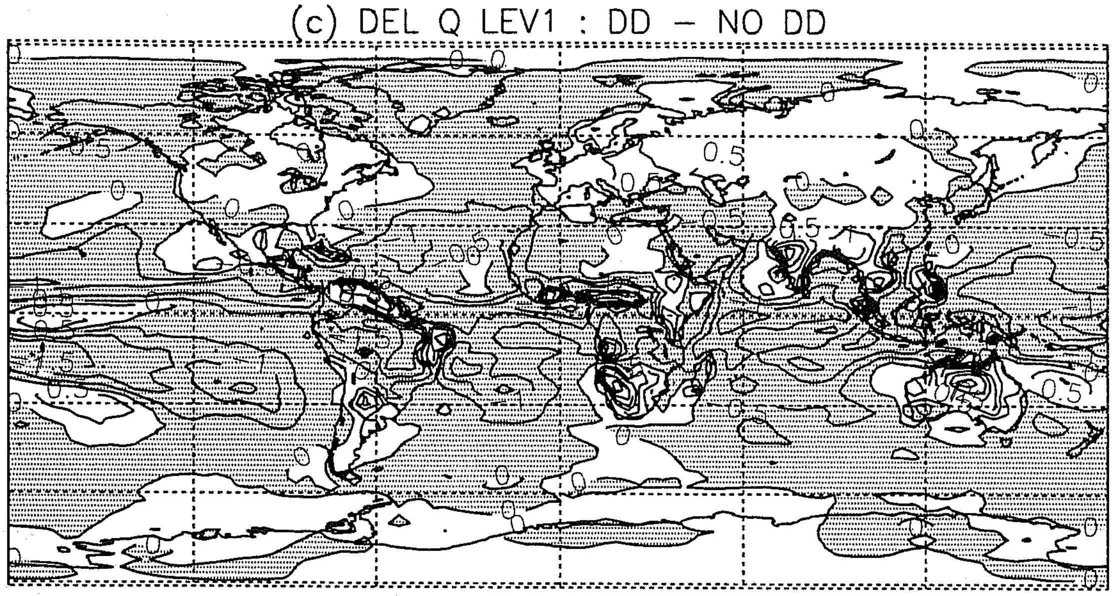


Figure 7 Simulation of December/January/February rainfall (3 year mean) by the Met Office Unified Model (a) without and (b) with convective downdraughts and (c) the impact of downdraughts upon lowest model level moisture and (d) surface latent heat flux.



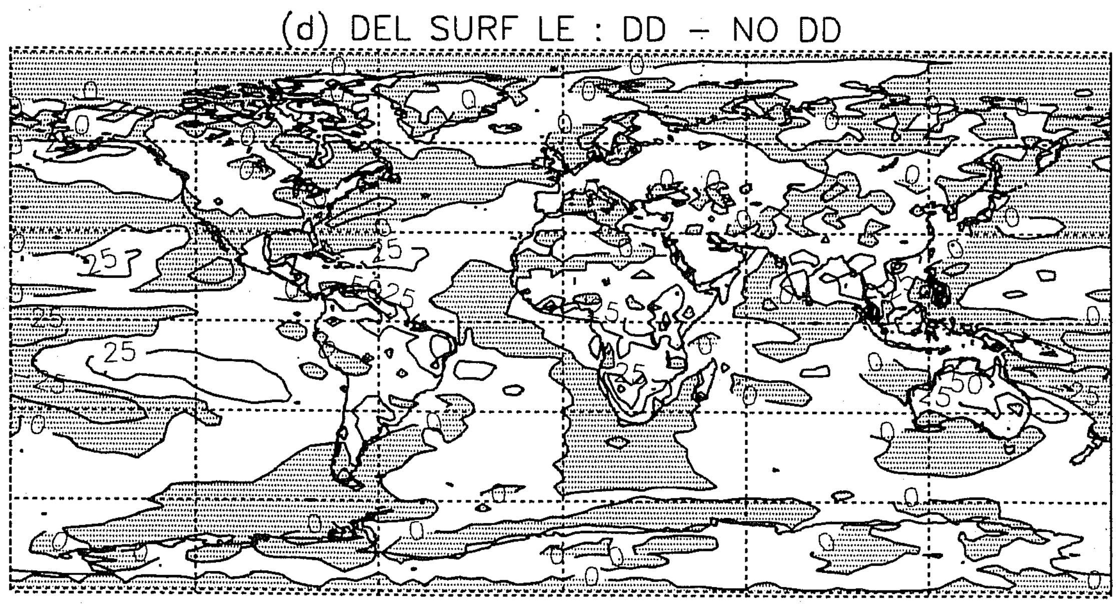


Figure 7 Continued

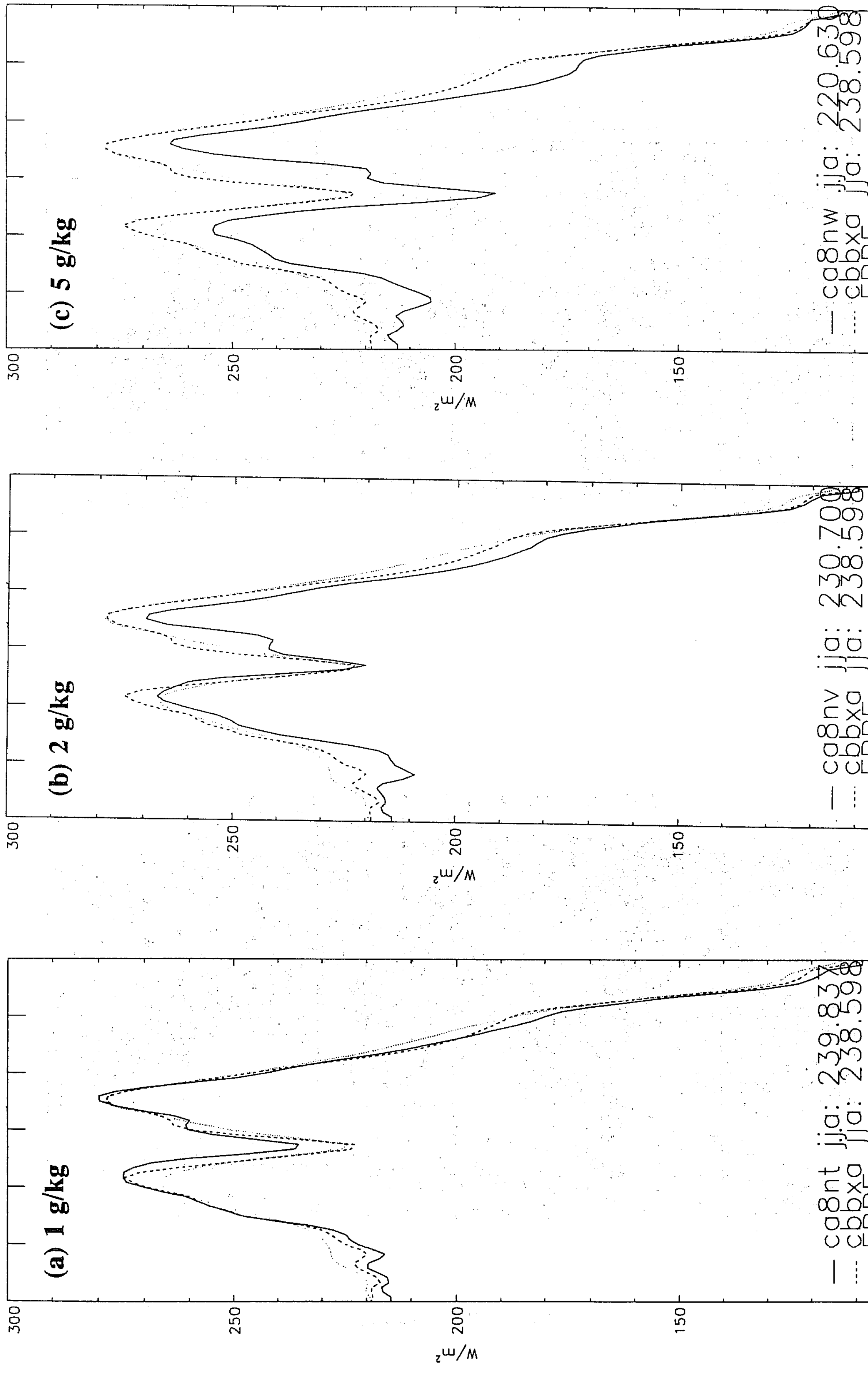


Figure 8 Sensitivity of OLR simulation (for June/July/August) by the UK Met Office Unified Model to amount of condensed water retained in the convective parcel : (a) 1, (b) 2, (c) 5 g/kg. (————) : simulation, (·····) : ERBE observations (1987-89)

current convection schemes.

2.4.3 The closure problem.

For mass flux parametrization schemes the closure problem is essentially the need to estimate the convective mass flux at the base of the cloud. This determines the magnitude of the convective heat release while the cloud model used determines how this heat is distributed in the vertical. No exact theory exists for the closure problem and several different methods have been used. However it is possible to broadly classify these various closures into "dynamical" and "adjustment" types.

(a) Dynamical closure

"Dynamical" closures relate the cloud base mass flux to the large-scale forcing and atmospheric structure, usually making some assumption regarding the "quasi-equilibrium" of the convecting atmosphere. An example of this type of closure is the moisture convergence closure of the Tiedtke (1989) convection scheme at ECMWF where the mass flux is calculated from the requirement that the sub-cloud layer moisture content is kept constant when convection operates. A more complex closure of this type is that of the Arakawa-Schubert (1974) scheme, the principle of which is outlined below.

Arakawa and Schubert (1974) considered the generation of vertical kinetic energy by buoyancy,

$$A=\int_{cloud} m_c \frac{(\theta^c-\theta)}{\theta}\frac{dp}{\rho g} \tag{17}$$

where A is termed the cloud work function. For simplicity virtual effects of moisture and the loading of condensed water have been neglected in this analysis.

Defining the normalised mass flux, η, as

$$m(p)_c = m_B \eta(p) \tag{18}$$

with m_B being the cloud base mass flux, equation (17) can be written as,

$$A=m_B\int_{cloud} \eta(p)\frac{(\theta^c-\theta)}{\theta}\frac{dp}{\rho g} \tag{19}$$

Arakawa and Schubert then assumed that the rate of change of the cloud work function with time is approximately zero, i.e.,

$$\left(\frac{\partial A}{\partial t}\right)=\left(\frac{\partial A}{\partial t}\right)_{LS}+\left(\frac{\partial A}{\partial t}\right)_{CONV}\approx 0 \tag{20}$$

where $(\ )_{LS}$ is the rate of change of A with time due to large-scale forcing and $(\ )_{CONV}$ is the rate of change of A with time due to convective processes. Thus a quasi-equilibrium is assumed to exist between large-scale and convective processes.

Differentiating equation (19) with respect to time and applying the condition of quasi-equilibrium as defined by equation (20) it can be shown, assuming steady state clouds, that

$$\int \eta(p)\frac{\overline{\theta}^{c}}{\overline{\theta}^{2}}L_{\theta}\frac{dp}{\rho g}+\int \eta(p)\frac{\overline{\theta}^{c}}{\overline{\theta}^{2}}Q1\frac{dp}{\rho g}=0 \tag{21}$$

where L_θ is the rate of change of potential temperature with time due to large-scale forcing.

Making the approximation that Q1 is well approximated by compensating subsidence,

$$Q1\approx m_{c}\frac{\partial\theta}{\partial p}=m_{B}\eta(p)\frac{\partial\theta}{\partial p} \tag{22}$$

then from equation (21) the cloud base mass flux is given by,

$$m_{B}=\frac{\int \eta(p)\frac{\overline{\theta}^{c}}{\overline{\theta}^{2}}L_{\theta}\frac{dp}{\rho g}}{\int \eta(p)^{2}\frac{\overline{\theta}^{c}}{\overline{\theta}^{2}}\frac{\partial\theta}{\partial p}\frac{dp}{\rho g}} \tag{23}$$

The assumption of the quasi-equilibrium of the cloud work function is well supported by observations. Figure 9 compares the rate of change of A with time against the rate of change of A due to convection calculated from Marshall Island data (from Arakawa and Schubert 1974). Although convective forcing provides a large tendency to change the cloud work function, the actual change of A is small.

(b) Adjustment closure

In schemes which use an "adjustment" closure, the cloud base mass flux is calculated from relaxing the large-scale atmosphere back to an equilibrium structure. The Fritsch and Chappel (1980) scheme provides an example of this type of closure, where the cloud base mass flux is calculated from the

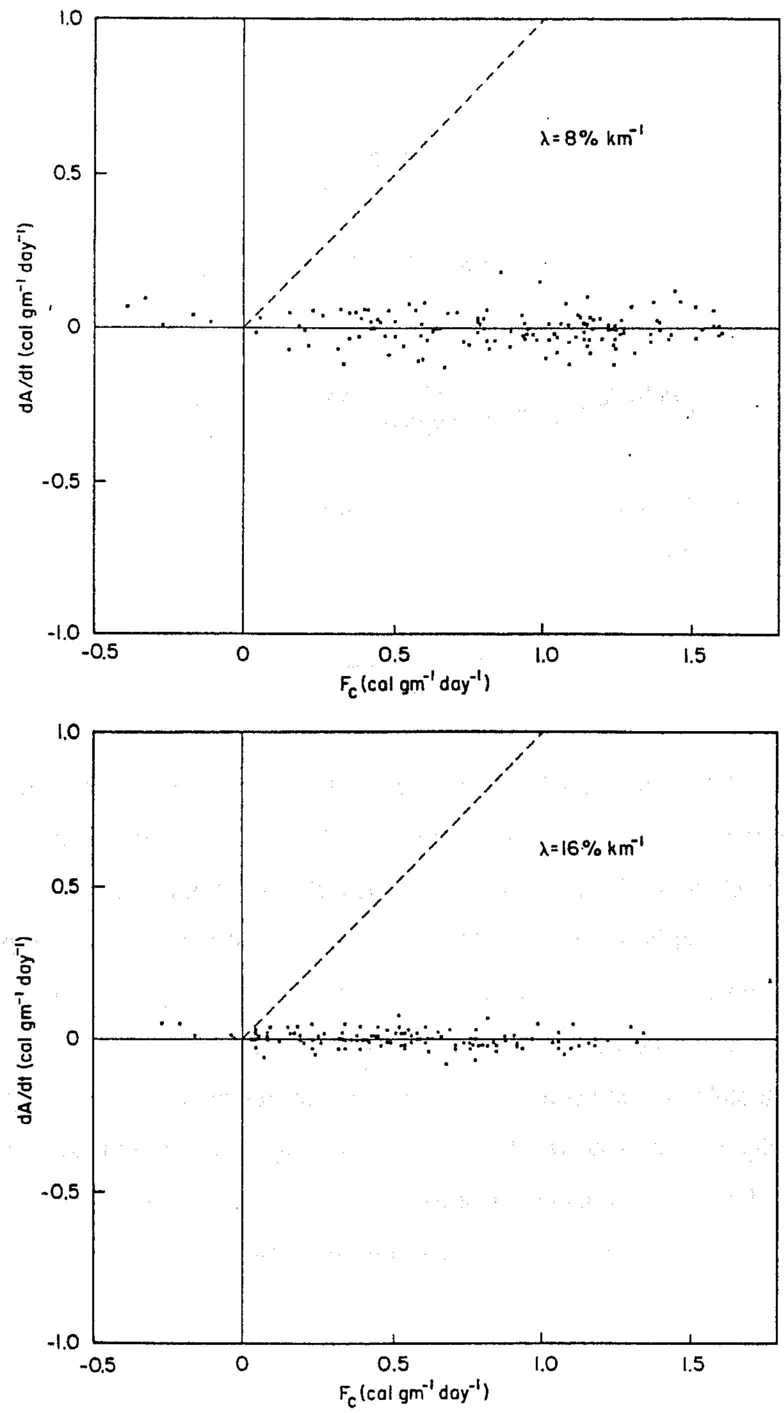


Figure 9 Change of cloud work function (A) with time compared to convective forcing (F_c) calculated from Marshall Island data for clouds with different entrainment rates (λ). (From Arakawa and Schubert 1974)

reduction of Convective Available Potential Energy (CAPE) over a time scale τ.

CAPE is defined as

$$CAPE=\int_{cloud}(\theta^{c}-\theta)\frac{dp}{\rho g} \tag{24}$$

Assuming steady state clouds, the rate of change of CAPE with time due to convective activity is given by,

$$(\frac{\partial CAPE}{\partial t})_{CONV}=-\int g(\frac{\partial\theta}{\partial t})_{CONV}\frac{dp}{\rho g}=-\int gQ1\frac{dp}{\rho g} \tag{25}$$

Approximating Q1 by a subsidence term as before and assuming that convection reduces CAPE to zero over a timescale τ, the cloud base mass flux is given by,

$$m_B=\frac{CAPE}{\tau}\frac{1}{\int\eta(p)\frac{\partial\theta}{\partial p}\frac{dp}{\rho g}} \tag{26}$$

Figure 10 shows the rainfall simulation from the Met. Office Unified Model at climate resolution (2.5x3.75 lat.-long. grid) averaged over a single June/July/August with the mass flux convection scheme using the above closure with an adjustment timescale of 2 hours together with the rainfall climatology of Legates and Willmott (1992). Although simpler that the dynamical type closure and having no information concerning the magnitude of the dynamical forcing other than the stability of the atmosphere in the vertical, the adjustment method is able to reproduce the mean distribution of rainfall well. The timescale for adjustment is resolution dependent due to the magnitude of the resolved vertical velocity on small horizontal scales increasing as resolution decreases (Nordeng, 1994). If a quasi-equilibrium is assumed to exist then moistening due to large-scale ascent is approximately balanced by compensating subsidence implying that

$$M_c\approx\rho\overline{w} \tag{27}$$

In general the resolved magnitude of the vertical velocity roughly doubles as the horizontal grid length is halved, implying that the timescale for adjustment must decrease as resolution increases.

(c) Choice of closure

Both the Arakawa-Schubert closure and the simpler adjustment method appear to perform well in determining the magnitude of the net convective heat release. Is there any reason why one is to be

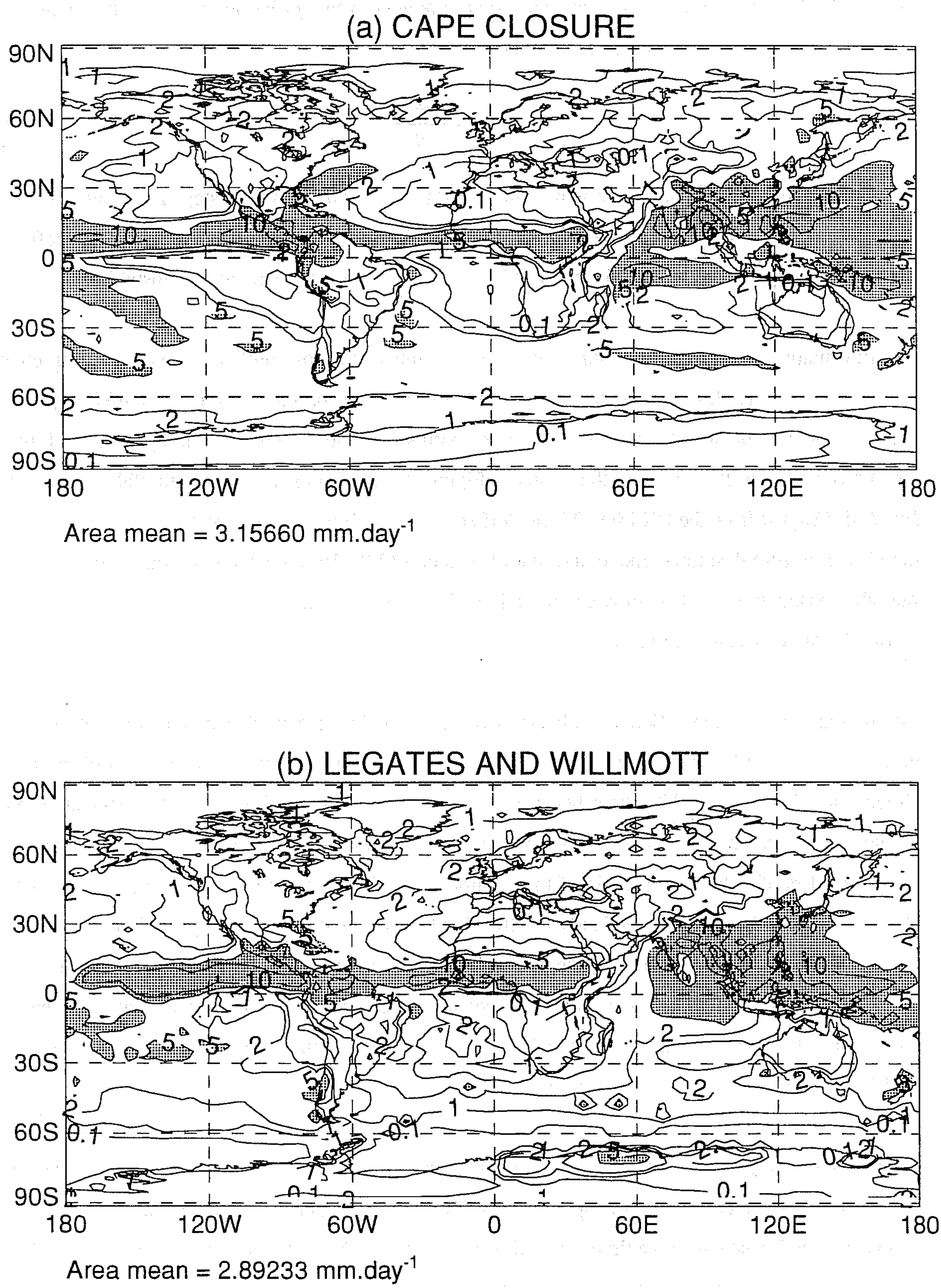


Figure 10 Comparison of rainfall from (a) a 90 day simulation (June/July/August) with a mass flux convection scheme using a CAPE adjustment closure with (b) the climatology of Legates and Willmott.

preferred over the other? Some insight into this question is obtained by considering the closure of the mass flux scheme presented in Gregory and Rowntree (1990) and used in the UK Met. Office Unified Model. In this scheme the cloud base mass flux is related to the initial buoyancy of the parcel lifted over a pressure interval Δp, usually one model level;

$$m_B = C\frac{Buoy}{\Delta p} \tag{28}$$

where C is an empirical coefficient which may need to vary with both time step and resolution (being 5.17×10^{-4} in the standard climate configuration of the UK Met. Office Unified Model). The coefficient C may be interpreted as a measure of the efficiency of convection at removing instability.

It is informative to consider how this simple closure relates to those outlined above. Having chosen the cloud base mass flux by another method the closure could be interpreted as a CAPE adjustment closure in which the adjustment timescale varies with space and time depending upon the stability of the atmosphere at the base of the cloud. Figure 11 compares the mean rainfall for a single June/July/August from the UK Met. Office Unified model at climate resolution from experiments using the CAPE closure described above and that of equation (27). Both closures reproduce the pattern of rainfall reasonably well compared to climatology with little difference being seen in the thermodynamic structure in the vertical.

Comparing to the Arakawa-Schubert closure, choosing cloud base mass flux by equation (27) does not imply a quasi-equilibrium of the cloud work function at any given time. However, as discussed by Gregory and Rowntree (1990) the scheme does come to equilibrium with imposed large-scale forcing within a single column model. Further evidence for this is provided by figure 12 which shows a plot of predicted rainfall through a cycle of the composite GATE easterly wave described by Thompson et al (1979), together with the tendencies of the cloud work function due to large-scale and convective forcing for a single column model integration. Ten hour averages are plotted, being averaged over four integrations through the wave cycle. The convective and large-scale forcing are clearly in balance throughout the wave cycle, even though the convection scheme is not constrained to such an equilibrium on a timestep to timestep basis.

These results suggest that although the Arakawa-Schubert closure hypothesis provides an accurate description of the state of the atmosphere when convection occurs, it is not essential to impose this upon a convection scheme on a timestep to timestep basis. In the real atmosphere convection is driven by buoyancy forces and does not have any information concerning the magnitude of the large-scale forcing other than how the forcing has affected the stability of the large-scale atmosphere. The quasi-

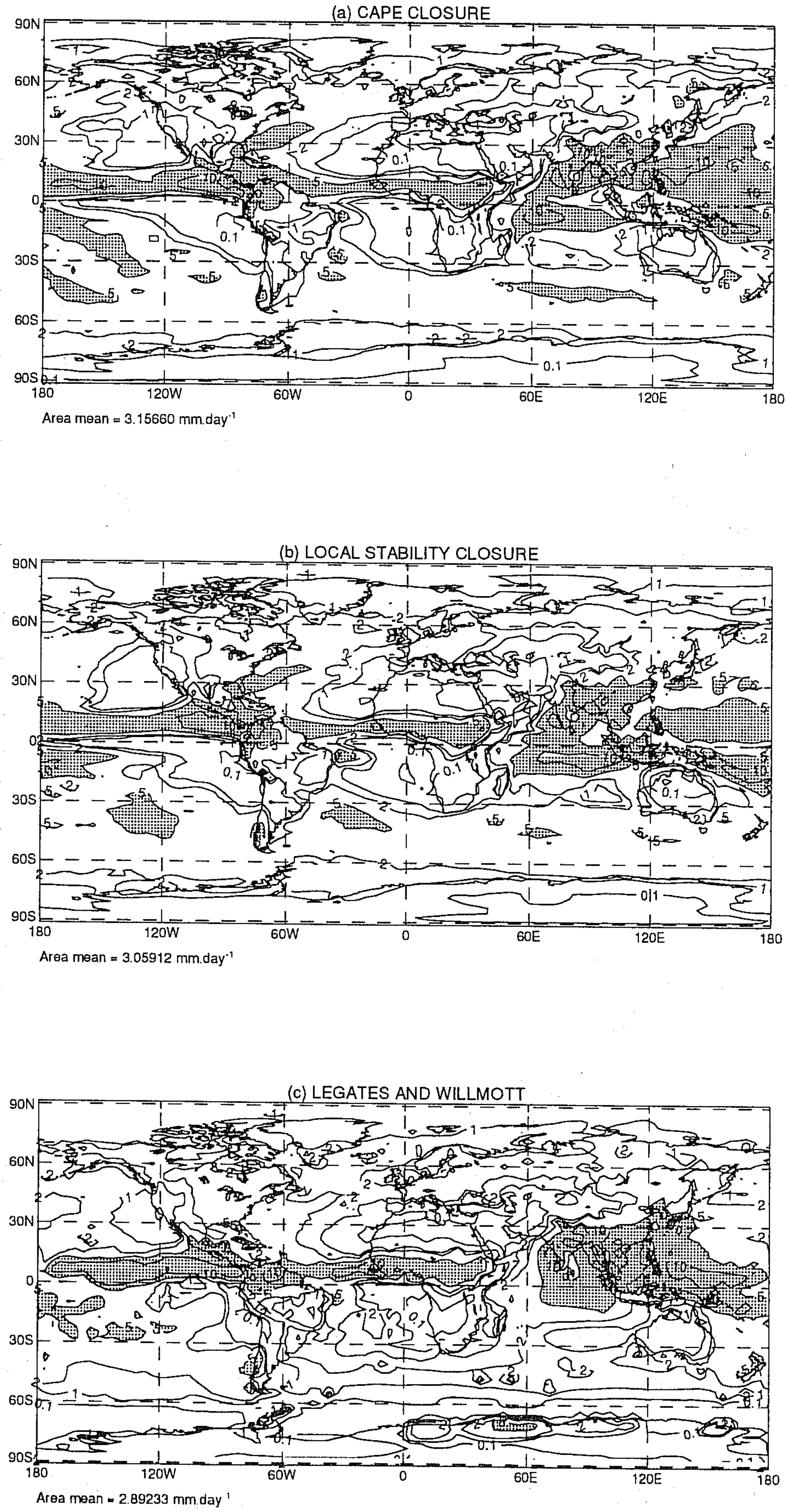


Figure 11 Comparison of rainfall from 90 day simulations (June/July/August) with mass flux convection schemes using (a) a CAPE adjustment closure and (b) a local stability closure with (c) the climatology of Legates and Willmott.

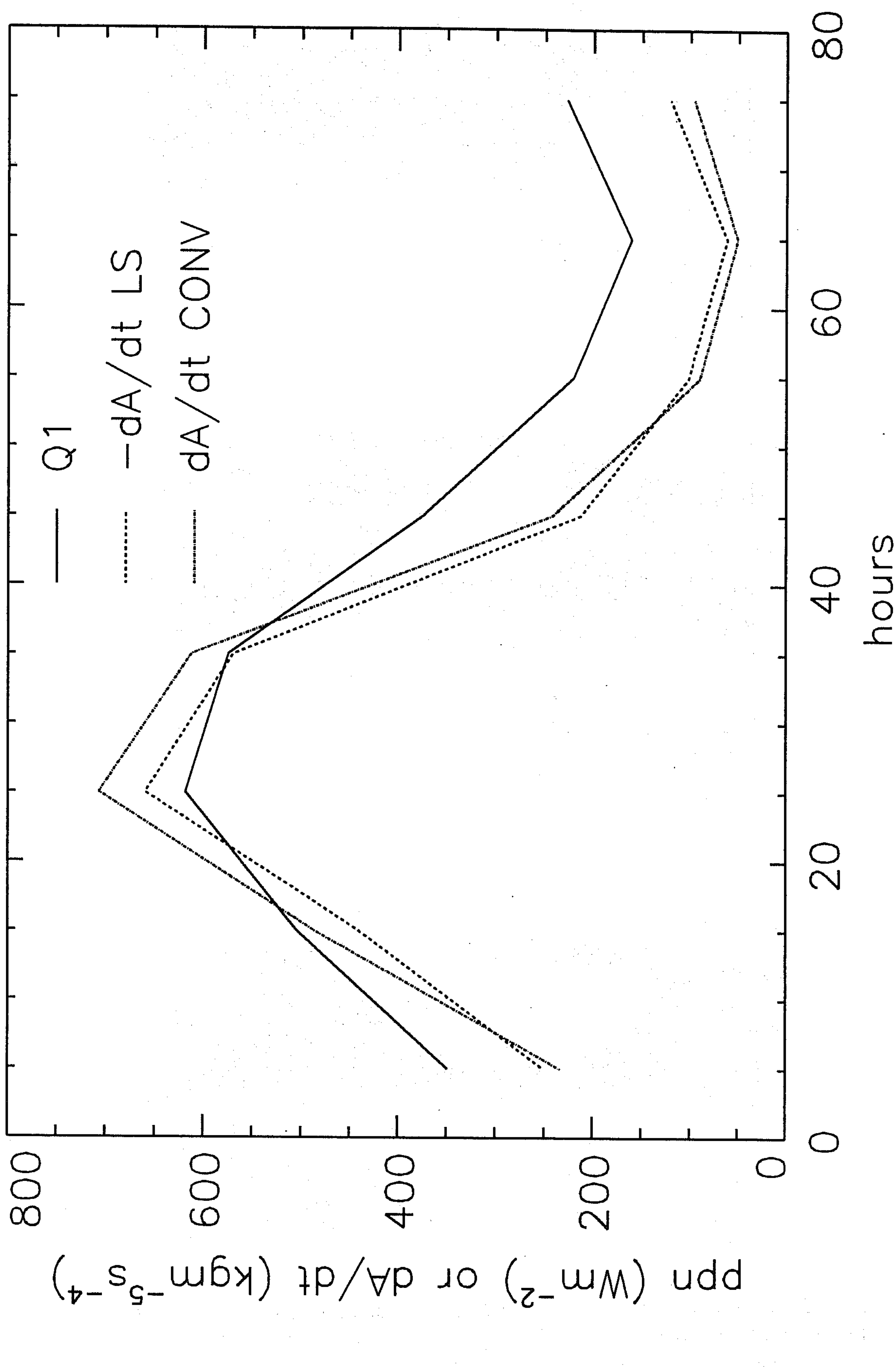


Figure 12 Q1, $-(dA/dt)_{LS}$, $(dA/dt)_{CONV}$ through a simulation of a GATE easterly wave using a mass flux convection scheme with a local stability closure.

equilibrium observed comes about through the modulation of the vertical stability of the atmosphere. This is essentially the route by which schemes that rely upon some measure of instability to determine the cloud base mass flux (see equations (26) and (27) above) achieve quasi-equilibrium. If the pattern of convective heating in the vertical matches the large-scale forcing the scheme will be in equilibrium, as defined by the Arakawa-Schubert scheme. If this is not the case then equilibrium is established through a modification of the thermodynamic structure of the atmosphere. Hence, as discussed in section 1.2 above, one characteristic of a good closure method is that the equilibrium is established with a realistic thermodynamic vertical structure.

(d) Comments upon moisture convergence closures

Other considerations must also be taken into account when choosing the closure of convection schemes. Recent work by Slingo et al (1994) has suggested that using a moisture convergence closure (Kuo type schemes) may lead to an underprediction of tropical transient activity. Figure 13 (from Slingo et al 1994) shows the square root of the variance of top of atmosphere OLR on a variety of timescales, ranging from synoptic to intraseasonal from two 360 day perpetual January integrations of the UGAMP GCM, one using the Betts-Miller convection scheme, the other a Kuo scheme . Transient activity is lower in the integration using the Kuo scheme for all timescales. Nordeng (1994) finds a similar result in the ECMWF model. He replaced the moisture convergence closure of the Tiedtke (1989) convection scheme with one similar to the adjustment closure described above and found improved simulation of tropical synoptic activity. However the failure of the moisture convergence schemes in these studies may be related to the exact method by which they are applied in the GCMs concerned. For example in the UGAMP GCM used in the Slingo et al (1994) study, the Kuo scheme was allowed to operate at all places where moisture convergence was present, caused either by surface evaporation or the atmospheric circulation. This led to the convection scheme acting over much of the tropical oceans, resulting in widespread weak precipitation and a poorly defined ITCZ. In the Bureau of Meteorology Research Centre (BMRC) GCM the Kuo scheme is only allowed to operate when the mean relative humidity in the cloud layer exceeds 80%, resulting in a more selective application of the convection scheme. Used in this manner the tropical transient activity appears reasonable (McAvaney, personal communication). It is clear that further clarification is required regarding the use of moisture convergence closures and analysis of the AMIP integrations may provide this.

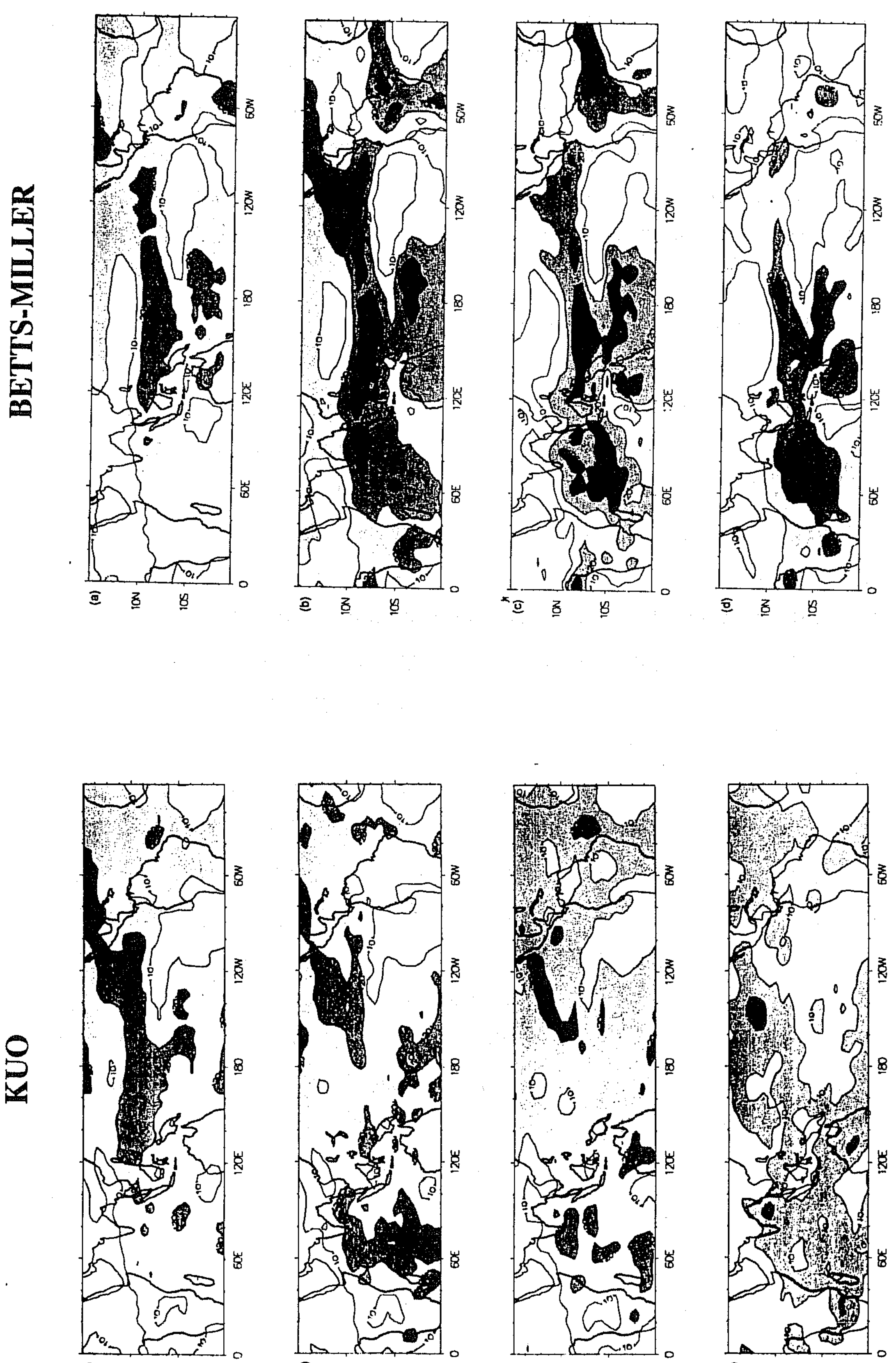


Figure 13 Square root of varience of the outgoing long-wave radiation (Wm^{-2}) for perpetual January simulations of the UGAMP GCM using a Kuo and the Betts-Miller convection schemes and explained by frequencies between (a) 2 and 6 days, (b) 6 and 14 days, (c) 14 and 30 days, and (d) 30 and 70 days.

3. Momentum transports

So far the discussion of the parametrization of convection has centred upon its impact on thermodynamic fields. However convection is also known to redistribute momentum in the vertical. Although difficult to carry out accurately, observational budget studies of GATE easterly waves (Stevens, 1979) and tropical cyclones (Lee, 1984) show residuals in the large-scale momentum budget which are associated with convective activity. However Thompson and Hartmann (1979) in a study of the Hadley circulation suggested that the vertical transport of momentum by convection was not a significant factor in determining the general circulation of the tropics. Hence the exact role of convective momentum transports in the general circulation of the atmosphere is not well understood. However several modelling centres report large impacts on the simulation of the tropical circulation when convective momentum transports are included into GCMs (for example, ECMWF - Nordeng, 1994). However the techniques used do not take into account the organisation of cloud systems which is known to determine their transport properties.

In principle the mass flux theory derived previously can be used to describe the transport of momentum by convection. Consider the momentum equation, in two dimensions and pressure co-ordinates. Averaging over a large area and expressing the fields in terms of the large-scale mean and un-resolved eddy contribution (see equation (1)), the momentum equation may be written as,

$$\frac{\partial\bar{u}}{\partial t}+\frac{\overline{\partial u^2}}{\partial x}+\frac{\partial\bar{\omega}\,\bar{u}}{\partial p}=-\frac{\partial\overline{\omega' u'}}{\partial p}=Q3 \qquad (29)$$

where Q3 is the apparent momentum source due to convection.

Q3 can be expressed in terms of cloud variables as in section (2) above;

$$Q3=-\frac{\partial\sigma(\overline{\omega u}^c-\bar{\omega}^c\bar{u})}{\partial p} \qquad (30)$$

As previously the incloud velocity can be obtained from a simple plume model of convection;

$$E\bar{u}-\Sigma_{\det}D_i\bar{u}_i^c+g\frac{\overline{\partial h}^c}{\partial x}+\frac{\partial\sigma\overline{\omega u}^c}{\partial p}=0 \qquad (31)$$

Hence the incloud velocity is seen not only to be affected by entrainment but also the presence of across cloud pressure gradients. It is unclear how this is to be accurately included into parametrizations of convective momentum transport as it will be influenced by the vertical wind shear and organisation of the cloud. The cloud organisation can also affect the direction of the momentum transport, with up-gradient transports occurring in some cloud types.

The earliest suggested scheme for convective momentum transport (Schneider and Lindzen, 1976) suggested that as ascent within cloud was rapid, the effect of across cloud pressure gradients could be ignored and that the in-cloud velocity should be kept fixed at the inflow value throughout the ascent. Diagnostics from cloud resolving models show that this is a poor assumption. Figure 14 compares the velocity within cloud updraught and downdraught with the domain averaged velocity for a simulation of convection in a cold air outbreak (Kershaw, personal communication). The incloud velocity is 1 to 2ms^{-1} lower than the large-scale value. Hence incloud pressure gradients must be accounted for in estimating the impact of convection upon large-scale wind fields.

Several methods have been suggested to accomplish this. The Tiedtke (1989) mass flux scheme in the ECMWF model uses increased entrainment and detrainment rates for velocity (Nordeng, 1994). This keeps the incloud velocity closer to the large-scale value although the method has not been validated against observations or detailed cloud models. Zhang and Cho (1991) develop a model for the flow around a convective cloud and deduce the cloud pressure gradient from this. They compare the results of their parametrization with one using the Schneider and Lindzen hypothesis and with momentum budget residuals from the GATE (figure 15). Accounting for the cloud pressure gradient does give a better fit to the observations, especially in the upper layers of the cloud.

Moncrieff (1981, 1985) has advocated the use of more dynamical cloud models in the estimation of momentum transports. These simple two dimensional models are able to represent the different flow configurations observed to exist with different vertical shears and so include the impact of cloud organisation upon both thermodynamic and momentum fluxes. Moncrieff (1992) also suggests a more radical approach to the parametrization of momentum transports; rather than representing the momentum flux due to individual clouds, the momentum flux associated with the mesoscale circulation of the convective system should be represented. He found that the momentum flux estimated from the analytical model of the mesoscale flow associated with the squall line was in better agreement with an explicit cloud model simulation than those estimated using the more "classical" approach described above.

Although there is great uncertainty in the estimation of convective momentum fluxes, there is also uncertainty in how to apply parametrized fluxes to the grid of numerical models. The convective fluxes represent "unbalanced" forcing, moving the flow away from a quasi-balanced state, e.g. geostrophic balance. However recent simulations with explicit cloud models have suggested that characteristic balanced flow evolves on the large-scale in the presence of convective forcing (Shutts and Gray, 1994). Because of the coarse resolution used in GCMs, the resolved dynamics may not be able to represent the adjustment to these balanced states correctly. Hence it may be necessary to

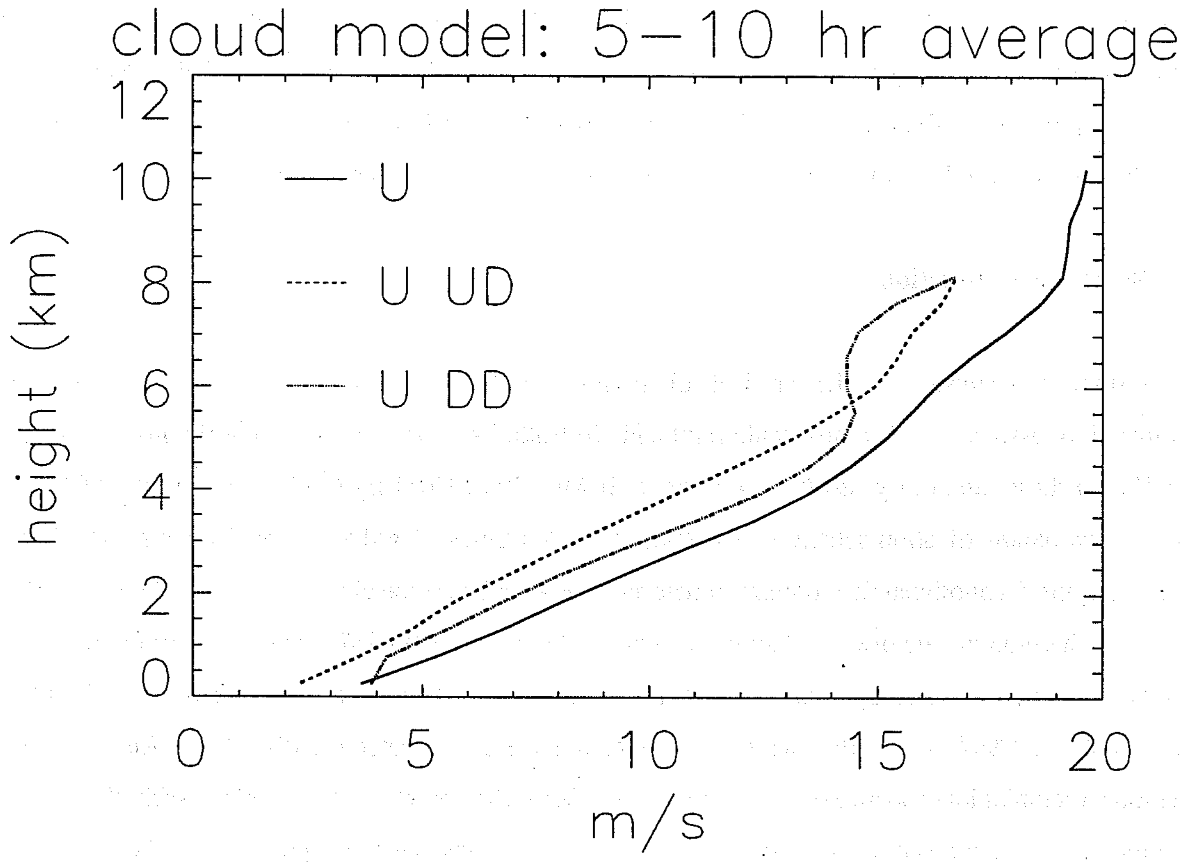


Figure 14 Domain averaged, updraught and downdraught horizontal velocity diagnosed from a simulation of deep convection in a cold air outbreak.

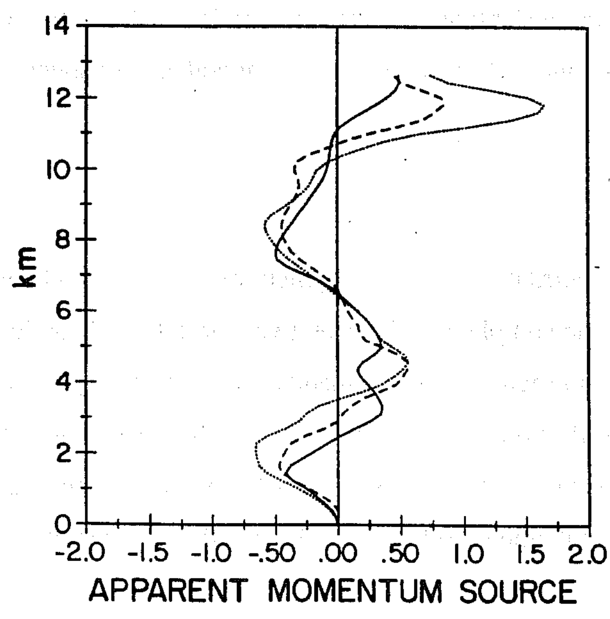


Figure 15 Vertical profiles of the x-component of the observed and parametrized apparent momentum sources for GATE convective cases. Solid line : observed; dashed line : parametrized as in Zhang and Cho (1991); dotted line : parametrized using Schneider and Lindzen's (1976) scheme. Units : 10^{-4} ms^{-2} (From Zhang and Cho, 1991)

parametrize the adjustment process itself, adjusting the atmosphere back to the balanced state in the presence of convection. However in a GCM, convection is rarely in isolation, occurring simultaneously at many grid points in close proximity. Hence the scale on which balanced structures exist may be larger than the gridscale of the model and so adjustment to these structures may be resolved by the model's dynamics scheme. This is a question for further investigation.

4. Slantwise Convection

A further mechanism for the vertical distribution of horizontal momentum may be slantwise convection, associated with mesoscale rainbands in frontal systems. Having a horizontal scale of 50 to 100km these are poorly resolved in current GCMs. Both Nordeng (1987) and Chou (1994) have developed parametrization schemes to develop these processes. Nordeng's approach was based upon adjusting the thermodynamic structure to neutrality using a Kuo type closure. This scheme considered only the thermodynamic impact of slantwise convection whereas the latter scheme, based upon a mass flux approach, considered dynamical transports. Studying the impact of this scheme upon the ECMWF model, Chou (1994) found that slantwise convection acted as a selective diffusion. Also in frontal regions precipitation production was transferred from the large-scale rain parametrization to the slantwise convection scheme (figure 16). This may have consequences upon the simulated moisture content of the mid-latitude atmosphere which is generally acknowledged to be larger than observations suggest. Removal of moisture by slantwise convection, which is a sub-grid scale process, may prevent excessive moisture, although Chou (1994) did not consider this point. Nordeng (1990) found that the inclusion of a parametrization of slantwise convection in a simulation of frontal precipitation enhanced rainfall rates locally and brought about a reduction in modelled atmospheric humidity.

4. Concluding comments

This paper has outlined the current state of development of convection schemes used in GCMs of the atmosphere. Compared to the complexity of convective systems in the atmosphere the schemes used are still relatively simple, but progress has been made over the past 10 years with schemes of a greater physical basis being included into many models, the mass flux approached discussed in detail above becoming more favoured in the modelling community. However much uncertainty remains and the simulation of the general circulation is still very sensitive to the representation of convective processes.

Although not considered here the tropical flow is very sensitive to the representation of shallow convection. The top of atmosphere radiation budget has been shown to be extremely sensitive to the supply of moisture to the upper troposphere by convection. Representation of the mesoscale

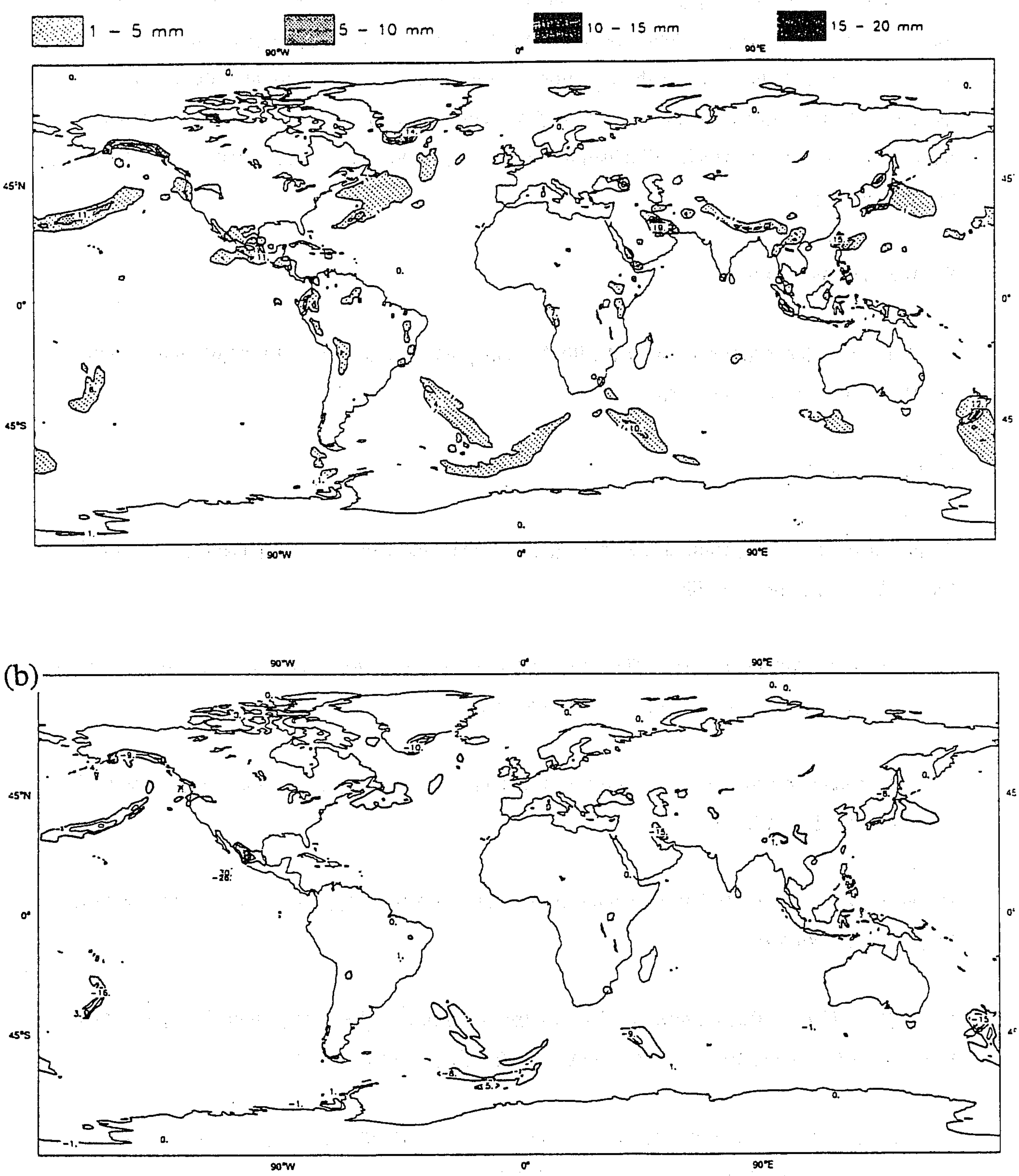


Figure 16 48 hour accumulated rainfall from the ECMWF model (a) due to slantwise convection and (b) reduction in large-scale rainfall when a parametrization of slantwise convection is included. (From Chou, 1994)

components of organised convection (anvils) is poor in many models, although recently some attempts have been made to parametrize these (Donner, 1993). The question of the importance of momentum transports and the impact of cloud organisation still remains unresolved. Greater understanding of these areas will only be achieved through more observational studies and through the use of detailed models of the processes concerned. Such work is being encouraged by the GEWEX Cloud Systems Study (Browning, 1994) which will hopefully bear fruit in improved convection parametrizations over the coming 10 years.

5. Acknowledgements

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