ERROR ESTIMATES FOR EXTENDED RANGE FORECASTS WITH DYNAMIC MODELS

J. Egger and H. D. Schilling

Meteorologisches Institut der Universität München
München, FRG.

1. INTRODUCTION

Up to know most extended range forecasts have been based on linear regression models. Dynamic models like GCMs have hardly been used, partly because of economic reasons and partly because the skill of such forecasts has been low so far. Nevertheless, there appears to be a growing interest in long range forecasts with dynamic models (e.g. Shukla (1981)).At this stage it may be worthwhile to estimate the errors inherent in extended range forecasts made with a dynamic model. To assess this error we adopt the following strategy: We restrict our attention to the barotropic modes of the atmosphere and assume that the barotropic vorticity equation captures the essential features of what goes on at the 500 mb surface. The equation is

$$\frac{\partial}{\partial t}(\nabla^2-\lambda^2)\psi + J(\psi, \nabla^2\psi + f) = -C\nabla^2\psi + F \tag{1}$$

ψ is the stream function, λ^{-1} is a radius of deformation and the term $\lambda^2\frac{\partial\psi}{\partial t}$ describes the influence of a free surface on the flow. The Coriolis parameter $2\Omega\sin\theta$ is denoted by f where Ω is the earth's rotation rate. The first term on the right hand side of (1) represents the effects of friction and F is a forcing term which is thought to describe baroclinic effects. Suppose now that we want to make a long-range forecast for the 500 mb surface. Then, we do not want to forecast the day-by-day variations of ψ nor do we want to resolve features with a scale of 1-2000 km. What we want is a forecast of the slowly varying part of the largest scales of motion. This is quite a reasonable program since data analysis has shown (e.g. Blackmon, (1976)) that the largest planetary scales contribute strongly to the low-frequent variance of the 500 -mb geopotential. If we could forecast the motions at these space and time scales we certainly would have a model with sufficient skill to be useful.

It is convenient to partition the stream function between a part ψ_e to be forecast in a extended range forecast and a part ψ_m which would have to be included in a medium range forecast but is not forecast in an extended range prediction:

$$\psi = \psi_e + \psi_m \tag{2}$$

The forecast equation for ψ_e is

$$\frac{\partial}{\partial t}(\nabla^2-\lambda^2)\psi_e + J_e(\psi_e, \nabla^2\psi_e + f) = - C\nabla^2\psi_e + F_e \tag{3}$$

$$- J_e(\psi_e, \nabla^2\psi_m) - J_e(\psi_m, \nabla^2\psi_m) - J_e(\psi_m, \nabla^2\psi_e)$$

The subscript e at the symbol J denotes the projection of a Jacobian on the spatial scales selected for the extended range forecasts. The three Jacobians on the right hand side of (3) describe the impact of the 'synoptic' scale modes ψ_m on flow on the largest planetary modes. Note that (3) does not ensure that only slow modes are forecast.

An extended range forecast of ψ_e by aid of (3) will contain gross errors. Some of these will be due to our ignorance with regard to the forcing F. The crude damping scheme is also a source of forecast error. But let us assume that the barotropic vorticity equation is the proper equation to use and that we have a perfect parameterization scheme for the forcing F and for the damping. Then, the only source of possible errors are the three Jacobians on the right hand side of (3). It is well known that no dynamic forecast modes can provide a good forecast for these terms beyond a few days (see also Section 2). Therefore these terms will act as a source of error in (3). What we want to do is to estimate the error of ψ_e forecasts induced by these interactions terms.

2. DATA

To assess the impact of the ψ_m-field on the ψ_e-field we have to evaluate the forcing Jacobians on the right hand side of (3) from data. We took two years of daily height observations (1972-1973) as provided by the German Weather Service. These height data were essentially expanded in terms of the eigenfunctions of the Laplacian on the sphere (the procedure actually used was slightly more complicated and is described in Egger and Schilling, 1982).

These eigenfunctions are the spherical harmonics

$Y_l^m = P_l^m(\sin\Theta)\, e^{im\lambda}$, (Θ latitude, λ longitude) so that

$$\psi = \sum_{m=0}^{M} \sum_{n=1}^{N} \psi_{mn} Y_{m+2n-1}^{m} + a\, u_0 \sin\Theta \tag{4}$$

with expansion coefficients ψ_{mn}. The term $a\, u_0 \sin\theta$ describes the superrotation. The earth's radius is denoted by a. As can be seen we admit only modes which are antisymmetric with respect to the equator. The zonal wave number is m and n is the number of zeros between the equator and the pole and may be called a meridional wave-number. We used the resolution $0 \leq m \leq 12$, $1 \leq n \leq 5$ for the expansion of the data fields.

To compute the term

$$J_e = -J_e(\psi_e, \nabla^2\psi_m) - J_e(\psi_m, \nabla^2\psi_e) - J_e(\psi_m, \nabla^2\psi_m) \tag{5}$$

we have to decide which scales have to be included in the ψ_e-field We chose

$$\psi_e = \sum_{m=0}^{5} \sum_{n=1}^{5} \psi_{mn} Y_{m+2n-1}^{m} + a u_0 \sin\Theta \tag{6}$$

, so that the ψ_e-field is composed of 31 modes. Of course,

$$\psi_m = \sum_{m=6}^{12} \sum_{n=1}^{5} \psi_{mn} Y_{m+2n-1}^{m} \tag{7}$$

With all the coefficients ψ_{mn} available on a day-by-day basis it is straightforward to evaluate J_e whereby J_e is expanded with expansion coefficients γ_{mn}.

Next the vorticity equation is projected on to the spherical modes. This procedure results in a forecast equation for the expansion coefificients of the ψ_e-field

$$\frac{\partial}{\partial t}\psi_{mn} + \ldots\ldots = -\tilde{C}\psi_{mn} + J_{emn} / (1 + \lambda^2/K_{mn}^2) - F_{mn} / (K_{mn}^2 + \lambda^2) \tag{8}$$

where k_{mn}^2 is the eigenvalue of the mode m,n:

$$K_{mn}^2 = (m + 2n - 1)(m + 2n)/a^2 \tag{9}$$

F_{mn} is the expansion coefficient of the 'baroclinic' forcing terms. Furthermore,

$$J_{emn} = -\delta_{mn}/K_{mn}^2$$
$$\tilde{C} = C/(1+\lambda^2/K_{mn}^2) \tag{10}$$

In (8) we have not written down the cumbersome interaction terms stemming from the term $J(\psi_e, \nabla^2\psi_e+f)$ on the left hand side of (3). Before we turn to integrations of (8) it is revealing to obtain some information on the statistical characteristics of these forcing terms. To that end, we discuss the powerspectra of J_{emn}. These spectra are essentially of the 'red noise' type. Therefore a first order Markov process M_j can be fitted to the time series $J_{emn}(j)$ where j is an running index in time increasing by one every day. The Markov process is

$$M_{j+1} = M_j e^{-b} + ((1-e^{-2b})R(0))^{1/2} W_j \tag{11}$$

R (τ) is the autocorrelation of the real or imaginary part of the forcing. W_j is a white noise process with power density 1. The decay rate b characterizes the autocorrelation through

$$R(\tau) = R(0)\, e^{-b|\tau|} \tag{12}$$

The corresponding fitted power spectrum is

$$\Sigma(\omega) = R(0)\, b/b^2+\omega^2 \tag{13}$$

The decay rate b and the autocorrelation R (τ) depend on the mode (m,n), of course.

Fig. 1 shows the parameter b for the imaginary part of the forcing terms, i.e. for the $\sin k_m x$ -forcing.

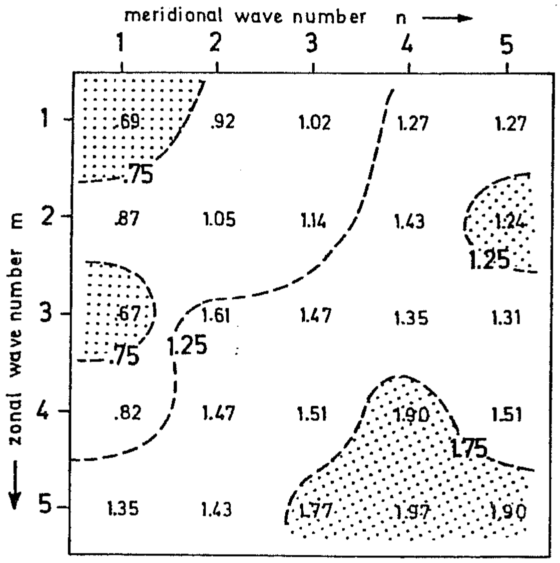

Fig. 1 Autocorrelation decay rate b (day^{-1}) for the $sink_m$x-forcing of the modes of the Ψ_e-field.

Roughly speaking the largest modes (m,n small) have the smallest values of b whereas the smallest modes (m,n large) have the largest values of b.

To demonstrate the quality of the fitting formula we show the average of all powerspectra with $1.25 \leq b \leq 1.75$ and the average of the corresponding fitted red noise spectra (Fig. 2).

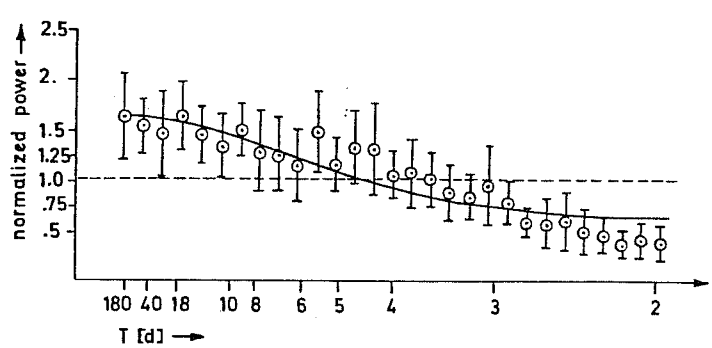

Fig. 2 Average over the powerspectra of all modes with $1.25 \leq b \leq 1.75$ d^{-1} and the mean fitting curve (solid) for these modes. The power is normalized with respect to a white spectrum for the same frequency interval.

The correspondence of the observed and fitted spectra is satisfactory although the fitted spectra overestimate somewhat the power of the forcing for high frequencies. Since (11) appears to capture the essential features of the synoptic scale forcing we can draw the conclusion that the forcing terms J_{emn} are essentially unpredictable beyond 2/b days, say. We can go on to say that even high resolution GCM's would not be able to give useful deterministic predictions for J_{emn} more than a few days in advance. In particular, the forcing must be seen as essentially white and totally unpredictable if we are concerned with longrange prediction on the scale of a month or more.

The variances R (o) has been computed as well and those for the $\sin k_m x$- modes are shown in Fig. 3.

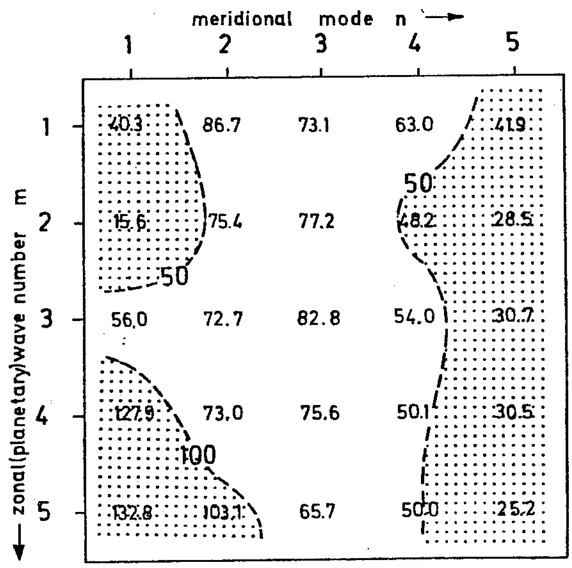

Fig. 3 Variance R(o) ($m^4 s^{-4}$) of the forcing for the imaginary part of the modes of the Ψ_e-field.

In general, the forcing has about the same intensity for all modes although the modes with $n \leq 2$, $m > 4$ receive the strongest input.

If we want to study the forcing of the planetary scale stream function at a certain locality we have to superimpose the contributions of all the various J_{emn} to the forcing at that point. Then we can compute the powerspectrum $\Sigma(\lambda, \theta, \omega)$ at any locality of the Northern Hemisphere. To display the results it is convenient to consider variances, i.e. the integral of the power over a frequency band. Following Blackmon (1976) we introduce the low-frequent variance of the forcing

$$\sigma_{fL}^2 = \frac{2}{\pi} \int_0^{\omega_{10}} \Sigma(\lambda, \Theta, \omega)\, d\omega \qquad (14)$$

This is the variance of forcing with periods $T = 2\pi/\omega$ larger than ten days. Fig. 4 shows the low frequent variance of the stream function forcing as obtained from the data.

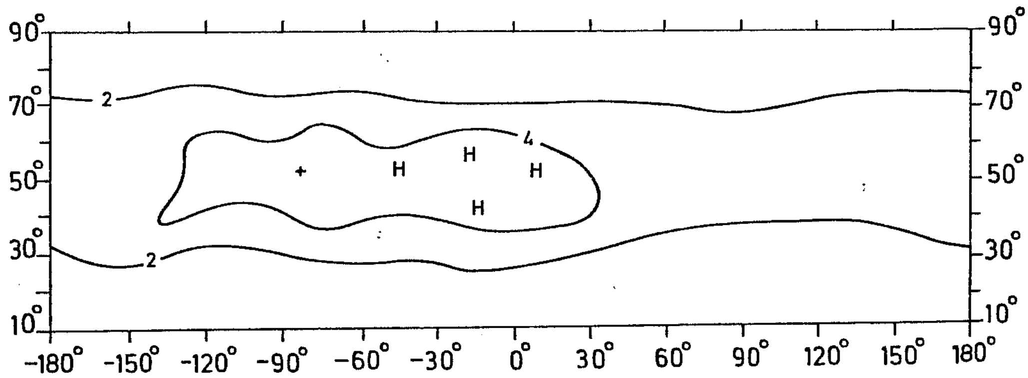

Fig. 4 Variance of the forcing for periods $T > 10$d in $10^2 m^4 s^{-4}$

The variances are generally lower at high and low latitudes than at mid-latitudes. There is a fairly broad maximum of the forcing over the western part of the hemisphere. The intensity of the forcing is of the order 2-4 x $10^2\ m^4\ s^{-4}$.

It remains to determine the long-term response of the Ψ_e-field to the forcing displayed in Fig. 4.

3. LINEAR RESPONSE TO THE FORCING BY SYNOPTIC MODES

(8) is the forecast equation to be integrated in time whenever a forecast has to be made. In that equation J_{emn} acts as a forcing term. This term is known for the years 1972-73 but it is not known when we have to make a forecast. In particular, as we have seen, we cannot forecast this forcing term beyond a few days even with a GCM.

In an actual situation we would have also great problems to determine the 'baroclinic' forcing F_{mn}. Also, the dissipative term would be a source of error. Let us, however, assume that we have a perfect parameterization of F_{mn} and that $-\tilde{C}\Psi_{mn}$ is the correct form for the damping. In other words we assume that (1) is the correct forecast equation for the 500 -mb flow. Then our lack of knowledge with regard to the forcing J_{emn} is the only source of errors in an integration of (8) (provided we know the initial state perfectly well). The error induced by these terms in an extended range forecast can be assessed by running (8) with J_{emn} prescribed on the basis of the observations. As we have seen J_{emn} has the characteristics of red noise. Therefore J_{emn} induces a response at all frequencies but the strongest forcing occurs for the smallest frequencies. It is the smallest frequencies we are concerned with in an extended range forecast. Therefore we have to expect that J_{emn} will induce a considerable error in longrange forecasts.

There are at least two methods how to determine the response. The first and most accurate one is to integrate (8) numerically whereby the forcing terms J_{emn} are updated from day to day according to the observations. The resulting time series $\Psi_{mn}(j)$ has to be analysed in order to find out the longterm variance of the error field.

Here we chose a simpler approach. We linearize (8) with respect to a zonal basic state. Then our problem is linear and we can find out the error by analytical means. In particular, we can put $F_{mn}=0$. To that end we linearize (8) with respect to the zonal mean flow $u_0 \cos\Theta$:

$$\frac{d}{dt}\psi_{mn} = -(i\omega_{Rmn} + \tilde{C})\psi_{mn} + J_{emn}/(1+\lambda^2/k_{mn}^2) \tag{15}$$

The symbol

$$\omega_{Rmn} = \frac{m}{a}\left(u_0 k_{mn}^2 - \left(\frac{2\Omega}{a} + \frac{2u_0}{a^2}\right)\right)/(k_{mn}^2 + \lambda^2) \tag{16}$$

denotes the Rossby frequency. In what follows we shall use a scale dependence of the damping based on a conventional Austausch Ansatz: $C = \nu k_{mn}^2$, ν constant.

Using (15) corresponds to assuming that the nonlinear interactions of the resolved modes are at least not more important than the linear wave dynamics. Such an assumption would be hardly tenable in a truly turbulent wave field but there are arguments that the regime of the largest atmospheric modes is a wave regime where linear dynamics may provide good guidance (Basdevant et al. (1981); see also White and Green (1981)).

Given J_{emn} as a function of time it is easy to solve (15) by analytical means. For our statistical considerations it is more convenient, however, to perform a Fourier transform of (15) in time where

$$P_{mn}(\omega) = \int_{-\infty}^{\infty} e^{i\omega t} \psi_{mn} \, dt$$
$$P(\lambda, \theta, \omega) = \int_{-\infty}^{\infty} e^{i\omega t} \psi_e (\lambda, \theta, t) \, dt \tag{17}$$

are the transform of the expansion coefficients and of the resolved part of the stream function, respectively.

Let j_{mn} (ω) be the transform of J_{emn}. Then we obtain from (15)

$$P_{mn}(\omega) = -i j_{mn} / (-\omega + \omega_{Rmn} - i\tilde{C}) \tag{18}$$

Switching back to physical space we have

$$P(\lambda, \theta, \omega) = \frac{1}{2} \sum_{m=1}^{5} \sum_{n=1}^{5} \left(P_{mn}(\omega) Y_{m+2n-1}^{m} + P_{mn}^{*}(-\omega) Y_{m+2n-1}^{*m} \right) \tag{19}$$

where P_{mn}^{*} is the complex conjugate of P_{mn}.

Since we know the Fourier transforms j_{mn} it is easy to compute the powerspectrum S $(\lambda, \theta, \omega) \sim pp^{*}$ of the response to the forcing. It is the power spectrum S $(\lambda, \theta, \omega)$ which contains most of the information we want since it tells us what the error will be at a certain locality on the sphere given the frequency ω of the atmospheric motion. For example, the variance

$$\sigma_{30}^{2} (\lambda, \theta) = \frac{2}{\pi} \int_{0}^{\omega_{30}} S(\lambda, \theta, \omega) \, d\omega$$

is the variance of the error of monthly mean forecasts induced by the synoptic scale forcing ($\omega_{30} = 2\pi/30 \ d^{-1}$ is essentially the maximum frequency resolved by a monthly mean forecast). In what follows we shall present the variance

$$\sigma_{L}^{2} (\lambda, \theta) = \frac{2}{\pi} \int_{0}^{\omega_{10}} S \, d\omega$$

which summarizes all the errors involved in extended-range forecasts which go beyond the 10-day limit of medium-range forecasts.

4. RESULTS

In Fig. 5 we present the extended-range error variance induced by the forcing by the Ψ_m-field.

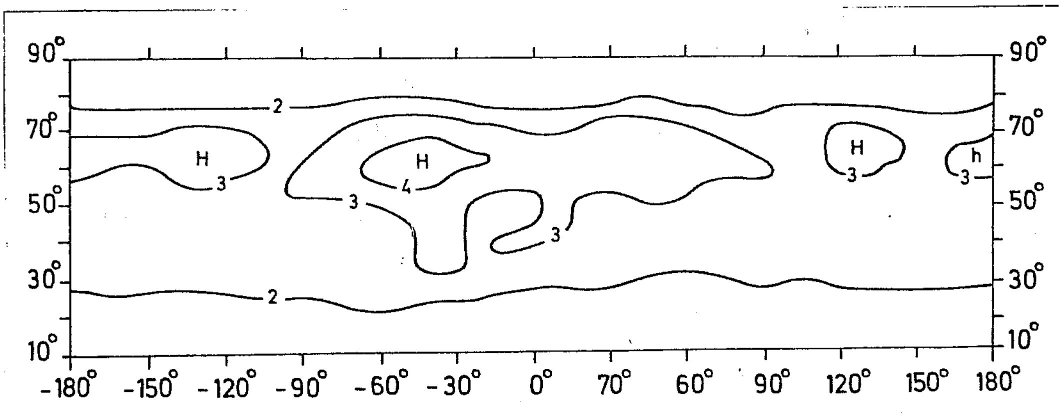

Fig. 5 Variance σ_L^2 of the 500 -mb stream function Ψ_e in $10^{13} m^4 s^{-2}$ for periods $T > 10$ d as induced by the forcing. The variance presented is an average variance where λ^2 and ν have been varied in the range $0.65 \le \lambda^2 \le 1 \times 10^{-12} m^{-2}$, $0.7 \le \nu \le 1 \times 10^6 m^2 s^{-1}$, respectively.

Actually, the variance σ_L^2 displayed in Fig. 5 is the average over seven different cases. (8) contains at least two important parameters neither of which can be determined with accuracy. Acceptable values of λ^2 and ν range over one order of magnitude at least. Although the results for the sphere appeared not to be overly sensitive to the choice of the parameters we decided to compute σ_L^2 for several combinations of these parameters and to average over the resulting variances.

We note that maximum variance is found over the Pacific, over eastern Canada and over the Atlantic. High values of the variance are restricted to the belt $50^0 \le \Theta \le 70^0$. Subtropical latitudes show a rather low level of error variance. Note the minima over the Rockies and over the Himalayas. The response at midlatitudes is of the order $3 \times 10^{13} m^4/s^{-2}$. This corresponds roughly to a root mean square error of about 50 m for the 500 mb extended range height forecasts.

It is most interesting to compare the error variances to the actual extended range variance of the atmosphere for the Ψ_e-modes. Blackmon's (1976) charts show this variance. Since Blackmon's data are not exactly compatible with our data basis we have computed σ_L^2 from our record. Fig. 6 shows the resulting variance of the 500 mb stream function for atmospheric motions with periods T $>$ 10 d.

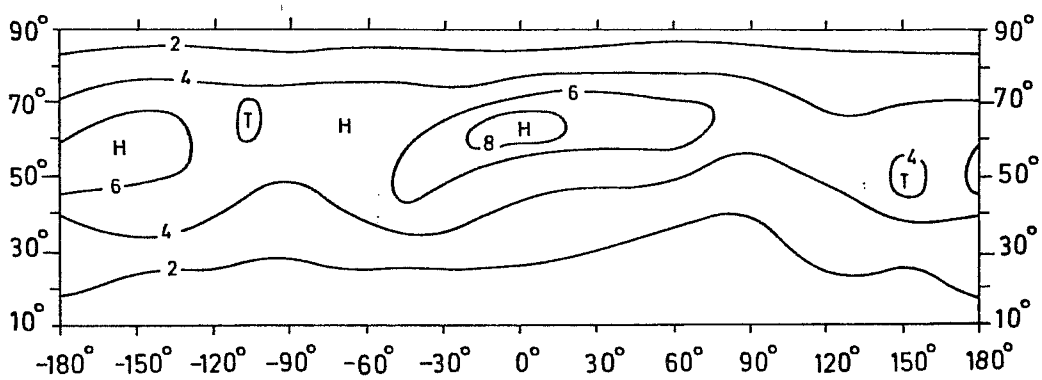

Fig. 6 Variance of the 500 - mb stream function $_e$ in $10^{13} m^4 5^{-2}$ for periods T 10 d as observed.

We have two prominent maxima of the variance, one over the Pacific and another one over the Atlantic. Low values of the variance are found over the Rockies and the Himalayas. The agreement of Fig. 5 and Fig. 6 is rather striking. If we accept this coincidence we have to conclude that the forcing by the Ψ_m - modes causes a main part of the observed longterm variance of the Ψ_e-field. Note that this part is certainly unpredictable. Therefore, there is little hope that an extended range forecast made with a dynamical model will show satisfactory skill. On the other hand, we have to be aware that the response to the synoptic scale forcing as displayed in Fig. 5 depends on the choice of the damping parameters. We can, therefore, not determine exactly what part of the observed long-term variance is due to the unpredictable synoptic forcing.

REFERENCES

Basdevant, C., B. Legras, R. Sadourny and M. Béland, 1981: A study of barotropic model flows: intermittency, waves and predictability. J. Atmos. Sc. 38, 2305-2326.

Blackmon, M. 1976: A climatological spectral study of the 500 mb geopotential height of the Northern Hemisphere. J. Atmos. Sc. 33, pp. 1607-1623.

Egger, J. and H.D. Schilling, 1982: On the theory of the atmosphere's long-term variability. Unpubl. manuscript.

Shukla, J. 1981: Dynamic predictability of monthly means. J. Atmos. Sc. 38, pp. 2547-2572.

White, A.A. and J.S.A. Green, 1981: A nonlinear atmospheric long wave model incorporating parameterizations of transient baroclinic eddies. J. Roy. Met. Soc. 108, pp. 55-85.