Eof-expansion of geopotential and wind data using a geostrophic constraint

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1. Introduction

One of the advantages of eof for representing the vertical structure of the atmosphere is the rapid convergence usually found when applying the method to wind data or to geopotential data. However, separate expansions are not directly applicable when it comes to construction of primitive equation models since there is no guarantee for internal consistency between these two separate expansions. An attempt has therefore been made to make a simultaneous expansion of wind and geopotential data using a geostrophic constraint in order to achieve an interdependence. I shall here briefly report on the method and on some of the results from this work that has been carried out at SMHI by Per Undén and myself.

2. Equations

Denoting by $\phi(x,y,t,p)$ the deviation of the geopotential from an horizontally averaged atmosphere and by $w = u + iv$ the complex wind vector we consider the following series expansions

(1) $$\phi(x,y,t,p) = \Sigma a_n(x,y,t)F_n(p)$$

(2) $$w(x,y,t,p) = \Sigma w_n(x,y,t)H_n(p)$$

where w_n and H_n are complex functions. With

$$D = \frac{\partial}{\partial x} + i\frac{\partial}{\partial y}$$

the equation for horizontal motion may be written

(3) $$ifw + D\phi = R$$

where the complex residual R vanishes exactly in the case of geostrophic balance. Since the left hand side of (3) is linear we may introduce one of the terms in the series (1) and (2) in (3) obtaining

(4) $$ifw_nH_n + F_nDa_n = R_n$$

Here the complex pressure gradient Da_n is not known from data. Its total value at different levels ΣF_nDa_n could be determined from analyzed fields but would not give the necessary separation in different modes.

We therefore consider

(5) $$Da_n = \alpha_n$$

as an unknown complex quantity which will have to be determined in the variational problem and rewrite (4) in the form

(6) $$ifw_nH_n + \alpha_nF_n = R_n$$

Taking now one of the terms in each of the expansions (1) and (2) we wish to vary a_n, F_n, w_n, H_n and α_n so that the variance of the following residuals

$$R' = \phi - s_nF_n$$

$$R'' = w - w_nH_n$$

is minimized at the same time as R_n in (6). Taking s to represent the variables x, y and t and taking complex quantities into account, we thus wish to determine extremes of the integral

(7) $$I = \int_s \int_p (\phi - a_nF_n)^2 + \mu \int_s \int_p (w^* - w_n^*H_n^*)(w^* - w_n^*H_n^*)\,ds\,p$$

$$+ \lambda \int_s \int_p (ifw_nH_n + \alpha_nF_n)(-ifw_n^*H_n^* + \alpha_n^*F_n)\,ds\,dp$$

where the coefficient μ is determined from data so that wind and geopotential are given equal weight.

Thus

$$\mu = \frac{\int\int \phi^2 ds\,dp}{\int\int ww^* ds\,dp}$$

The Lagrange multiplier λ has the character of an eigenvalue giving for the eigenfunctions a certain but different weight to the requirement of geostrophy.

Varying now a_n, F_n, w_n, H_n and α_n in (7) we obtain a system of five integral equations which can be solved by an iterative procedure starting with a first guess on F_n and H_n and where the normalizations

$$\frac{1}{P} \int F_n^2 dp = 1 \qquad \frac{1}{P} \int H_nH_n^* dp = 1$$

are also applied.

A detailed description of the equations and the method of solution will be given in a research report from SMHI.

3. Results

The data used in a first calculation were all simultaneous wind and radiosonde measurements to 100 mb available from stations in WMO region IV (Europe and surrounding areas) and from one week each of the months January, April, July and October 1976. Altogether about 2x2000 observations were included.

Figures 1-4 show the vertical functions obtained for the first four modes in the expansion. The full line represents F(p), the line with circles the real part of H(p) or Re H(p) and the line with stars Im H(p). The functions F(p) and Re H(p) are very similar to those obtained in previous calculations where wind and geopotential data have been expanded separately. Even if in the present expansion geostrophy is predominant one can for instance in $F_2(p)$ and Re $H_2(p)$ see typical deviations where surface friction, at least partly, is of importance. The form of Im $H_2(p)$ also reflects the effect of surface friction.

The convergence in the expansion is very rapid as is seen from table 1 where the relative contribution to the total variance is given for geopotential and wind separately.

Mode no.	Relative contribution to total variance			
	geopotential		wind	
	%	acc %	%	acc %
1	95.7	95.7	35.9	85.9
2	3.2	98.9	6.4	92.3
3	0.8	99.7	3.0	95.3
4	0.2	99.9	1.5	96.8

Table 1

Already two terms in the expansion cover almost 99 per cent of the variance in the geopotential. The convergence in the wind is slower. This is certainly due to a number of different factors such as large scale ageostrophic wind, measurement errors, local circulations which are not reflected in the geopotential field etc.

In a few cases the functions F(p) and H(p) have been used for expansion of independent data. The results obtained so far show almost the same convergence in the series indicating that the expansion may be useful in the construction of a primitive model. It may also be used as a filter in order to obtain a suitable initial field for more conventional primitive equation models.

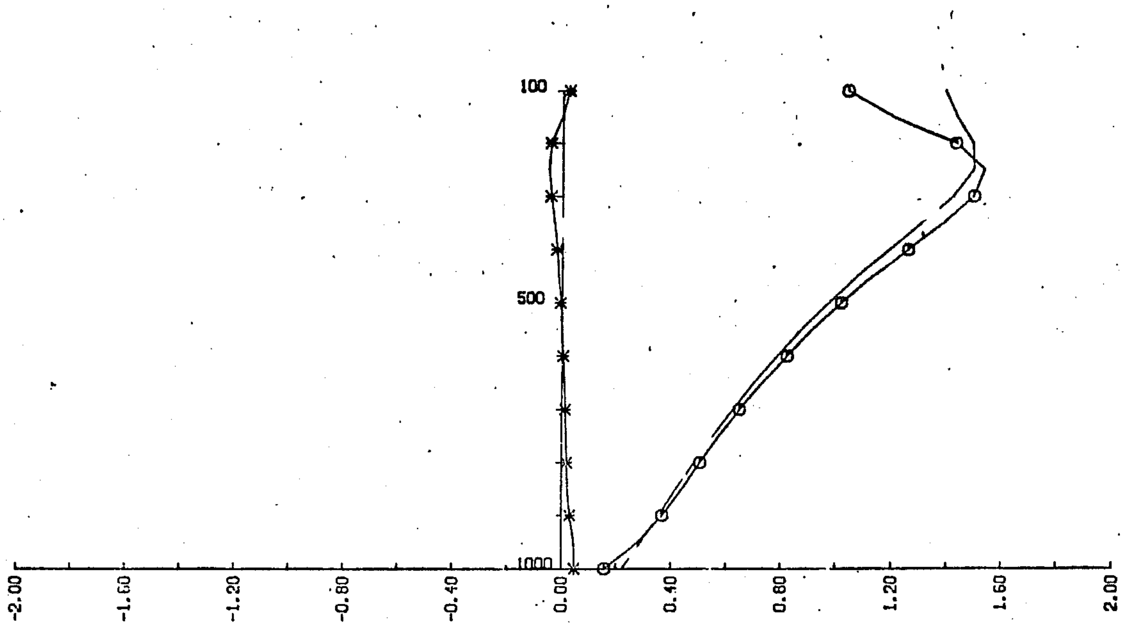

Figure 1

The functions $F_1(p)$ and $H_1(p)$

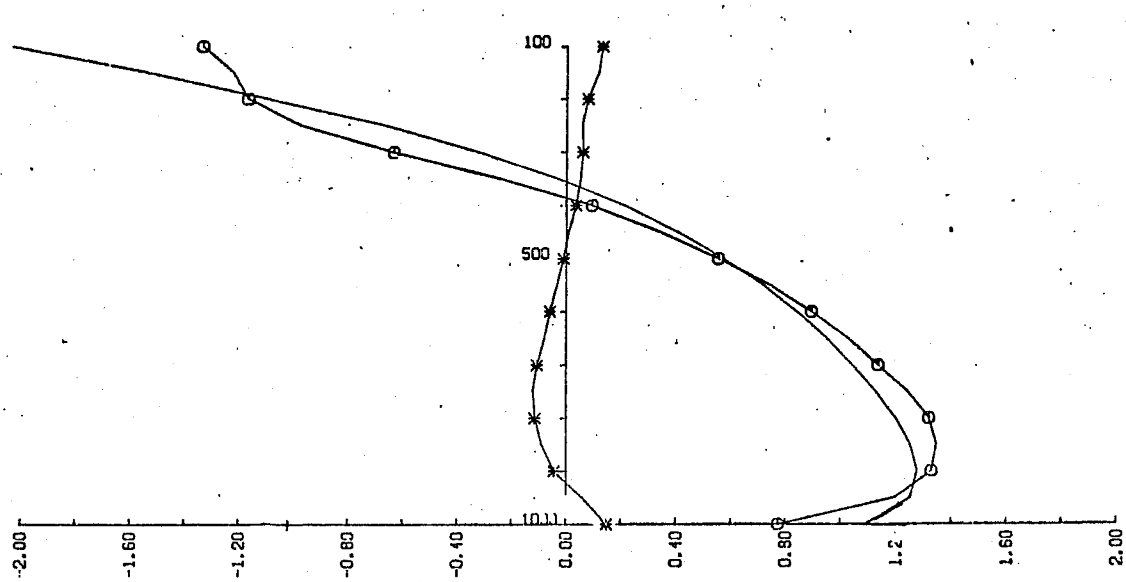

Figure 2

The functions $F_2(p)$ and $H_2(p)$

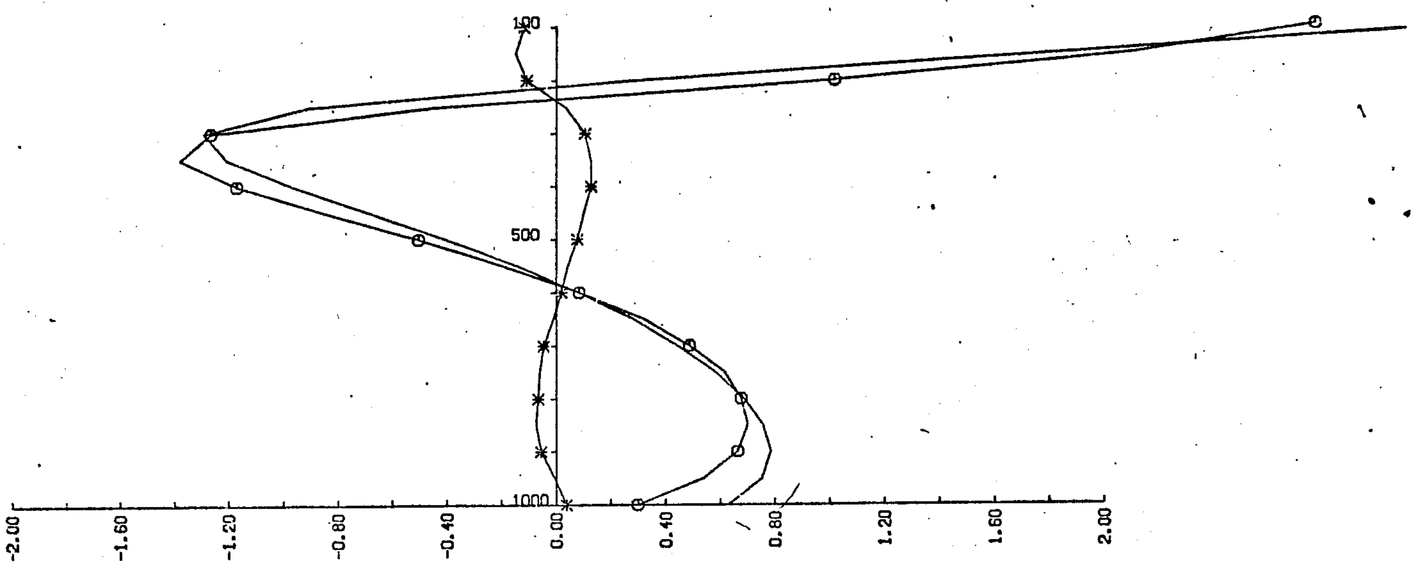

Figure 3
The functions $F_3(p)$ and $H_3(p)$

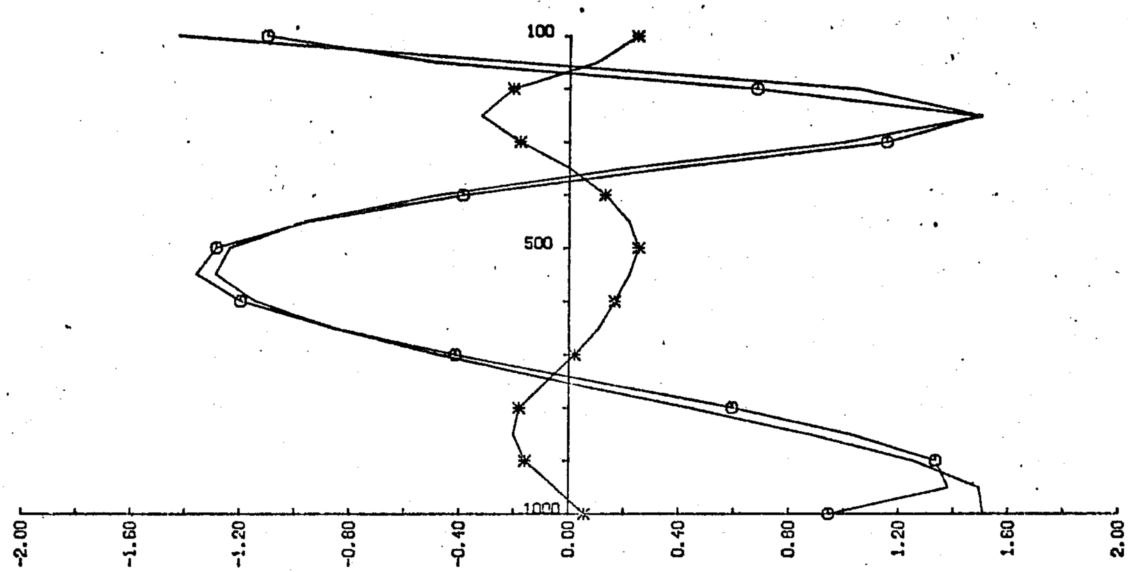

Figure 4
The functions $F_4(p)$ and $H_4(p)$