FINE–SCALE MODELLING OF ORGANIZED DEEP CONVECTION IN THE CONTEXT OF PARAMETERIZATION

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Summary: The role of deep, precipitating, organised convection as a sub–grid scale process in high resolution general circulation models raises new issues in scale interaction and parameterization. Theoretical developments are necessary because this type of convection involves episodic, spatially isolated phenomena of a scale not well separated from the mesh size. This is distinct from the statistical homogeneous realisation approach identified with other sub–grid scale convection theories. This paper presents idealised mathematical models to understand the primary dynamical processes and attendant flux laws involving organised convection. Emphasis is on momentum fluxes by travelling convective systems. An archetype analytical model compares well with observations both in terms of dynamical structure and flux profiles. This archetype theory can be extended to a hierarchy of models applicable to a range of mesoscale convective system regimes. The approach identifies parameterization concepts with the basic dynamics of convection and the interactions among convective, mesoscale and large scale processes.

1. INTRODUCTION

The role of mesoscale phenomena in the large scale circulation of the atmosphere is a fundamental but poorly understood problem. Basic research into organised convection and its scale interactions is of intrinsic scientific merit. It can improve the way that scale interactions involving complex multi–phase, nonlinear processes are approximated and in practical terms provide new insight into the long–standing problem of convective parameterization.

The type of convection addressed herein is the organised, deep, and heavily precipitating type associated with the meso-β scale (20–200 km) and strong wind shear. It is epitomised by mesoscale convective systems (MCSs). The horizontal scale of MCSs is not well separated from the grid scale and this raises new challenges in sub–grid

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scale transport (parameterization) theory. This aspect is pertinent as the resolution of large scale models improves and moist convection needs to be more realistically represented. General circulation models (GCMs) do not meaningfully resolve the meso–β scale and parameterization schemes do not adequately represent the physics involved. It is sobering that organised convection cannot be adequately resolved even in mesoscale models with a horizontal resolution of 12.5 km (Zhang et al., 1988). GCMs will not be able to achieve this resolution in the foreseeable future, giving more urgency to the formulation of new ideas and theories. This is particularly true for momentum transport because of its inherent dynamical nature. Herein, emphasis is on the archetype mathematical models of momentum fluxes and their mean flow representation. The study of momentum fluxes requires dynamical models and brings a new basic perspective to parameterization.

The interaction of convective fluxes with the forcing circulations, manifested by mass and moisture convergence and other processes, is the crucial aspect in parameterization and not the fluxes *per se*. Communication between convective elements and large scale circulations frequently involves intermediate scales and in this regard the meso–β scale plays a vital role. Diabatically–driven circulations in the tropics down to scales of several hundred kilometers, the lower end of the meso–α scale (200–2000 km), are presumably resolved in GCMs that have a mesh length of (say) 60 km. Explicit representation of organized convection in these models will not be realistic until the resolution is O(1 km), thus not in the foreseeable future. With a 60 km grid, however, the interaction of (parameterized) organised precipitating convection and the larger scale processes can be studied on a case study basis *e.g.* highly convective synoptic conditions over summertime continents and the tropical western Pacific. The latter is a primary example of a hierachy of organised convection on scales spanning O(10–1000 km).

The role of organized precipitating convection in the general circulation of the atmosphere raises concepts that depart from the statistical basis of the current approach to parameterization. The distinction arises from the dynamical nature of organised convection and the attendant control over the fluxes. Models of greater dynamical sophistication than the rudimentary plume models currently used in convective parameterization schemes are necessary. This raises fundamental issues on how these systems should be parameterized, one of the main subjects addressed herein. In order to make progress it is necessary to understand the complex, non–linear, multi–scale dynamical interactions associated with organised systems. *However, flux representations of minimal complexity are necessary in parameterization schemes and this requirement has largely motivated the archetype dynamical model development described herein* .

There have been considerable advances in the theory, modelling, and observational definition of organised convection (MCSs in particular) during the last decade or so (Rutledge, 1991). However, there is a dearth of understanding of the interaction of MCSs with the larger scales of motion. The necessity for including convectively–induced circulations in GCMs has not yet been adequately explored. A mathematical theory for the fluxes is necessary for studying the interaction of convection with other scales of motion. Flux formulation in terms of mean flow variables is a fundamental problem throughout fluid dynamics. The MCS is particularly challenging because multi–phase physics in a shear flow is involved. The magnitude of this task is put into perspective by noting that there is a wealth of unsolved scientific problems associated with the physically simpler single– phase phenomena in unsheared base states (*e.g.* geophysical turbulence). Arguably, there is an analogy between the role of coherent structures in geophysical fluid dynamics and organised deep convection in a field of random cumulus in the sense that both require an understanding of persistent isolated entities embedded within a stochastic field of motion.

1.1 Fine–scale Modelling as a Parameterization Scheme Test Facility

Fine–scale models in the present context are defined as models that have a mesh length of about 1 km and domains of at least 100 km. In terms of parameterization, fine–scale (convection resolving) modelling is currently used more as a facility for testing existing schemes than as a procedure to develop new ones. The verification of the horizontally averaged quantities such as the apparent sources of heat, moisture, momentum (the so–called Q_1, Q_2 and Q_3 quantities, respectively) in different meteorological conditions against parameterized and observed values is a valuable application of these models. For example, a two–dimensional model has been extensively used to test the quasi–equilibrium hypothesis that is the backbone of the Arakawa and Schubert (1974) scheme (Lord, 1982; Krueger, 1988; Xu 1990). This approach, in concurrence with the basic concepts behind that scheme, largely involves statistical analyses of cloud fields and an evaluation of the closure hypothesis. The physical basis of this approach has much in common with the large eddy simulation (LES) widely used in boundary layer modelling.

1.2 Fine–scale Modelling in Parameterization Research

Fine–scale models having large domains are useful for detailed studies of the effect of fluxes on the larger scales because the interaction among convective clouds, meso–β scales and the larger scale are thereby well represented. However, these models are no panacea for rapid progress in parameterization unless the void between the inherent complexity of fine scale modelling and the required simplicity of model realisations can be bridged. Indeed, how should fine–scale modelling be usefully applied to parameterization research? This question is of practical as well as theoretical significance because the computational resources required for three–dimensional, finely resolved, lengthy integrations can be comparable to those required for general circulation modelling. However, several fine–scale models can now resolve convection and its attendant mesoscale circulation in adequately large domains and thus address scale interaction directly. Note that microphysical and sub–cloud scale turbulence processes will always have to be parameterized in models of this type and improvements in the parameterization of these processes are welcomed.

1.3 Ordinary versus organised convection representations

Current convective parameterizations distinguish only between deep and shallow cloud categories and stratiform clouds are largely represented as a grid–scale process. Parameterizations either do not use cloud models (adjustment schemes) or, if used, these are of the one–dimensional entraining plume type (mass flux schemes) in which a single cloud realisation is used. Cloud models of this type are likely to be too rudimentary to represent organised cloud systems within a mesoscale ensemble, especially the momentum transport.

The concept of the classical entraining plume representation of buoyant elements within a convective ensemble (Ooyama, 1971) has been widely used in parameterization. It is, for example, applied in the Arakawa and Schubert (1974) scheme. In that case, each member of the cloud ensemble (the number of clouds is at most equal to the number of grid intervals in the convecting layer) entrains at all levels but detrains only within the grid interval containing the top of each cloud element. The mass flux $\eta(z,\lambda)$ for each cloud in the ensemble, normalised by the cloud base mass flux, is defined by $\eta(z,\lambda) = e^{\lambda(z-z_B)}$ for $z \in [z_B, z_D]$, where z_B and z_D are the cloud base and top, respectively. The value of λ is piecewise constant (i.e $\lambda = \lambda_i$, where the subscript denotes the i–th cloud in the ensemble). In practice, λ is calculated iteratively by finding the moist static energy (say) consistent with each cloud top. This simple representation is an appropriate model of transient convection in small windshear.

The perplexing aspect of the ensemble approach relates to the effect of mesoscale flow organisation on the fluxes. The theory developed herein should be considered as a broad strategy for an improved understanding of fluxes by organised convection and their induced mesoscale circulations. It is acknowledged that *individual clouds or weakly interacting cloud ensembles* as represented by the Arakawa–Schubert scheme are important realisations of 'ordinary' atmospheric convection. However, it is considered that *strongly interacting cloud ensembles* especially those associated with in marked wind shear may be an important missing link in the parameterization puzzle. If this is so, the meso–β scale is the primary interaction scale to understand. The importance of this strong interaction is expected to become more evident as the horizontal resolution of GCMs improves. More important, in the near future better precipitation forecasting products for input to hydrological models will be required. Clearly, highly organised, convective events are the most difficult to forecast accurately.

Organised convection largely occurs when the mean flow has a substantial shear and is, by definition, a less stochastic process than 'ordinary' convection. Arguably, it should be parameterized as a dynamical event as opposed to a statistically homogeneous realisation. Dynamical models based on exact solutions of the equations of mass, momentum, and thermodynamics have been derived by the author. The cloud mass flux can be calculated directly from these models, an approach that is somewhat different from an entraining plume model briefly summarised above. Miller and Moncrieff (1983) used a physically simple organised regime (the so–called 'classical model' of Moncrieff, 1981) that does not trasport momentum because it has vertically orientated updraught and downdraught branches. The entrainment/detrainment structure of this model is quite different from that in the entraining plume. This particular dynamical model (essentially a weak interaction regime) could be used to represent the deepest cloud in an ensemble representation in conjunction with entraining plume cloud elements.

The use of more than one type of cloud model in a parameterization scheme requires more sophisticated flux laws and criteria for regime selection. However, a small number of cloud categories should be adequate. The organisation of individual clouds into an ensemble raises the concept of a *cloud system representation* and attendant dynamical processes in parameterization theory. A paradigm is the travelling organised convective system, the ubiquitous MCS.

2. MESOSCALE CONVECTIVE SYSTEMS

The MCS is a particularly energetic and organised type of deep convection. Mesoscale convective complexes (MCCs), as defined by Maddox (1981), and squall lines are both subsets of the MCS family. Collectively, these systems are the major producers of warm season precipitation over the central USA (Fritsch et al., 1986) and other regions. However, they are not properly represented in GCMs and certainly not as organized entities. They are represented to a degree in regional mesoscale (limited area) models but their occurrence is highly dependent on the choice of convective parameterization scheme (Zhang et al., 1990).

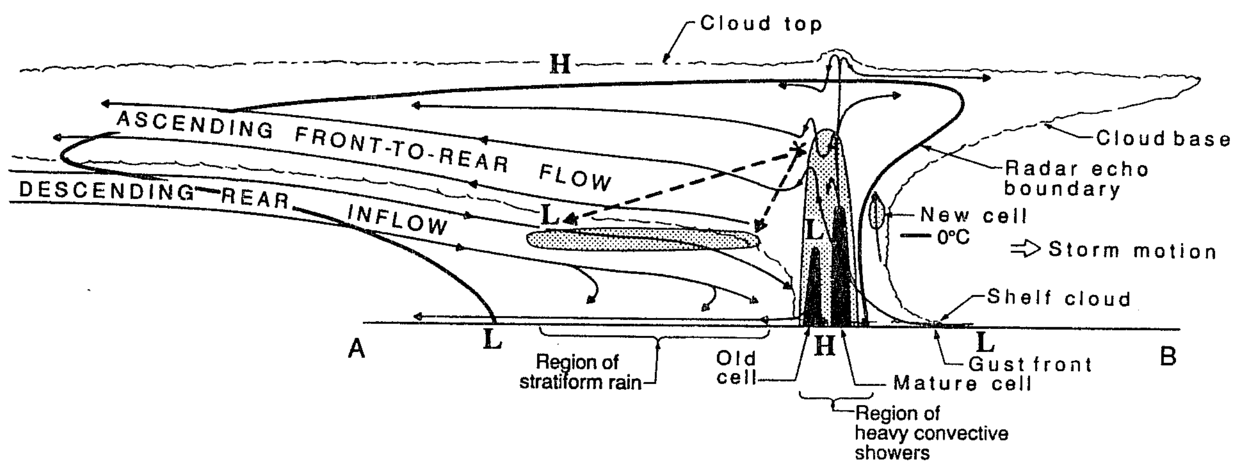

Fig. 1. Schema of the relative airflow and physical processes associated with a squall line type MCS. This is a vertical cross section orientated perpendicular to the line. Note the distinct scales of the convective and stratiform regions. [From Rutledge, 1991].

This implies that the heating profiles as represented by current parameterization schemes operating in areas where MCSs are active may be seriously flawed; the *direct* momentum fluxes are almost always omitted and, if included, are usually misrepresented. MCSs have extensive cirrus/stratus cloud shields (see satellite images) and an attendant mesoscale circulation. However, the area of strong convection and heavy rainfall (see radar scans) is relatively small. Figure 1 schematises this dual–scale structure. Consequently, although the effects of mesoscale convective systems certainly extend for hundreds of kilometers, their mesoscale circulation is primarily driven by the baroclinic effects arising from the heating due to an ensemble of deep convection cells of scale O(1–10 km), as shown by Lafore and Moncrieff (1989). *The hypothesis that this relatively small scale of strong convection effectively drives a mesoscale circulation implies that the parameterization of these systems will not be circumvented by explicit resolution in GCMs nor, for that matter, in limited area models.*

However, there is one caveat to this statement. If a MCS evolves into an MCC, it tends to generate a gyre of vertical vorticity extending throughout a substantial part of the troposphere (McAnnelly and Cotton, 1989). The dynamical scale of this gyre is comparable to the Rossby radius of deformation and the system evidently attains a degree of geostrophic adjustment. Consequently, it is then essentially a 'quasi–balanced' system for which classical potential vorticity theory may be more appropriate (Hertenstein, 1988) than the 'unbalanced' process considered herein. Presumably, high resolution GCMs can explicitly resolve (albeit crudely) such balanced systems *provided they are spun up by the convective parameterization scheme used, although this probably does not often happen.* Operational GCMs need to adequately represent both types of precipitating systems, for example to improve precipitation forecasting on mesoscales.

MCSs significantly affect scales smaller and larger than their characteristic scale in a more energetically and dynamically interactive fashion than 'ordinary' convective clouds: (1) Convective scale downdraughts disturb the boundary and surface layers over a large area because the systems are long–lived, are several hundred kilometers in length and propagate relative to the earth; (2) mesoscale downdraughts (largely dynamically forced descent) warm and dry the middle and lower troposphere; (3) the deep and extensive stratiform region behind many MCSs produce upper tropospheric stratiform (stratus and cirrus) decks that are radiatively significant; and (4) the dynamical organisation of the airflow produces anisotropic momentum fluxes that cause a distinctive flow acceleration, O(10 $ms^{-1}hr^{-1}$). MCSs are ubiquitous (Fig. 2a) and occur with a marked interannual climatic variability (Fig. 2b) as shown by Tollerud and Rodgers (1991). Note that Fig. 2 includes only MCCs, so an analogous treatment of MCSs occurrence could reveal even more striking statistics. None of these processes is adequately represented in current parameterization schemes, an inadequacy that can only become more acute as GCM resolution inevitably improves.

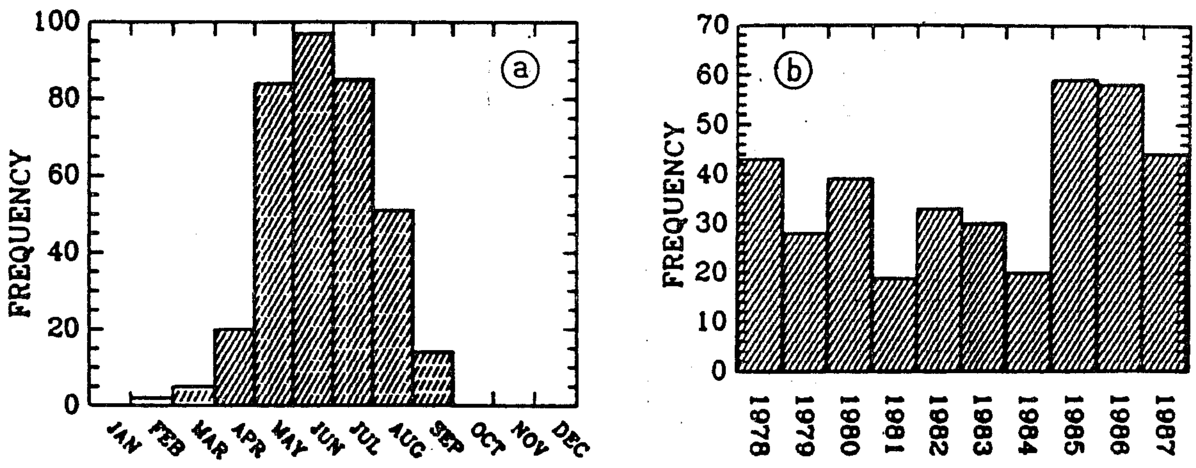

Fig. 2 (a) Number of MCC occurrences over the central USA for each month during the period 1978–87. (b) Annual variability of MCCs over the central USA during 1978–1987. [From Tollerud and Rodgers, 1991]

2.1 Limited-area Modelling

Zhang et al. (1988) showed that it is necessary to include parameterized convection to achieve a realistic simulation of a meso–β scale convectively disturbed weather system even when a grid length of 12.5 km is used in a limited area model. This aspect was further illustrated in Zhang et al. (1989) where, despite a fairly sophisticated large scale representation of the condensation/evaporation process, parameterized convection was required to produce sufficiently strong boundary layer (localised downdraught) cooling to initiate new convective activity and result in a realistic system life cycle. Limited area models with a mesh of (say) 10 km can reasonably well simulate meso–β circulations in highly convective conditions provided a suitable parameterization scheme is used. It will be interesting to see if this behaviour is emulated in very high resolution GCMs. *Note that the meso–β scale is essentially absent from current general circulation models with the result that an important flux* *interaction scale is artificially truncated.*

2.2 Fine-scale Modelling

The utility of fine scale modelling has emerged since the realisation that coarse horizontal resolution (about 1 km) could be advantageously employed in the simulation of deep convection (Miller and Pearce, 1974). This is especially true in circumstances where a high degree of dynamical control exists (*i.e.* in a shear flow) because fluxes are concentrated on well defined dynamical scales and dominate those on smaller scales. Provided the major energy sources are parameterized (condensation/evaporation and radiation) the major structural features are remarkably well reproduced. Thus, crude parameterizations of the evaporation and condensation processes are useful in fine scale models of organised convection. Moreover, to the extent that the phenomenon is largely conservative, dissipative processes are by definition of secondary importance. As a result, significant progress has been made with quite crude microphysics and turbulence parameterizations in models having meshes of O(1 km). The degree to which the introduction of more sophisticated (and computationally expensive) schemes will modify the overall dynamical structure of the systems (and hence the mesoscale fluxes) is currently unclear.

Fine-scale modelling has contributed significantly to the extensive progress that has been made over the last decade or so in understanding travelling organized convection (Moncrieff and Miller, 1976; Thorpe et al., 1982; Dudhia et al, 1987; Redelsperger and Lafore 1988; Crook and Moncrieff, 1988; Lafore and Moncrieff 1988; Nicholls, 1988; Rotunno et al., 1988; Fovell and Ogura, 1988; Tao and Simpson, 1989, among others). These simulations emulate observed system structure quite well, particularly squall line type MCSs consisting of an ensemble of strong convection cells that periodically develop (periods ranging from 20 to 35 min) over the leading edge of a cold pool that is maintained by evaporatively-driven downdraughts. Figures 3 and 4 show that these cells have a horizontal scale O(10 km) and travel backwards relative to the leading edge of the cold pool which moves in the direction of system propagation. These cells are embedded in a mesoscale circulation (ascent and descent) of horizontal scale O(100 km). This distributes diabatic heating over a wide area and helps establish baroclinic vorticity generation that is largely responsible for driving the mesoscale airflow. The Lafore and Moncrieff (1989) model study illustrates this type of behaviour in an African squall line case. This two–dimensional study used an interactively–nested model (Clark and Farley, 1984). In this simulation three nested domains were used (1, 2, and 3) having horizontal meshes of 6 km, 3 km, and 1 km, respectively, configured as in Fig. 3. Consequently, a well–resolved convective region was simulated in a large outer domain (480 km) that contained the mesoscale circulation. Several shear profiles and differing thermodynamic soundings were used in integrations extending up to 16 hours.

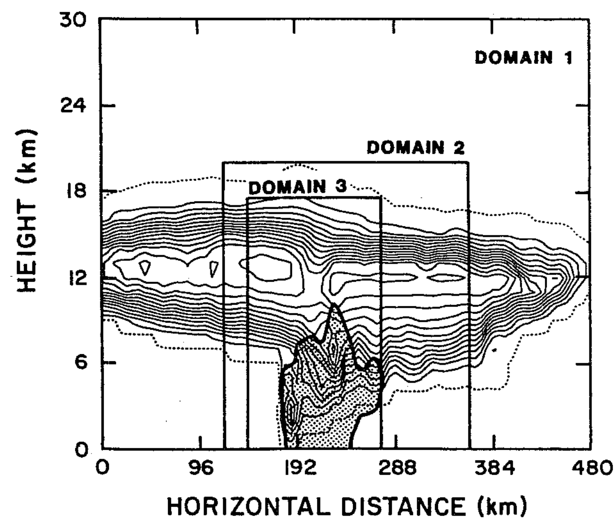

Fig. 3. The three–domain nested model cloud mixing ratio field q_c after 12 hours of simulation. The domains of the inner models are drawn and the heavy contour outlines of the heavy precipitation zone $q_r \geq 0.125$ g kg^{-1}. The q_c isoline is 0.25 g kg^{-1}. [From Lafore and Moncrieff, 1989].

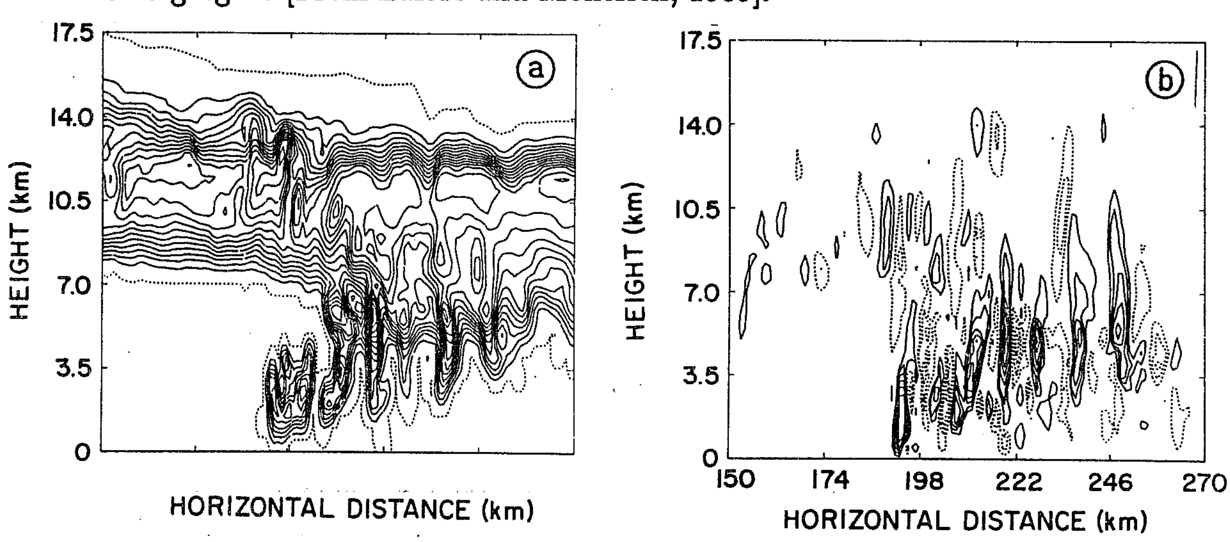

Fig. 4. The simulated convective scale structure of an MCS. (a) Cloud water in the inner domain with isoline interval 0.25 g kg^{-1} and (b) vertical velocity in the inner domain with isoline interval 1 m^{-1} [From Lafore and Moncrieff, 1989].

2.3 Dynamical Models of the Mesoscale Circulation

It is becoming clear from the analysis of data encompassing many field experiments in diverse geographical regions (not to mention satellite images) that MCSs are ubiquitous. Their contribution to the net convective flux divergence could be substantial. A simple but nevertheless dynamically precise formulation emulates the observed momentum flux profiles; a hierarchy of regime archetypes will now be summarised.

2.3.1 *Conservative hydrodynamic archetype model*

Figures 1, 3, and 4 show the basic morphology of line type MCSs. The hydrodynamic archetype solution developed in Moncrieff (1991), in which density is constant and no baroclinic sources of vorticity exist, is depicted in Fig. 5. The jump updraught branch (rear–to–front flow) is a fundamental component of the system, the structural integrity of the three–branch morphology and the mesoscale momentum fluxes. Despite its dynamical simplicity, this model captures the essential features of the observed momentum flux profile (see later). The far–field archetype is useful because it defines a single nondimensional number $E = \Delta p/\frac{1}{2}\rho U_0^2$, where U_0 is the constant inflow to the jump inflow branch and Δp is the pressure change across the system.

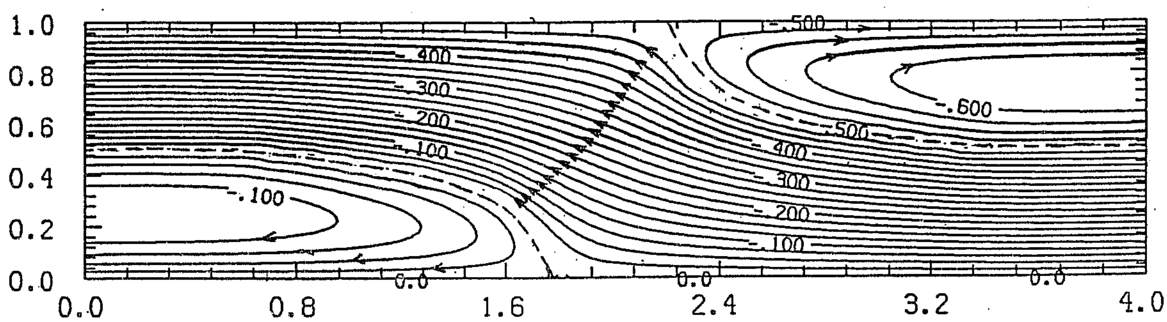

Fig. 5. Conservative hydrodynamic archetype MCS model. Streamfunction field is obtained from a numerical two–dimensional solution of an exact vorticity equation defining a free–boundary problem. The approximate shapes of the two free boundaries are shown. The flow is unstratified and $h_0 = h = 1/2$ in this example. Units are $U_0 H$ and the isoline interval is 0.025. [From Moncrieff, 1991]

This quantity is fundamental to the dynamics of isolated, organised propagating convection in a shear flow. Formally, it is the ratio of the work done by the pressure field to the kinetic energy of the jump inflow per unit volume. Note that vorticity generation by horizontal potential temperature gradients arising from latent heat release, evaporation and stratification are excluded from the archetype. A rationalisation of this strategy and a full discussion of the far–and near–field models is given in Moncrieff (1991).

The regime diagram for the so–called symmetric and asymmetric regimes that result from the theory is shown in Fig. 6. The domain–integrated horizontal momentum equation is the primary constraint on the solution domain. Only flow morphologies that have a certain functional relationship between the jump updraught depth (h_0) and the downdraught depth (h) can exist. This relationship for the asymmetric regime is given by $h_0 = (1-h)/(3-4h)$, the curve plotted in Fig 6. It can be shown that $h_0 \in [\frac{1}{3}, 1]$ corresponding to $E \in [-8, \frac{8}{9}]$, or approximately $\Delta p \in$ [–4 hPa, $\frac{1}{2}$ hPa] for an inflow speed U_0 of 10 ms^{-1}. Alternatively, the values of h and h_0 can be expressed solely in terms of E, as $h = \frac{1}{4}\left[3-(1-E)^{-\frac{1}{2}}\right]$ and $h_0 = \frac{1}{4}\left[1+(1-E)^{\frac{1}{2}}\right]$. The functional relationship for the symmetric regime is $h_0 = 1-h$, hence its name.

The near-field solution (a free boundary type of problem) for the example of $h = h_0 = 1/2$ is shown in Fig. 5a. The archetype, as the nomenclature suggests, is the most elementary dynamical representation of the finite–amplitude mesoscale circulation occurring within an MCS. The presence of the ascending rear–to–front flow is the fundamental component that promotes stationarity and is typical of both observed and modelled MCSs. In the absence of this flow branch, transient behaviour predominates (Moncrieff, 1978). Its presence, intimately involving the parameter E, requires mean flow (synoptic) conditions in which strong shear is confined to low levels.

The observed momentum flux profile is remarkably well produced by the archetype flux realisation, including mean flow enhancement and associated upgradient transport (see later). This model can be extended at the expense of a progressively increasing mathematical complexity by including (1) shear in the jump updraught inflow; (2) the cold pool (density current) using the theory of Moncrieff and So (1989); (3) latent heating in the overturning updraught branch; and (4) evaporative cooling in the downdraught. The latter two extensions require the full conservation equations of Moncrieff (1981). Note that the inclusion of compressibility is relatively easy, requiring merely a transformation of vertical coordinate from height to a form of normalised pressure (Moncrieff, 1991). However, it is physically inconsistent to include latent heating in a jump updraught that does not overturn trajectories. For this reason, the following elaboration to the archetype hydrodynamical model is required.

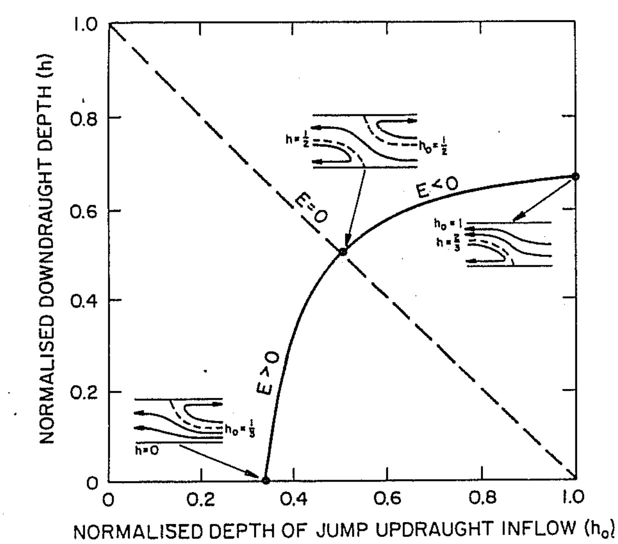

Fig. 6. Regime diagram for the conservative hydrodynamical archetype model showing the functional relationship between h and h_0. The insets show the flow regime associated with limiting regions of parameter space. Broken and full lines represent the symmetric and asymmetric regimes of behaviour, respectively. [From Moncrieff, 1991]

2.3.2 *Conservative convective archetype*

As previously described, observations and numerical models both illustrate that many MCSs are characterised by a transient cellular convective region above the leading edge of the cold pool. These cells are embedded within a slowly varying mesoscale circulation (see Figs. 3 and 4). *A vital point is that flow trajectories do not overturn in the jump updraught of the hydrodynamic archetype.* This morphology is consistent with a two–dimensional jump updraught having a convective Richardson number equal to or less than zero, namely a non–buoyant or forced updraught.

Consider another type of jump updraught in which there is a release of convective available potential energy (CAPE) so the convective Richardson number is positive. Allow airflow trajectories within the jump updraught branch to overturn as in Moncrieff and Miller (1976). This regime can be considered to be an idealisation of steady 'supercell' type of organised three–dimensional overturning. As in the hydrodynamical model, total energy, mass, and thermodynamic (Lagrangian) properties are conserved along trajectories. However, a more general energy conservation equation has to be used for the jump updraught (Moncrieff, 1981). The set of solutions is again constrained by the (eulerian) domain–integrated momentum integral.

The mathematical problem can be treated in considerable generality but the special case of $h_0 = h = 1/2$ serves as an illustration. The dynamical problem is defined in terms of the two non–dimensional numbers $E = \Delta p/\frac{1}{2}\rho U_0^2$ and a convective Richardson number for the jump updraught, namely $R_J = CAPE/\frac{1}{2}U_0^2$. These two nondimensional numbers are related by the expression

$$R_J = \frac{1}{2}\epsilon(2+\epsilon)(2-\epsilon) \tag{1}$$

where $\epsilon = 1 + \sqrt{1-E}$. Solutions exist within the range $1 \leq \epsilon \leq 2$. This defines low values of the Richardson number ($0 \leq R_J \leq \frac{3}{2}$). Moreover, $0 \leq E \leq 1$ so a positive pressure anomaly exists behind the system. This implies a high jump inflow speed (large U_0 and a rapidly travelling system and/or small positive values of CAPE). Low values of the convective Richardson number are associated with a high degree of dynamical organisation. This aspect was demonstrated for strictly two–dimensional overturning by Moncrieff (1978), albeit for a different convective regime. The asymptotic (remote flow) structure in the limit of $R_J = 0$ in the above formula ($\epsilon = 2$) is the hydrodynamic archetype. The internal (near–field)structures of the two models are, however, quite distinct due to the inherent topological difference between two– and three–dimensional flow fields in the interior of the system (near field).

2.3.3 *Non–conservative archetype*

The above archetypes are consistent with an organised convective region embedded within a comparably well organised mesoscale circulation. However, a more 'chaotic' or transient behaviour can be modelled by assuming non–conservation of energy along trajectories. This distinction is somewhat analogous to discrete wave propagation theory (e.g. dissipative hydraulic jumps versus undular bore behaviour in shallow fluids). A more detailed discussion of the latter two dynamical models is beyond the scope of this paper. Flux laws will be derived only for the hydrodynamic archetype.

3. MESOSCALE MOMENTUM FLUX

The primary objectives of this section are, first, to formulate momentum flux laws for line–type MCSs using the dynamical model described in 2.3.1 and, second, to validate the flux laws.

3.1 Formulation

Non–linear conservation properties have been shown to model distinct nonlinear regimes of convection and its fluxes (Moncrieff, 1981, 1990, 1991). These are particularly useful in two–dimensions because analytic solutions are obtainable. The two–dimensional solutions orientated *perpendicular* to the low–level wind (or shear) are finite–amplitude representations of line type MCSs. Unless otherwise stated, velocities are relative to a frame of reference travelling at the earth–relative velocity of the convective system. However, system relative coordinates will be used throughout this paper, unless otherwise stated.

Define $\Delta_s Q(z) = [Q]_0^{L_s}$ to be the far–field difference in an arbitrary scalar quantity Q where $s = x$ or y. The system spans the space $x \in [0, L_x]$ $y \in [0, L_y]$ and is taken to be of unit transverse dimension ($L_y = 1$). The system is of unit depth and L_x is called its *effective width.* Define the *mesoscale mass flux,* as $\mathcal{M} = L_x < \rho w_m(z) >$ where on using the mass continuity equation,

$$\mathcal{M}(z) \quad = \quad - \int_0^z \Delta_x(\rho u_m) \, dz \ . \tag{2}$$

Now consider the formulation of the momentum flux. Integrate the Euler equation over the area $a = L_x \times L_y$ and define the corresponding *mesoscale momentum flux* to be $L_x < \rho u_m w_m(z) >$. On integrating the horizontal component of the relative momentum equation, the momentum flux divergence is shown in Fig. 7a and is

$$L_x \frac{\partial}{\partial z} < \rho \mathbf{v}_m w_m(z) > \ = \left(- \Delta_x(\rho u_m^2 + p_m) \ , \ 0 \right) \tag{3}$$

where p_m is the pressure deviation from a hydrostatic base state and in the archetype model $p_m = \Delta p$, the reference pressure perturbation arising in the definition of E. Clearly, the momentum flux is the integral of this equation and is shown in Fig. 7 b. Since $w_m = 0$ at $z = 0$ and H, the integral momentum constraint ('flow force')

$$\int_0^1 \Delta_x(\rho u_m^2 + p_m)\, dz = 0 \tag{4}$$

must be satisfied. Thus in the absence of turbulent mixing or surface drag, momentum can be redistributed but not generated by the convection. This is quite different from the problem of orographic gravity waves because in that case mean flow momentum can be removed by the form (pressure) drag effect.

It follows that the mesoscale mass flux and momentum flux divergence can be written in terms of far–field variables (Eqs. 2 and 3, respectively). These quantities are analytically defined by the dynamical models. The reorganisation of the mass and momentum fields are thereby mutually consistent because they are determined from mass and energy conservation properties together with the integral momentum constraint.

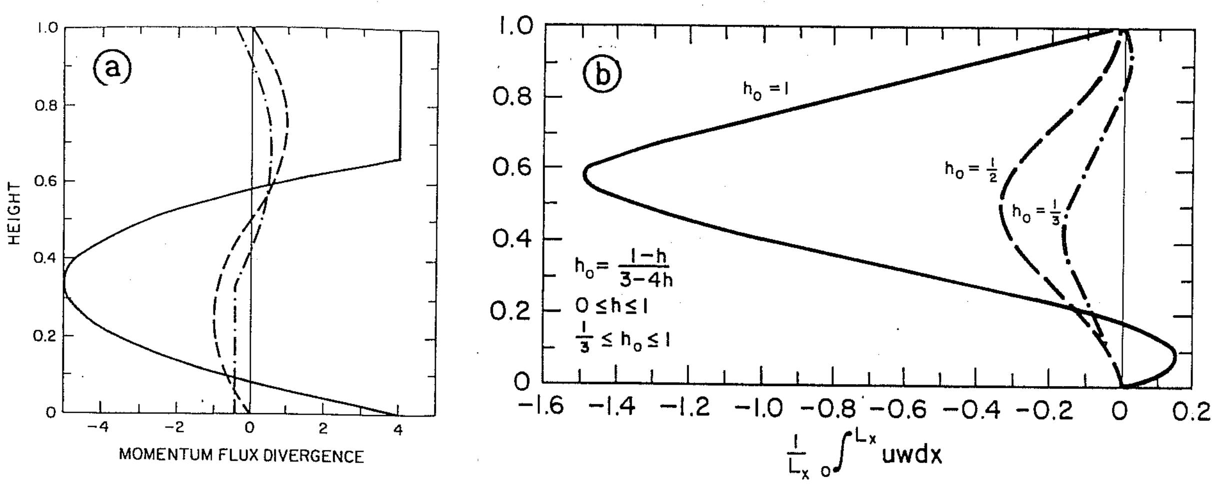

Fig. 7 (a) Momentum flux divergence and (b) momentum flux by the conservative hydrodynamical archetype model. Units are $\rho U_0^2/H$ and ρU_0^2, respectively.

3.2 Comparison with Observations

Analysis of a GARP Atlantic Tropical Experiment (GATE) squall line by LeMone (1983) demonstrated the upgradient nature of the momentum transport by organised convection of the type considered herein. The convective momentum tendency (negative of the momentum flux divergence) is shown in Fig. 8a for an eastward–moving case. The easterly component of momentum at upper levels is enhanced, while the westerly component is enhanced at lower levels. This behaviour is typical of squall line type MCSs as has been subsequently found in field experiments in diverse geographical locations, spanning from mid-latitudes through sub-tropics to tropics. This gives credence to the idea that a universal theory of the dynamics of these systems and their transports is an achievable proposition.

Lafore et al. (1988) performed a dual–Doppler radar analysis of a westward moving squall line that occurred during the Convection Profonde Tropicale (COPT) experiment in west Africa and the momentum flux profile is shown in Fig. 9a. The tropical squall system studies of LeMone (1983) and Lafore et al. (1988) give broadly similar results.

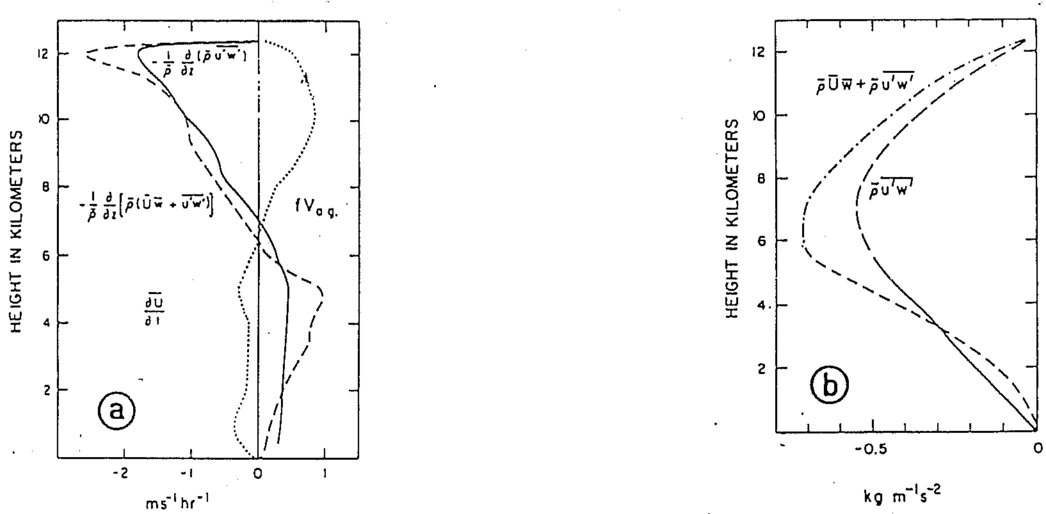

Fig. 8. (a) Convective momentum tendency due to an eastward–moving GATE squall line in GATE; (b) corresponding momentum flux. [From LeMone, 1983]

A dual–Doppler study of a middle latitude squall line that occurred over Oklahoma, USA was performed by Smull and Houze (1987). The convective momentum tendency obtained from this case study is shown in Fig. 10. Note that the archetype model represents mesoscale as opposed to convective scale momentum fluxes. It is hypothesised that the mesoscale fluxes will often dominate due to the coherent nature of the mesoscale airflow. This is given credence by the Smull and Houze (1987) analysis, because the mesoscale fluxes clearly dominate those on the convective scale.

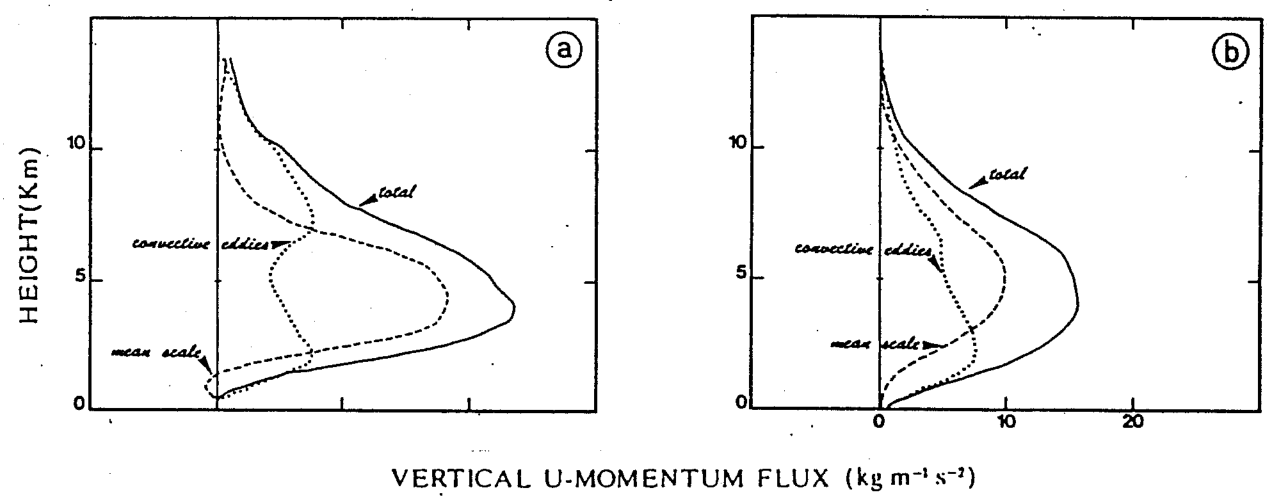

Fig. 9. (a) Convective momentum flux due to a westward–moving west African squall line; (b) corresponding fluxes derived from a fine–scale modelling simulation data set. [From Lafore et al. 1988].

The form of the momentum flux is similar in both tropics and middle latitudes and are in overall agreement with the theoretical model, especially for the cases having downdraughts of finite depth. However, the relative importance of the convective and mesoscale fluxes varies from case to case. This could be a real feature but could also be a product of the inadequate resolution of the radar and aircraft data used in the studies. Clearly, more precise field measurements using modern technology is necessary to quantify this problem.

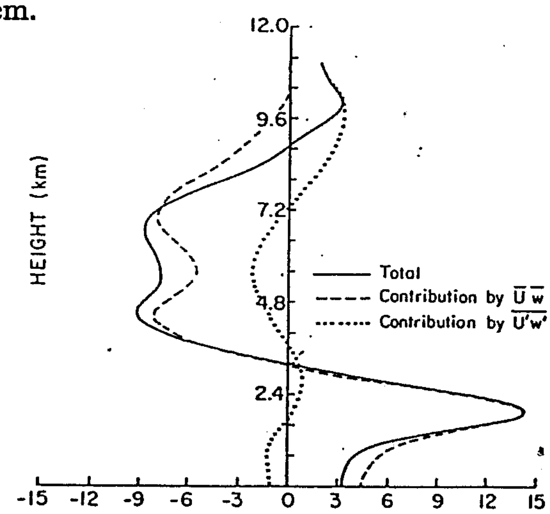

Fig. 10. Convective momentum tendency due to an eastward–moving Oklahoma squall line. [From Smull and Houze, 1987]

3.3 Comparisons with Fine–scale Models

Lafore et al. (1988) modelled the same west African squall line that they observationally analysed. The profiles of observed and modelled mesoscale momentum fluxes (Fig. 9 a and b) are similar. Both are consistent with the theoretical model. Due to the practical difficulties in obtaining accurate momentum fluxes from radar or aircraft observations in strong MCSs, extensive analysis of fine–scale numerical simulations are necessary to test the momentum (and thermodynamic) flux formulae. One important aspect that requires study is the scale dependence of the fluxes, in particular the relative contributions of the convective and mesoscale, namely O(10 km) and O(100 km), respectively.

3.4 A Limited–area Simulation of an MCS

The mesoscale momentum budget for a squall line was obtained by Gao et al. (1990) using a data set from Zhang et al. (1989). The Penn State University/NCAR regional mesoscale model (MM4) was used with the Fritsch and Chappel (1980) parametrization scheme to represent convective–scale thermodynamic fluxes; momentum fluxes were not parameterized. The initial conditions were obtained from an analysis of standard observational data for the case study of 10–11 June 1985 during the Preliminary Regional Experiment for Stormscale Operational and Research Meteorology (PRESTORM) field experiment. The mesoscale relative flow field evolved to the three–branch morphology reminiscent of the archetype model. The relative momentum flux transverse to the squall line, also shown in this figure, has the negative values typical of a system having an eastward–moving component of motion (the system moved southeastwards). Figure 11 shows that the horizontally–averaged momentum flux is similar to those typical of the analytic model.

This study raises the interesting problem of the relative contributions of resolved versus parameterized convection. For example, the results of Zhang et al., (1989) and Gao et al., (1990) show that the momentum field in an MCS can be crudely represented by *resolved or implicit* thermodynamics, albeit using fairly complicated schemes for both sub–grid scale and grid scale moisture. However, *parameterized* convection could not be neglected because it was required to give a realistic system life cycle and system structure. Note that momentum flux was not parameterized in Zhang et al., (1989). This illustrates the problem of scale separation because although the *direct effect* of momentum was omitted, the *indirect effect* brought about by flow response to (parameterized and resolved) convective heating is represented.

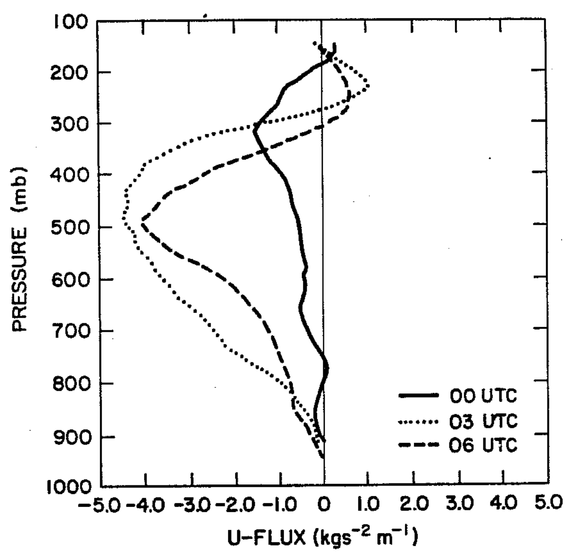

Fig. 11. Area–averaged momentum flux derived from a regional model at 0000, 0300 and 0600 UTC on 11 June 1985. Averages computed over a (450 km × 300 km) area. [From Gao et al., 1990].

Clearly, finer resolution can always be expected to produce a more realistic growth rate and dynamical structure. However, the real question is *to what extent more stringent conditions and essential processes exist in fine scale (explicit resolution) models that are not included in current parameterization schemes for convection in GCMs.* Parsons (private communication) suggests that criteria for activating parameterization of squall lines should include a representation of the effects of gust front dynamics due to its convection initiation properties.

3.5 Incorporation of Momentum Fluxes in the Large–scale Equations

Consider the x–component of the grid scale momentum equation. In the absence of other sub–grid scale processes, Moncrieff (1991) represented the convective momentum induced tendency as

$$\frac{\partial}{\partial t}(\overline{\rho}\,\overline{u}) + div(\overline{\rho}\,\overline{u}\,\overline{\mathbf{v}}) - \overline{\rho} f \overline{\mathbf{v}} + \frac{\partial \overline{p}}{\partial x} = \epsilon \frac{\delta}{\delta t}(\rho u_m) \tag{5}$$

where the overbars represent grid scale variables and the convective momenntum tendency $\delta/\delta t$ is the negative of the momentum flux divergence. The dimensionless closure (amplitude) parameter ϵ is a function of the mass flux and the grid resolution. The *total* mesoscale effect must be implemented on the right–hand–side of Eq. (5) and this implies that the mesoscale perturbation is unresolved in the corresponding large scale model. For reasons outlined in the introduction this is quite true for virtually all existing global *and* mesoscale models because a grid of O(1 km) is necessary to capture the convective scale processes that are primary to the generation of the mesoscale circulation. This is considerably different from (say) the Schneider and Lindzen (1976) method.

Application of the mesoscale transports to convective parametrization in large–scale models requires considerations in addition to flux approximations. First, a closure that establishes the amplitude (ϵ) of the sub–grid scale tendencies in Eq. (5). Second, a procedure to initiate or activate the convection scheme; in this regard current parcel lifting methods may not be optimal but are acceptable considering the imprecision in representing the physical processes. The above tendencies should be shear dependent *i.e.* applied only when the low level shear is concentrated in low levels.

3.5.1 *Closure*

A mass flux type of closure is attractive. Define $\mathcal{M}_g = L_g < \rho w_m(z_R) >$ to be a *reference mesoscale mass flux*, the mass flux corresponding to a mesoscale circulation that fully spans the grid scale (L_g), so that $L_x = L_g$ in Eq. (3). The reference height $z = z_R$ can be arbitrary in the range (0,1) because the analytically determined mass and momentum fluxes are dynamically consistent. Let M be a mesoscale mass flux that is independently determined and related to the conventional cloud mass flux (M_c). Define $\epsilon = M/\mathcal{M}_g$ to close the mesoscale parametrization. Note that ϵ can be of order unity and is a function of h (or E) and M_c. It has an intrinsic large–scale grid dependence, namely $\epsilon = \mathcal{F}(h, L_g, M_c)$. The ensuing momentum parametrization scheme (flux approximation, closure and initiation) should be compared to conventional schemes. Moreover, the functional form of M and ϵ should be investigated using fine–scale numerical models.

4. MOMENTUM FLUXES USING CLOUD MEAN PROPERTIES

4.1 Schneider and Lindzen Scheme

Schneider and Lindzen (1976) approximate the convective momentum flux divergence as

$$\frac{1}{\rho}\frac{\partial(\overline{\rho u'w'})}{\partial z} = \frac{1}{\rho}\frac{\partial}{\partial z}\left[M_c(\mathbf{v_c} - \bar{\mathbf{v}})\right] \tag{6}$$

where $\bar{\mathbf{v}}(z) = \frac{1}{a}\int_{grid} \mathbf{v}\, da$, M_c and $\mathbf{v}_c(z)$ are the grid scale mean of horizontal velocity, the convective mass flux per unit volume and the cloud–scale mean of the horizontal velocity, respectively. The negative of this quantity is the 'cumulus friction' or the convective momentum tendency applied to the right–hand–side of the momentum equation. The mass flux is defined in the usual way as $M_c(z) = a\overline{\rho w'} = \int_a \rho w' da$, where the cloud area is a. Equation (6) requires that the horizontal eddy flux divergence is negligible on the grid scale, an approximation that is consistent with the basic assumptions of this approach. However, these assumptions are questionable when the parameterized scale is comparable to the mesh size.

The methods used to determine the cloud–mean velocity $\mathbf{v_c}$ distinguishes individual applications of that scheme. Schneider and Lindzen (1976) assumed that the cloud scale horizontal momentum is conserved so $\mathbf{v_c}$ is constant, say, the grid scale horizontal velocity at cloud base. However, horizontal momentum is conserved only under exceptional conditions. In particular, the role of the horizontal pressure gradient should not be neglected in deep convection, especially squall line systems. Recognizing the weakness of the momentum conservation assumption employed in the Schneider and Lindzen scheme, *ad hoc* attempts have been made to more realistically calculate the cloud–mean momentum (*e.g.* Flateau and Stevens, 1978; Shapiro and Stevens, 1980).

It is illuminating to demonstrate the effect of the original Schneider and Lindzen (1976) method on a mean flow profile typical of situations in which MCS convection prevails. This mean wind profile has strong shear in low levels and small shear in upper levels. Take the idealised (earth–relative) flow profile $\overline{u}(z) = 4z$ for $z \in [0, 1/2]$ and $\overline{u}(z) = 2$ for $z \in [1/2, 1]$ as an illustration. This is similar to one of the profiles used in simulations by Thorpe et al. (1982), and others. Selecting the (constant) cloud mean velocity as $u_c = u_0(1/4) = 1$. Use the mass flux calculated from the analytic model (Fig. 12, the two-dimensional model). The convective momentum tendency is shown in Fig. 13a.

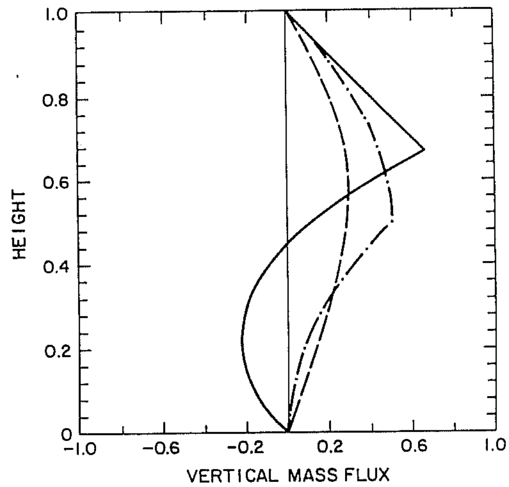

Fig. 12. Mass flux calculated from the archetype hydrodynamic model. In units of ρU_0. [From Moncrieff, 1991].

4.2 Application of the Two–dimensional Conservative Archetype Model

A limitation of the original Schneider and Lindzen (1976) scheme is the use of a constant cloud mean momentum. Since the two–dimensional flow field for the conservative hydrodynamical archetype has been obtained (Fig. 5 a) it can be used to calculate the cloud mean momentum without approximation. Note that the effect of the pressure field is automatically included in the two–dimensional solutions.

The cloud mean momentum is now defined as the horizontal mean of horizontal velocity component derived from the solution of the free boundary problem ($u = \partial\psi/\partial z; w = -\partial\psi/\partial x$) and therefore a function of height. The mean flow profile, $\overline{u}(z)$, is as defined in section 4.1 because the steady two–dimensional flow regime is a legitimate time–asymptotic state of the initial value problem having the velocity profile defined above as an initial state (*e.g.* Thorpe et al. 1982). *Thus the full effect of the finite–amplitude system that develops in wind profiles of this type is represented by the flux divergence formulation.* It is stressed that this regime of convection exists in quite general mean wind profiles, provided that the shear is strong in low levels. There is an important distinction to be made between the mean momentum application of the dynamical model and the conservative flux law developed in section 3.1. That is, the latter is determined solely by far–field (remote flow) properties, while the former requires near–field (fully two–dimensional) flow solutions that in turn require the solution of a difficult free–boundary problem for each individual case. The convective momentum tendency using the model–derived cloud mean momentum is as shown in Fig. 13b.

4.3 Flux Law Comparisons

Note that the relative and absolute divergence profiles give identical results in the cloud mean momentum approach and neither reproduces the observed structure in Figs. 8, 9, or 10. The two cloud mean momentum methods differ in detail but yield broadly similar momentum flux tendencies (Figs. 11a and b). It was expected that the difference would be small in this example because the absence of baroclinic effects precludes the production of vorticity. Consequently, the flow acceleration (and the internal pressure field perturbation) is produced solely by dynamical effects.

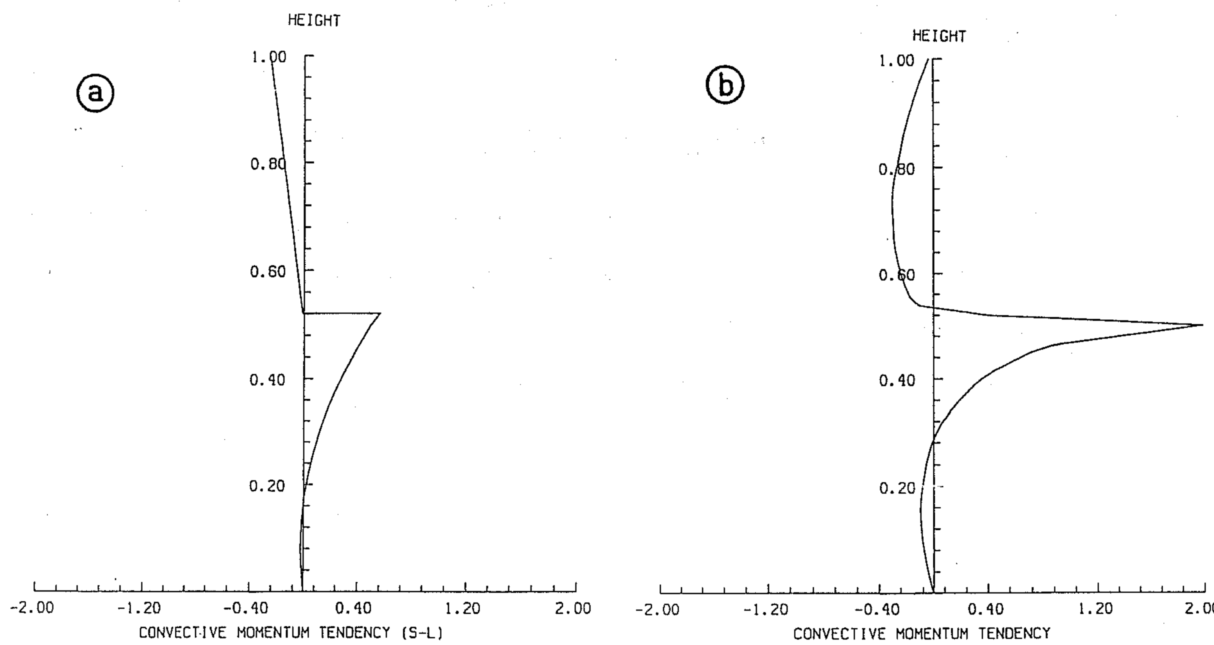

Fig. 13. Convective momentum flux tendency using: (a) Schneider and Lindzen (1976); (b) Schneider and Lindzen with cloud mean momentum calculated from the two–dimensional model. Abscissa in units of U_0^2/H and ordinate in units of H.

The profile obtained using the conservative method (Fig. 14) is quite different from either of the above tendencies and in good general agreement with the observationally determined profile. Clearly, it is superior to either cloud mean momentum approaches (*c.f.* Figs. 8, 9, 10, 13, and 14). This result encourages further development of the conservative approach and its application in a large–scale parameterization. Note that in a parameterization scheme the amplitude would be determined by a closure assumption, such as that defined in section 3.5.

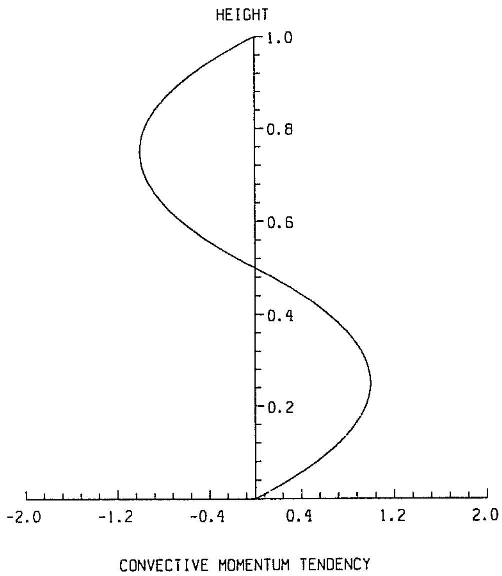

Fig. 14. Convective momentum flux relative to the system calculated from conservative hydrodynamical archetype. Abscissa in units of U_0^2/H and ordinate in units of H.

5. CONCLUSIONS

Momentum fluxes by organised convection have been formulated using a dynamical model developed from exact far–field mass, energy and momentum conservation principles. The mesoscale mass and momentum fluxes are mutually consistent in the conservative approach and this property has been employed to define a physically based closure. The realistic nature of the modelled momentum fluxes, despite the simplicity of the archetype model, can be explained in elementary terms. Momentum transport by organised eddies in a shear flow is largely determined by their orientation relative to the mean flow shear *c.f.* phase tilt in wave dynamics. This is an intrinsic property of the dynamical structure of the relative flow involving the pressure field. In developing an elementary explanation there is no loss in generality in considering an eastward–moving system; an analogous argument applies to westward–movement. It is obvious that if the eddy orientation axis tilts upshear then $<\overline{(\rho u'w')}(z)>$ is negative. Thus, provided the eddy orientation is correctly modelled the momentum fluxes should be well represented. *This orientation is indeed reproduced by the two–dimensional solution and is consistent with the physical integrity of the* *system*. Inclusion of the omitted baroclinic processes neglected in the archetype are not expected to fundamentally alter this basic conclusion. Note the opposite (downshear) orientation prevails in the constant shear analytical model of Moncrieff (1978), which has $< \overline{(\rho u'w')}(z) >$ positive. However, it that case a dynamically and thermodynamically consistent stationary structure could not exist. *i.e.* transient behaviour was necessary. This proves that other regimes of organisation can have a quite different momentum flux profile, so a strong mean flow regime dependence must be allowed for in the generalisation of the flux laws.

Herein, the conservative dynamics of strictly two–dimensional convection has been addressed and is part of a long–term objective to develop a comprehensive theory of organised convection parameterization. A general dynamical model of an MCS was formulated by Moncrieff (1990) where it was shown that the macroscale dynamics of MCSs can be expressed in terms of six nondimensional numbers – a set of three Richardson numbers, measuring the ratio of the available potential to available kinetic energy for each branch; a set of two Froude numbers defining the density current nature of the problem and associated with cold air outflow and mean flow stratification; and the parameter E whose physical interpretation has already been defined. However, a parameter space of such complexity and nonlinear relationships among the nondimensional numbers precludes a simple flux representation. The hydrodynamical archetype is a first step in defining a rigourous dynamical approach to flux theory for organised convection in a shear flow.

It is not obvious which aspect of the archetype need to be improved *in the content of parameterization* because this depends on the resolution of the large scale model *e.g.* limited area models are more sensitive to localised dynamics than coarse mesh GCMs. The next steps in theoretical development of the flux formulae are to: (1) include shear in the jump updraught inflow of the hydrodynamic archetype; (2) extend the archetype approach to include the convective jump and non–conservative models briefly summarised in sections 2.3.2 and 2.3.3; (3) partition the fluxes into the mesoscale and convective scale components; (4) formulate anisotropic fluxes, which is required by noting that the line–parallel fluxes are downgradient, as distinct from the largely upgradient character of the line–normal fluxes (LeMone et al., 1984); (5) explore the relationship of the closure hypothesis of section 3.5 to classical parameterization theory (if one exists); and (6) to include thermodynamic fluxes in the theory. These considerations significantly complicate the mathematical problem and the flux formulation but there is no basic reason why these extensions cannot be realised.

General circulation model tests (both idealised and complex) should be performed to assess the impact of convective momentum transport by organised convection on a global scale *c.f.* orographic wave drag evaluation. Evidently, general circulation models can be sensitive to convective momentum transport (M.J. Miller and Michael Tiedke, private communications). There is no reason why the momentum flux formula could not be applied in conjunction with an existing thermodynamic scheme (say using the closure in section 3.5) to examine the sensitivity of GCMs to organised momentum fluxes. The effect may be strong in synoptic regions where the low-level, large-scale shear is large compared to that in the upper troposphere, say over large areas in the tropics and the continents in summer.

Fine-scale numerical models, that can adequately span the several interacting scales involved in this complicated sub-grid scale problem, is an obvious utility for exploring the many aspects mentioned in the text. Indeed, analogous investigations to those outlined for the theoretical development should be performed on fine-scale model data sets. Both theoretical and numerical model flux analyses should be more rigourously compared against the real atmosphere. In this regard, the few existing data sets that currently exist are inadequate. New data sets that span scales O(10–1000 km) will be very useful.

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