PARAMETERISATION

OF THE

PLANETARY BOUNDARY LAYER IN GLOBAL MODELS

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APPENDIX

SURFACE LAYER THEORY FOR MODELLERS

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1. Introduction

As we have already seen from Dr. Hide's lectures, boundary layers in rotating fluids can have radical effects on the entire regime of flow. This is no less true in the atmosphere than in other rotating systems.

The atmospheric boundary layer supplies roughly 50% of the Internal Energy in the atmosphere through the water vapour budget. The water vapour itself plays an important part in maintaining the static stability of mid latitudes at a value well removed from the dry adiabat.

The greater part of the atmospheric kinetic energy is dissipated in the boundary layer. In parallel with this process the boundary layer forces vertical velocities and fields of convergence and divergence in the free atmosphere which maintain the atmosphere in a state of approximately rigid rotation, in that $U / (a\Omega) \ll 1$ where U is a typical atmospheric velocity, a is the radius of the earth and Ω is the rate of rotation of the earth.

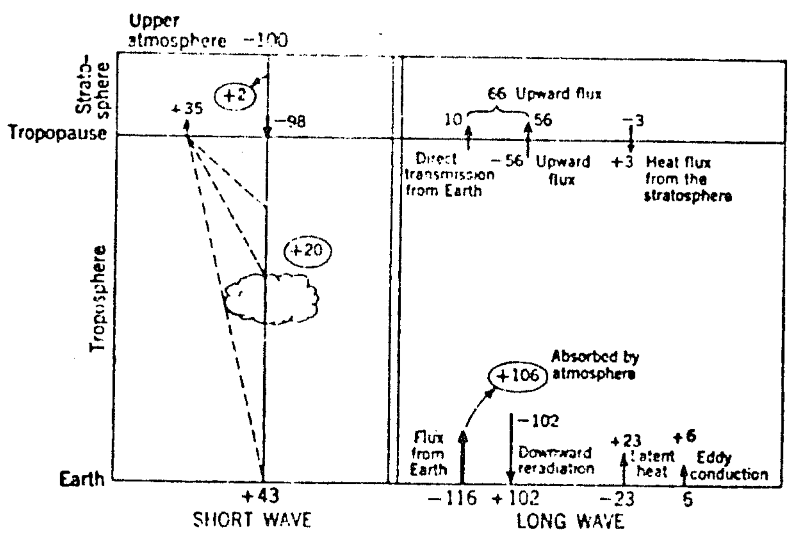

Fig. 1 The heat balance of the earth-troposphere system.

Fig. 1 taken from Haltiner and Martin (1957) will be familiar to you, and shows a schematic of the energy balance of the atmosphere. Of the incident solar energy roughly 35% is reflected or back scattered, 20% is directly absorbed by the atmosphere and 43% is absorbed by the surface.
Since on average the gound temperatures are stable over periods of several years this energy is returned to the atmosphere in the form of radiant energy (14%), latent heat (23%) and sensible heat (6%).

The presence of the latent heat has profound effects on atmospheric dynamics in a number of important ways. One could argue that the most fundamental of these is through its effect on the mean static stability of the atmosphere.

The atmosphere is a rapidly rotating fluid, the perturbations away from a state of rigid rotation are relatively small. At mid latitudes the zonal flow relative to the earth are 20 - 30 m/sec. while the solid body rotation velocity is ~ 300 m/sec. Thus the flow is certainly quasi-geostrophic. Moreover, we know from the theory of baroclinic instability that in order to get this instability in the form we are familiar with we must have

$$RiRo^2 \sim 1$$

$$Ri = \frac{N^2}{(U_z)^2}$$

$$Ro = \frac{U}{fL}$$

and N is the Brunt-Väisälä frequency, U is a typical velocity, f the Coriolis parameter, L a typical length scale and U_z the vertical wind shear.

In practice this means that we must have a large static stability. The tropospheric lapse rate is ~6.5°K/km. Below about 500 mb, depending on the temperature, the moist adiabats have a lapse of ~ 5°K/km. Above 500 mb, depending on the temperature, the moist adiabats change character until they become parallel to the dry adiabats with a lapse of g/Cp ~10° K/km. This poses the question of the influence of water vapour on the mean tropospheric lapse rate.

Once after listening to a lecture on planetary atmospheres by Professor Hide, I looked up a table of L_v for various substances. The largest was H_2O and has a value of ~600 cal/gm, the next was ammonia with ~300 cal/gm. I then asked what would the moist adiabat look like if we reduced L to 50% of its present value.

The results are shown in Fig. 2.

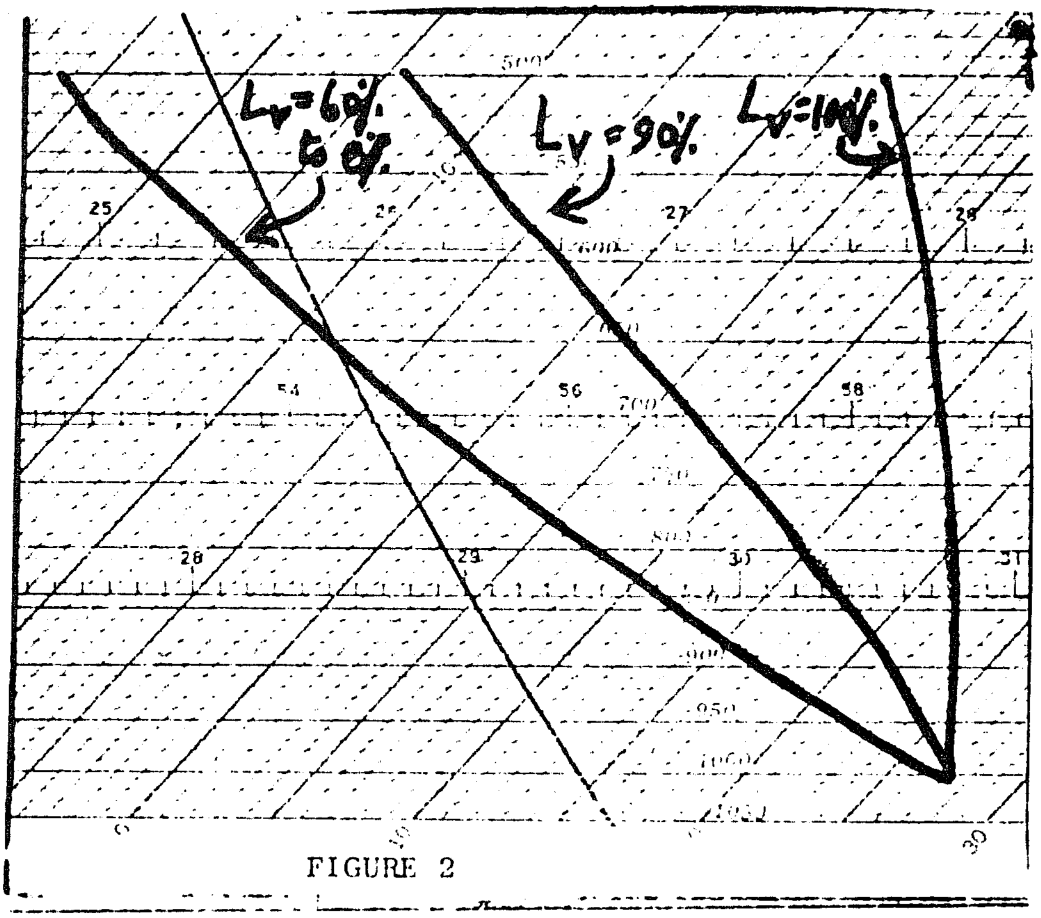

FIGURE 2

They demonstrate the importance of the large value of $L_v(H_2O)$ for the saturated adiabat. The question (posed in Hide, 1967) as to whether the large static stability of the atmoshere in mid latitudes is due to the activity of baroclinic waves or to moist processes is still open. It seems reasonable to suppose that the large latent supply to the atmosphere through the boundary layer has some dynamic implications in addition to the energetic considerations.

It is essential for a model of the atmosphere that the boundary layer formulation treats the following areas in a realistic manner. It must specify (1) the surface values of the turbulent fluxes of momentum heat and moisture, (2) their variation in the vertical, (3) the height of the top of the boundary layer and (4) the vertical velocity at the top of the boundary layer.

2. Geostrophic models

The very first atmospheric models used for numerical weather prediction were based on barotropic vorticity equation models.

Charney and Eliassen (1949) considered forecasts of the January mean 500 mb anomalies taking account of vorticity advection and the vertical velocity induced by mountains. They found that their solutions were much improved if they took account of the forced motions due to boundary layer convergence. They used Ekman layer theory to derive the relationship

$$\omega_0 = -\frac{1}{\varrho_0}\,\mathrm{div}\,\underline{D} = \frac{H}{f} F \zeta_{g0}$$

where ω_0 is the vertical velocity at the top of the boundary layer, ζ_{g0} the geostrophic vorticity, $\underline{D}$ is the vertically integrated mass transport in the boundary layer, H is the depth of the boundary layer and

$$F = \frac{\sin 2\alpha}{\sqrt{2}} \frac{\sqrt{Kf}}{H}$$

Fig. 3 taken from their paper shows the results of including this effect in a prediction of the anomalies in the 500 mb height field for January at 45° N. The anomalies were assumed to be due to mountains and Ekman layer convergence. The inclusion of the Ekman layer term has a marked effect on the perturbations.

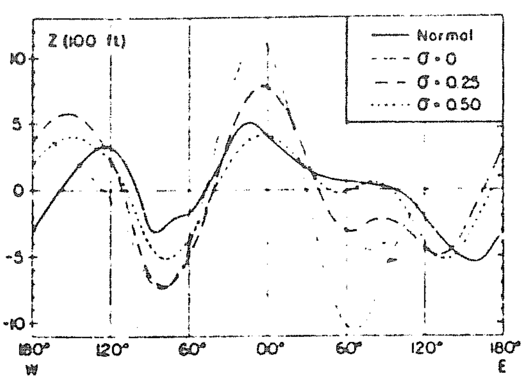

Fig 3 Normal height profile of the 500 mb surface at 45° N for January together with computed stationary profiles for σ = 0 (no friction), for σ = 0.25 (moderate friction) and for σ = 0.50 (strong friction). For purposes of comparison the heights are represented as deviations from their respective means

With the advent of multi-layer or baroclinic quasi-geostrophic models this parameterisation of Charney and Eliassen was found to work quite well and was used in most models.

The other crucial area of boundary layer control, through the effect of the fluxes of heat and moisture on the static stability was cheerfully circumvented by quasi-geostrophic theory which, at least in its simpler forms, specifies the static stability as a function of height and, possibly, time, but not as a function of horizontal position.

3. Primitive Equation Models

With the advent of Primitive Equation Models and more powerful computers a number of new possibilities become available.

Since the flow in the free atmosphere was no longer assumed to be quasi-geostrophic one could try to resolve the boundary layer rather than parameterising it by means of the Charney Eliassen scheme. This was the approach used by Smagorinsky ,Holloway and Manabe (1965). We discuss their boundary layer formulation in some detail, as most other formulations have a similar general structure.

The vertical grid uses unevenly spaced levels in the vertical (TABLE I) .

TABLE 1

LEVEL	$\sigma=p/p_*$	Height (m)
1	.0159	27,900
2	.07	18,330
3	.165	12,890
4	.315	8,680
5	.5	5,430
6	.685	3,060
7	.835	1,490
8	.940	520
9	.990	80

At the lower boundary they used the following conditions for the vertical fluxes of momentum, sensible heat and latent heat:

$$\tau = \rho(h)\, C_D(h)\, |\underline{V}(h)|\, \underline{V}(h)$$

where $C_D(h) = \{k_0 / \ln(h/z_0)\}^2$, $h = 75\,m$

$z_0 = 1\,cm$

$$H = c_p \rho(L) C_D(L) |\underline{V}(L)| \{T_* - T(L)/\sigma(L)^{\kappa}\}$$

$$(LH) = L_v E$$

where

$$E = \rho(L) C_D(L) |\underline{V}(L)| \{q_s(T_*) - q(L)\}$$

Here k_0 is von Karman's constant, T_* is the surface temperature, q is the specific humidity, q_s its saturated value and ρ is the density.

Vertical transfer of momentum and moisture is included for the layer below ~700 mb. The calculation is made according to the mixing length theory for neutral stratification. The vertical transfer of heat is not included except through the convection scheme. The vertical convergences of u,v momentum and of specific humidity are written

$$\begin{Bmatrix} F_\lambda \\ F_\theta \\ F_q \end{Bmatrix} = \frac{\partial}{\partial\sigma} l^2 \left|\frac{\partial \underline{V}}{\partial\sigma}\right| \frac{\partial}{\partial\sigma} \begin{Bmatrix} p_* u \\ p_* v \\ p_* q \end{Bmatrix}$$

where the mixing length l is specified by

$$l = \begin{cases} k_0 z & z \le h \\ k_0 \dfrac{h(H-z)}{H-h} & h \le z \le H \\ 0 & H < z \end{cases}$$

h=75 m   H=2.5 km $k_0 = 0.4$ z_0 = 1 cm

In particular the boundary layer height is constant.

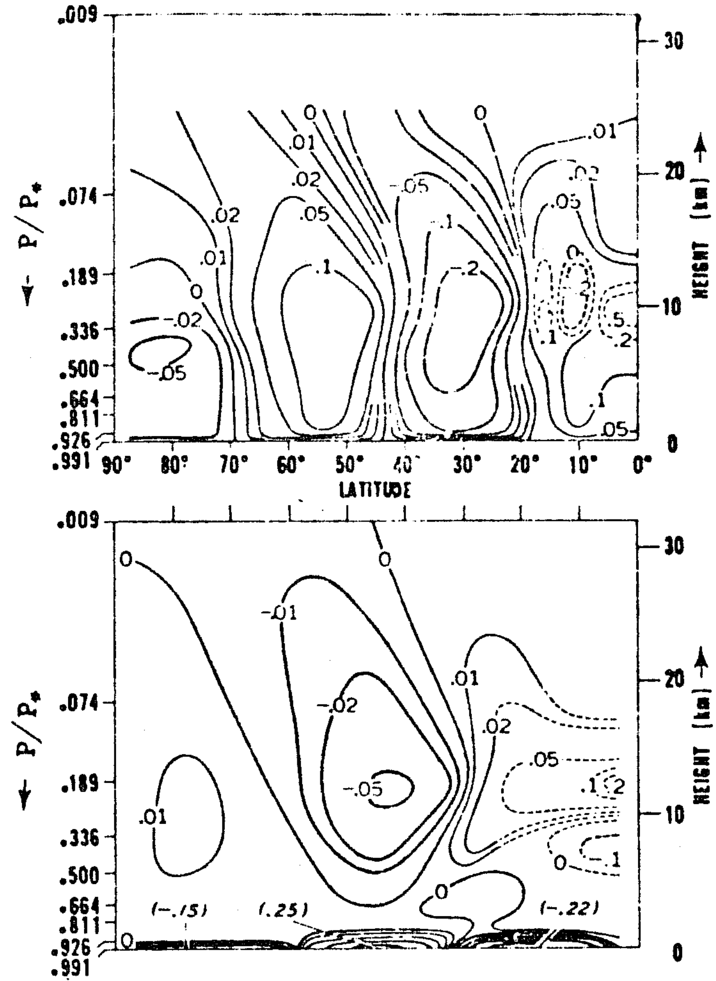

FIGURE 4' —The zonal mean of the vertical component of the wind (cm./sec.) and of the meridional component of the wind (m./sec.) of the model atmosphere are shown in the upper and lower parts of the figure, respectively. Positive values are upward and northward.

Figure 4 shows the mean meridional circulation in their model. The authors showed that the eccentricity of the circulation is due to the assumed form of the variation of the mixing length coefficient with height.

We turn now to a consideration of the boundary layer formulations in other models. Bhumralkar (1975) provides a convenient digest.

As regards the surface fluxes there are many ways of prescribing them. Some models, such as the GFDL model, ignore the effect of stability on the momentum fluxes. Others take account of this in a variety of ways using empirical constants based on measurements. Some also assume that the stress acts at an angle to the wind at lowest level in the model.

Table II taken from Bhumralkar's paper summarises the position in detail.

TABLE II

COMPUTATION OF SURFACE FLUXES IN VARIOUS GCMS

Model	Fluxes Calculated from Eq. (11) or Eq. (12)	No. of Levels Coor-dinate System	Location of Level s Lowest GCM Level/Layer	Values for $C_D/C_H/C_E$	Determination of $\vec{V}_s$	Determination of T_s q_s	Determination of T_b	Determination of q_b	Remarks
1. GFDL (1971) Holloway & Manabe (1971)	Case I Eq. (11)	9/σ	75 m 75 m	$C_D = C_H = C_E$ (i) $C_D = 2 \times 10^{-3}$ (both land and sea) (ii) (C_D)ocean $= 1.1 \times 10^{-3}$ (C_D) land $= 4.3 \times 10^{-3}$	All three calculated from <u>prognostic</u> equations at the level s		Sea: specified Land: heat budget equation at the surface	Saturated q at T_b	GFDL (1974): 18 levels in the vertical; level s at 36 m
2. NCAR (1971) Kasahara & Washington (1971)	Case I Eq. (11)	6/Z	No explicit level s in the GCM Lowest GCM level = 3 km	$C_D = C_H = 3 \times 10^{-3}$ $C_E = 0.7\, C_D$ to reduce evaporation.	All three calculated <u>diagnostically by</u> equating surface fluxes to the fluxes in layer from s to the lowest GCM level (at 3 km) (see text)		Same way as GFDL	Same as GFDL	
3. RAND (1971) Gates et al. (1971)	Case I Eq. (11)	2/σ	No explicit level s in the GCM Lowest GCM level = 800 mb = σ = .75 layer = 600 mb = σ = .50	For neutral surface layer (C_D)ocean $= \min[(1 + .07\lvert\vec{V}_s\rvert)\, 10^{-3}, .0025]$ (C_D)land $= .002 + .006\left(\frac{Z_s}{5000\ m}\right)$ also $C_D = C_H = C_E$	$\vec{V}_s$ obtained by linear extrapolation of predicted velocities at levels 1 & 3 to the lower boundary	T_s, q_s are obtained by equating surface fluxes of sensible heat and moisture to the same fluxes from s to the lowest GCM level (at σ = .75	Prognostic equation	Saturated q at T_b	
4. GISS (1974) Somerville et al. (1974)	Case I Eq. (11)	9/σ	No explicit level s in the GCM Lowest GCM level ~ 1 km layer ~ σ = .9	For <u>neutral</u> surface layer same as in RAND GCM For <u>non-neutral</u> conditions (for all surfaces) $C_D = (C_D)\text{neutral}\ \frac{1}{\sqrt{1 - 7\,\Delta T/\lvert\vec{V}_s\rvert^2}}$ for $\Delta T < 0$ (stable) $C_D = (C_D)\text{neutral}\ (1 + \sqrt{\Delta T/\lvert\vec{V}_s\rvert^2}$ for $\Delta T > 0$ (unstable) $\Delta T = T_b - T_s$ also $C_D = C_H = C_E$	$\vec{V}_s$ obtained by extrapolating velocities in layers 8 and 9 to the lower boundary	T_s, q_s same as in RAND except the surface fluxes are equated to fluxes from S to σ = .9, the lowest GCM level	Prognostic equation	Saturated q at T_b	
5. UCLA (1974) Arakawa and Mintz (1974)		3/σ	No explicit level s in the GCM Lowest GCM level σ = 8/9 Layer σ = 7/9	C_D and C_H have a functional dependence on bulk Richardson number and (h/Z_o) h is the depth of b.l. Z_o is roughness: $\begin{cases} .45\ m\text{(land} \\ 2.5 \times 10^{-4}\ m\text{(oceans)} \end{cases}$	See text				

TABLE II (CONTINUED)

COMPUTATION OF SURFACE FLUXES IN VARIOUS GCMs

Model	Fluxes Calculated from Eq. (11) or Eq. (12)	No. of Levels Coordinate System	Location of Level a Lowest GCM Level/Layer	Values for $C_D/C_H/C_E$	Determination of				Remarks
					$\vec{V}_a$	T_a q_a	T_b	q_b	
6. UK I (1972) (Similarity theory) Corby et al. (1972)	Case II Eq. (12)	5/σ	No explicit level a in the GCM Lowest GCM level at 1 km (also the top of the b.l.)	$C_D = C_H = C_E$ α Land: $\begin{cases} 9.3 \times 10^{-3} \text{ stable} & \pi/6 \\ 4.0 \times 10^{-3} \text{ unstable} & \pi/18 \end{cases}$ Ocean: $\begin{cases} 0.2 \times 10^{-3} \text{ stable} & \pi/9 \\ 2.0 \times 10^{-3} \text{ unstable} & 0 \end{cases}$ α is the angle between the surface stress and the wind at level a. Z_o: roughness parameter Z_o = 10 cm for land; 1 cm for sea	Computed prognostically at 1 km. $\Delta\theta_e = (\theta_e)_a - (\theta_e)_b$ $\Delta q = (q)_a - q_b$	Uses equivalent potential temperature (see text) $Q_o = \rho C_p C_D \left[\lvert \vec{V}_a \rvert + A \left(\frac{\Delta\theta_e}{\theta_e} \right)^{\frac{1}{2}} \right] \cdot \Delta\theta_e$ where θ_e = equivalent potential temperature. $E_o = \rho C_D L \left[\lvert \vec{V}_a \rvert + A \left(\frac{\Delta\theta_e}{\theta_e} \right)^{\frac{1}{2}} \right] \cdot \Delta q$	Prognostic equation	RH = (75% over ocean / 60% over land)	Formulation for Q_o is modified in order to establish a near moist adiabatic lapse in lowest km over typical oceans (as is observed) and at the same time allow for a steeper lapse rate to exist over treated land areas (see text).
7. UK II (1974) (Similarity theory) (Only ocean-covered planet considered) Pearce (1974)	Case II Eq. (12)	5/σ	No explicit level a in the GCM Lowest level at 850 meters	$C_D = \left(\frac{U_*^2}{\lvert \vec{V}_a \rvert^2} \right)$, C_H, C_E, α are obtained from nomograms (Clark, 1970) which are plots of U_*, angle α, C_H, C_E as functions S and R_o.	Given by prognostic equation at 850 m. (This is required to obtain Q_o and E_o). (i) Surface stress given by $\tau_o = \rho U_*^2$ (ii) τ_o, Q_o and E_o are zero when $\lvert \vec{V}_a \rvert < 5 \frac{cm}{sec}$	T_a, q_a are referred to the lowest GCM level, i.e., at 850 m to obtain $\Delta T = T_a - T_b$ $\Delta q = (q)_a - q_b$	Prescribed as function of latitude	Saturated q at T_b	S = stability parameter = (ρ ... + (gΔq)/... V_a $\Delta T = T_a - T_b$ R_o = surface Rossby No. = ... θ = potential temperature is used instead of Temperature f corresponds to 21° latitude between 21° and equator
8. USSR (1974) Fux-Rabinovich (1974)	Case II Eq. (12)	5/ζ	No explicit level a in the GCM Upper limit of the b.l. = 1.5 km fixed	C_D, C_H, C_E and α determined from nomograms (same as in UK II above (7)).	Details not available				ζ, coordinate in vertical, takes into account the global orography $S = \frac{g}{\gamma} \frac{(T_o - T_h - h\gamma_h)}{f \lvert \vec{V}_a \rvert}$ h: b.l. top (first GCM level) γ = lapse rate at h = 6°C/km

As regards the specification of the turbulent fluxes above the surface layer the majority of the formulations are of the following kind :

$$\tau = \rho K_m \frac{\partial \bar{V}}{\partial z}$$

$$H = -\rho C_p K_H \left(\frac{\partial \theta}{\partial z} - \gamma_{CG} \right)$$

$$E = -\rho K_E \frac{\partial q}{\partial z}$$

where K_H K_m K_E may be functions of stability and γ_{CG} is the counter gradient factor introduced by Deardorff. Some models (e.g. NCAR) have no explicit boundary layer and use the formulation $\rho_s C_D |V_s| V_s = \rho_s K_H (V_1 - V_s)/(z_1 - z_s)$

assuming no flux degree in the boundary layer, where s refers to the surface and 1 to the lowest model level. One assumes that all the diffusion coefficients are equal and takes them to be functions of stability:

$$K_H = d\left[A_1 + A_2\left\{1 - \exp\left\langle A_3\left(\frac{\partial \theta}{\partial z} - \gamma_{CG}\right)\right\rangle\right\}\right], \quad \text{unstable}$$

$$= d\left[A_1/(1 + A_4 Ri) + A_5\right] \quad \text{stable}$$

where Ri is the Richardson number.

Other models (e.g. Delsol, Miyakoda and Clarke 1971) calculate an explicit boundary layer

$$K_m = l^2 \left|\frac{\partial V}{\partial z}\right| (1 - \alpha_c S) \quad \text{unstable}$$

$$= l^2 \left|\frac{\partial V}{\partial z}\right| (1 + \alpha_c S)^{-1} \quad \text{stable.}$$

where S is a stability parameter

The notable exception from all of this is the UCLA model which uses the methods discussed by Deardorff in this seminar series. They need not, therefore, be dealt with here.

Finally, in quite a different vein we mention the method used by Fischer (1976).

In this approach the model is restricted to the free atmosphere. Normal σ-coordinate models use the lower boundary condition

$$\dot{\sigma} = 0 \text{ on } \sigma = 1 = P/P_*$$

Fischer instead uses

$$\dot{\sigma}_H = \frac{1-\sigma_H}{p_*}\left\{\int_0^{\sigma_H} \nabla \cdot p_* \underline{V}\, d\sigma + \frac{g}{f} J(p_*, \phi - RT)\right\} - \frac{g\,\sigma_H}{f\,p_*}\,\mathrm{Curl}\,\underline{\tau}_0$$

$$\frac{\partial p_*}{\partial t} = \frac{1}{\sigma_H}\int_0^{\sigma_H} \nabla \cdot p_* \underline{V}\, d\sigma - \frac{p_*}{\sigma_H}\dot{\sigma}_H \quad , \quad \sigma_H \simeq 900\,\mathrm{mb}.$$

so that he imposes a vertical velocity at the top of the boundary layer proportional to the curl of the surface stress, following Charney and Eliassen. We discuss the effects of these representations in the next section.

4. Tests of the Parameterisations

Given this wide range of prescriptions can we decide which is the best one from the point of view of accuracy and economy ? It is clear, a priori, that most are deficient because of the implied assumptions of stationarity and homogeneity. The data necessary to test the formulations in detail away from the surface layer is only slowly emerging. There are indications already from the observational studies presented by other speakers at this meeting that the presence of clouds in the form of strato cumulus or small cumulus has a profound effect on the vertical fluxes in the P.B.L. The work of Mahrt (1975) indicates that non-linear effects, such as advection, which allows the fluid to remember its past history, is significant in the b.l. particularly near the equator.

In order to test the parameterisations, one can run forecast models with differing prescriptions and compare the results with real data and with each other. One can then try to identify the features of the parameterisations to which the model is most sensitive. This is a difficult matter because the parameterisations for all of the physical effects interact. Thus changes in the boundary layer formulations may not affect the free atmosphere directly, but rather indirectly through its effect on some other process such as convection or changes in the radiative field.

Comparisons of the effect of different boundary layer prescriptions have been published by Delsol, Miyakoda, Clarke (1971) (DMC) and by Fischer (1976) and others.

DMC made a number of forecasts from the same set of real data, varying special aspects of the boundary layer parameterisation.

In the first experiment there were three runs to examine the surface layer formulation. In run A the standard GFDL formulation was used. In run B everything was as in A except that the drag coefficient C_D was $4.3 * 10^{-3}$ over land and $1.1 * 10^{-3}$ over sea. In run C the Monin-Obukhov formulation for the surface layer was used. (See appendix for a summary of the Monin-Obukhov theory of the surface layer). Over land the roughness length was taken as 16.8 cm giving a drag coefficient of $4.2 * 10^{-3}$. Over sea z_o was specified as $z_o = .032\ |V*|^2/g$, following Charnock.

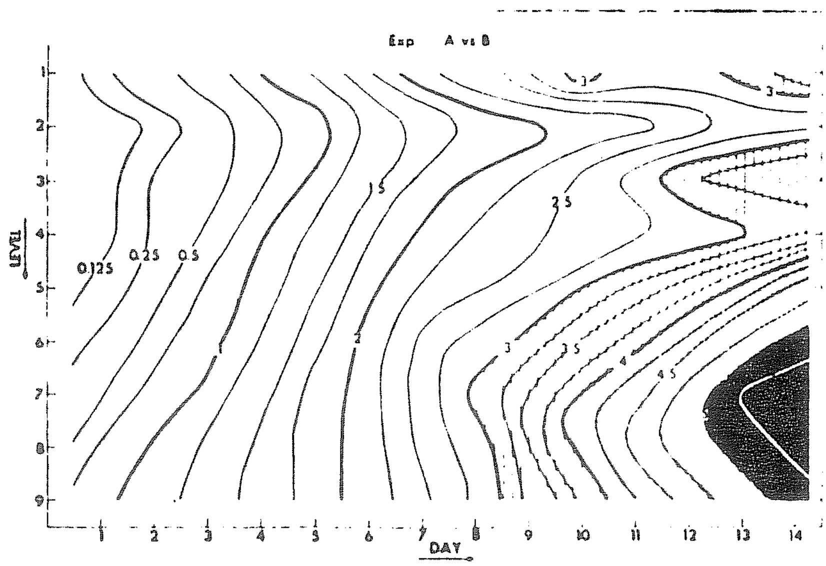

Figure 5 Time evolution of the discrepancy between Experiments A and B. The contoured variable is the r.m.s of the temperature difference of the two experiments averaged spatially in units of degrees C. The abscissa is time and the ordinate is the vertical level

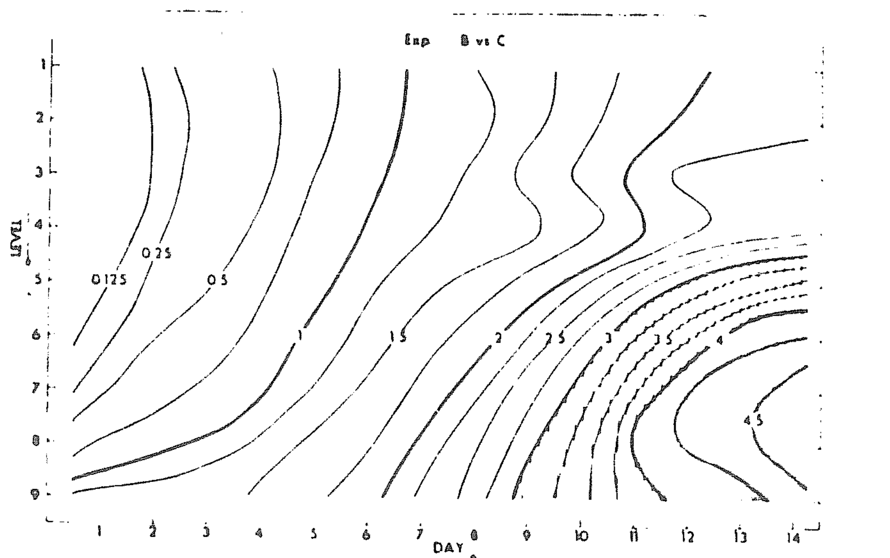

Figure 6 The same as in Fig 5 but for Experiments B and C

DMC provide a discussion of the synoptic differences in the forecasts. Over the oceans the cyclones in B are deeper than in A on days 6 - 8 . The wind intensity over land at the lowest level is lower in B than in A, with the converse situation over the ocean. An overall view of the differences is presented in Figures 5 and 6 which give a time height plot of the r.m.s. temperature differences between the three experiments for the domain north of 20^o North.
"The differences between A and B are appreciably larger than those between B and C. B and C both produce appreciably larger differences from A but have smaller differences between themselves suggesting that the use of values of C_D varying between land and sea produces a larger effect than the change in the sophistication of the turbulent transfer processes in the surface layer".

In a further set of integrations DMC studied the effect of a change in the Ekman layer, or Planetary boundary layer parameterisation with the same initial data.

The first run, run D, was exactly as run C except for the inclusion of a diurnal cycle. The Ekman layer formulation was

$$K_m = \ell^2 \left| \frac{\partial V}{\partial z} \right|$$

$$K_m = K_h = K_q, \quad \ell = 30\text{m for } 75\text{ m} \leq z \leq 2.5\text{km}, \quad = 0 \text{ for } z > 2.5\text{ km}$$

Run E, like run D, included a diurnal cycle but the formulation of the exchange coefficient took account of the stability.

In unstable situations $K_m = \ell^2 \left|\frac{\partial V}{\partial z}\right| (1 - \alpha S)$

In stable situations $K_m = \ell^2 \left|\frac{\partial V}{\partial z}\right| (1 + \alpha S)^{-1}$

$$K_m = K_H = K_q$$

$$\alpha = 18$$

$$S = \frac{(g\ell)^{\frac{1}{2}} \frac{\partial \theta}{\partial z}}{\theta \left|\frac{\partial V}{\partial z}\right|}$$

$$\ell = k_0 z / \left(1 + \frac{1480}{|V|} f z\right)$$

In addition heat conduction is permitted in the soil according to

$$\frac{\partial T_s}{\partial t} = K_s \frac{\partial^2 T_s}{\partial z^2}$$

while the surface heat flux is $H_{soil} = - \left(\rho_s C_s K_s \frac{\partial T_s}{\partial z}\right)_{z=0}$

with $K_s = 7 * 10^{-3}\ cm^2 S^{-1}$, $C_s = .2\ Cal\ g^{-1}\ {}^oK^{-1}$ $\rho_s = 1.5\ g\ cm^{-3}$

Figures 7 and 8 compare r.m.s. temperature differences between experiments C,D,E. The introduction of the diurnal cycle in the standard GFDL formulation for the Ekman layer produces relatively minor changes (fig.7). The changes between D and E (fig.8) are a good deal larger.

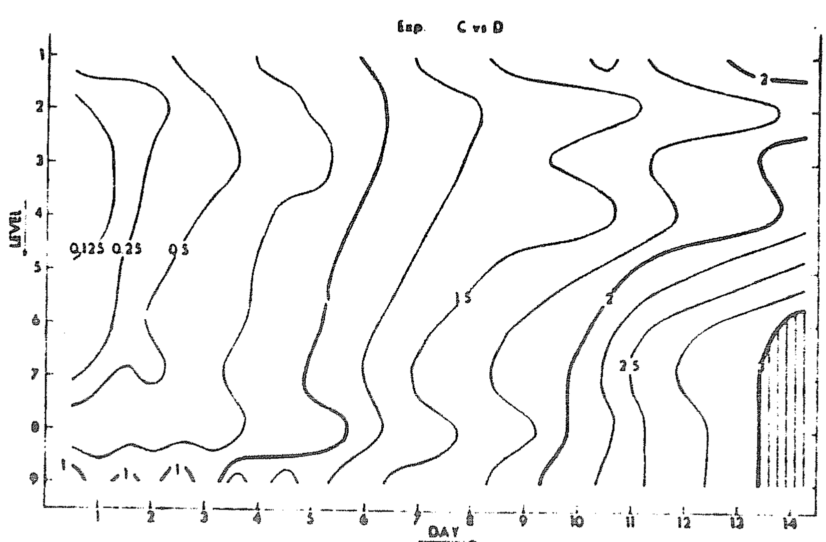

Figure 7 Time evolution of the temperature difference between Experiments C and D. See Fig. 5 for further details.

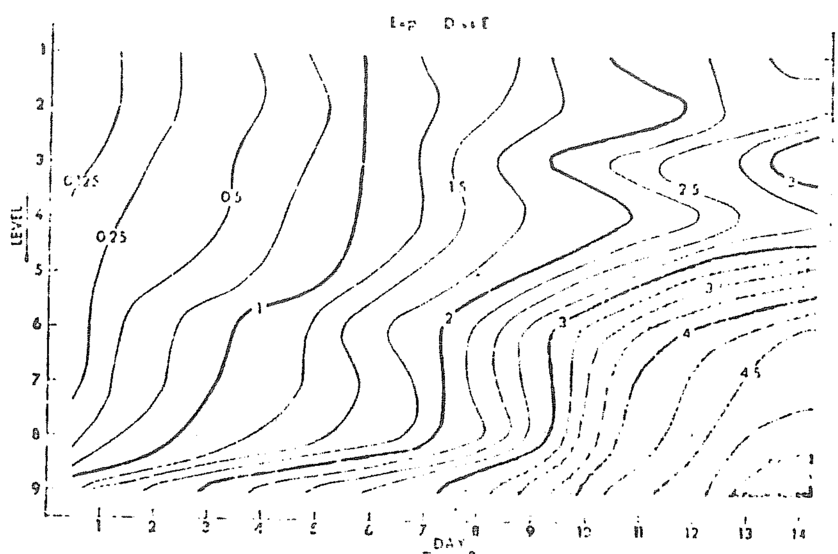

Figure 8 The same as in Fig. 7 but for Experiments D and E

The synoptic charts at 500 mb and 1000 mb at day 6 in experiments A and E, i.e. the original GFDL version and the version with soil conduction, Monin-Obukhov surface layer and stability dependent Ekman layer, show differences that are hardly noticeable after six days. However, after 10 days the differences were more sizeable.

In summarising their results DMC observed that of the various processes relating to the boundary layer that they studied, the most substantial effect was produced by varying the surface drag coefficient between sea and land. As pointed out by Bengtsson (per.comm.) this may be because of a direct effect on the longest waves, due to the distribution of continents and oceans. Second in importance is the Ekman layer parameterisation and third is the Monin-Obukhov formulation for the surface layer. The diurnal cycle was least important although the effect was greatly amplified when the Ekman layer transfer process has a Richardson number dependence.

In contrast to these rather moderate effects of the boundary layer parameterisation the results of Fischer (1976) are striking.

Fischer considered flow in an f-plane channel starting from a weak analytically specified barotropic disturbance superimposed on a zonal jet. Apart from the surface effects to be described the flow is adiabatic.

The model is a 5 level p.e. model with equally spaced levels. One supposes that the flux divergence of momentum $-g\frac{\partial \tau}{\partial p}$ is non-zero only at the lowest level (900 mb).
In the first formulation one approximates

$$-g\left(\frac{\partial \tau}{\partial p}\right)_{900} = - g\, \tau_0 /200 \text{ mb}.$$

where τ_0 is the surface wind stress. The second formulation uses a frictionally induced vertical velocity at 900 mb which carries the vertical transport of momentum into the boundary layer so that

$$\omega_H = - g/f \text{ Curl } \underline{\tau}_0$$

This is added to the vertical velocity at 900 mb and the surface pressure tendency equation suitably modified (para. 3 above). Three prescriptions for $\underline{\tau}_0$ were used in formulation I:

a) bulk aerodynamic formulation
b) resistance law formulation
c) the formulation used in the NCAR model.

The prescriptions a and b were used in formulation II.

Table III (adapted from Fischer 1976) shows the deepening of the low centre in 100 hours in the adiabatic experiment (0) and in the five other experiments. The remarkable feature is the extent to which the Ekman pumping formulation has reduced the growth by almost half. The reason for this is unclear and worthy of further study. The result is at variance with Barcilon's results (1964) for baroclinic instability on an f-plane.

TABLE III

Expt.	Change in central pressure after 100 hours	
0	-29	mb
Ia	-25.6	"
Ib	-22.4	"
Ic	-23.8	"
IIa	-14.5	"
IIb	-17.3	"

Barcilon's equations were quasi-geostrophic and he had much more resolution in the vertical. This latter point is perhaps the more important source of the difference.

5. Theoretical Studies of Effect of Boundary Layer on Large Scale Flow

From the point of view of the numerical forecaster faced with producing medium range forecasts, there is one further question to be considered. This has to do with the vertical resolution to be used given available resources. We are currently addressing ourselves to this problem at ECMWF from the following point of view. Dynamicists have shown that friction can be modelled in the fairly simple way introduced by Charney and Eliassen.

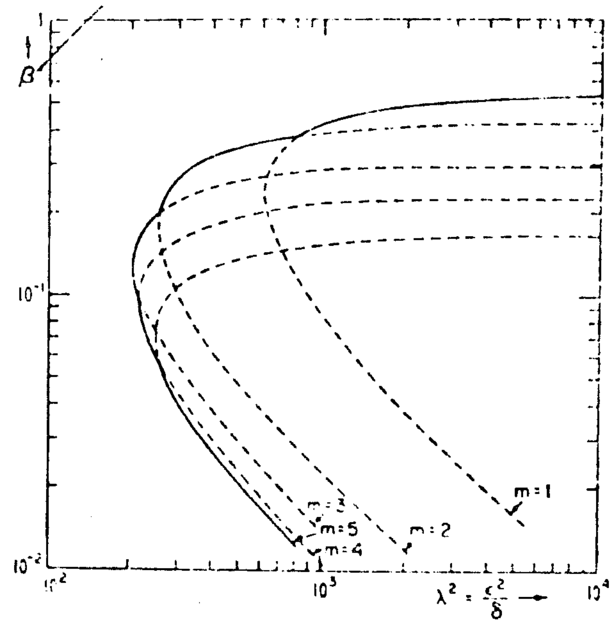

Fig. 9 Transition curve for the case $b=3a$. The transition curve (solid line) is the envelope of the transition curves for discrete values of m (dotted lines) which are integral multiples of $2(b-a)/(b+a)$. The transition curve shown here is for an annulus with $b=3a$.

Barcilon (1964) studied the effect of friction in an annulus and showed that the transition curves found experimentally could be reproduced using this formulation. Fig. 9 taken from his paper shows an example of the sort of result he found. He implicitly assumed that time dependent effects in the boundary layer were unimportant so that the boundary layer flow was instantaneously adjusted to the interior flow.

Charney and Eliassen (1964) in an important paper used the same Ekman pumping formulation to study the growth of the hurricane depression by the CISK mechanism.

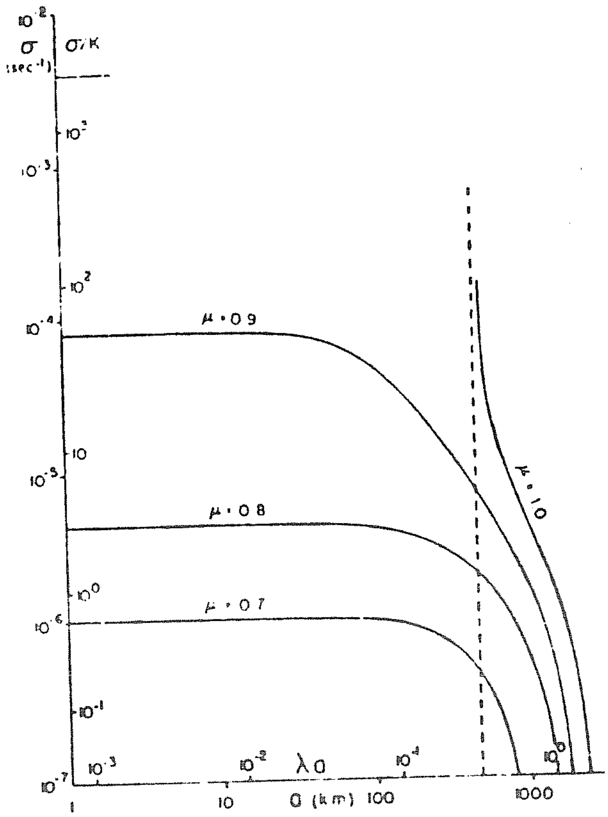

FIG. 10 The growth rate of the tropical depression as a function of the radius of the cloud region.

Fig. 10 shows an example of their results relating the growth rate to the horizontal scale of the disturbance for various values of the parameter

$$\mu = \bar{q}/q_s$$

where $\bar{q}$ is the horizontally averaged humidity in the disturbance and qs is the saturation value for the horizontally averaged temperature.

The growth rate curves level off at a radius of ~ 100 km corresponding to hurricane dimensions. Thus this study, like Barcilon's, used an instantaneously adjusted boundary layer and like Barcilon's has a substantial measure of success in elucidating physical mechanisms.

Many studies since then have investigated the time dependent effects in the boundary layer in response to an imposed external forcing, e.g. Bates (1973), Kuo (1973).

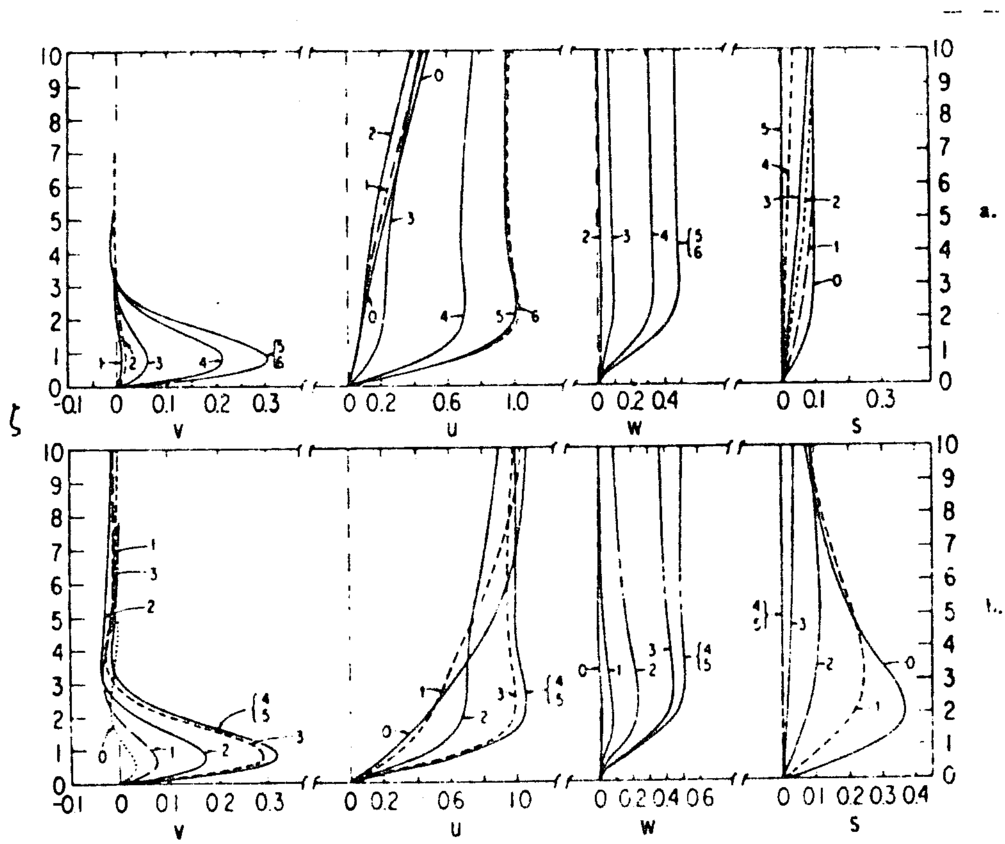

FIG. 11. Steady solutions for various values of α with $S=8\times10^{-a}$, a., and oscillatory solutions for various values of α with $S=8\times10^{-a}$, $\beta=0.05$, b.

Fig. 11 from Kuo's paper shows how the time dependence of the boundary layer can circumvent the damping effect of the static stability on the vertical velocity out of the boundary layer. In Fig. 11 u,v are horizontal velocities, w is vertical velocity, s is essentially perturbation temperature.

We are undertaking a study to determine the implications, if any, of these time dependent effects for the required vertical resolution, using models linearised about a realistic basic state and 2-dimensional non-linear models. These permit much higher resolution than three-dimensional models while still retaining a good deal of verisimilitude.

What we hope to learn from these studies is:

1. The significance of time dependent boundary layer effects on the large scale flow.
2. The vertical resolution necessary to resolve these effects, should it prove necessary.
3. What parameterisations would adequately represent these effects with reduced vertical resolution.
4. The sensitivity of the interior flow to the different prescriptions for the boundary layer discussed above.

Some of this work will be discussed by J.-F. Louis in this seminar. This work will have significance also for the vertical resolution required in other parts of the atmosphere, such as the tropopause, where a great deal of energy dissipation occurs.

Conclusion

The subject of boundary layer studies per se is the subject of this seminar as a whole and needs no further comment here. As regards the effect of boundary layer formulations in Numerical Prediction experiments the literature is rather sparse. Results to date indicate that changing the formulation results in significant alterations in the forecast only after six or seven days.

Theoretical studies of the interaction of the boundary and the large scale flow have been largely limited to studying one way interactions, i.e. the boundary layer is assumed to adjust instantaneously to the interior as in CISK studies or the interior flow is specified independent of the boundary layer behaviour as in studies of thermally or mechanically forced boundary layers. There is evidence to believe that further work is necessary on the question of the mutual interaction of the boundary layer and the large scale flow, particularly as regards time dependent effects.

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APPENDIX: SURFACE LAYER THEORY FOR MODELLERS

(by J.-F. Louis)

1. Introduction

When reviewing the various papers presented at the ECMWF seminar, we found that none of the speakers had presented in detail the theoretical background to support the methods of parameterisation of the surface fluxes in an atmospheric model. Treatment of this problem can be found in various text books such as Sutton (1953), Priestley (1959), Lumley and Panofsky (1964) or Monin (1973).

We found none, unfortunately, that we thought was appropriate for our purpose, and that we would have liked to recommend to those who want to understand the parameterisation methods used in models without necessarily becoming specialists of micrometeorology. Thus, for the sake of completeness, we include this chapter in the proceedings for the seminar.

One of the problems of parameterising the boundary layer in a large scale model of the atmosphere is to express the surface fluxes of momentum, heat and water vapour in terms of large scale variables, namely the winds, temperature and humidity given by the model at grid points at the ground and at some distance above the ground.

The fluxes are those created by the correlation between the vertical velocity and the fluctuations in horizontal velocities, temperature and humidity during turbulent motion. Let us consider for example the flux of sensible heat:

$$F_H = \rho c_p \overline{w'\theta'} \qquad (1)$$

The overbar represents a space-time average over the grid volume of the model, and the primes represent sub-grid scale deviations from the average. We must express this flux F_H in terms of the quantities given by the model : the potential temperature at the ground Θ_o and at the first level above the ground Θ_1, and the wind velocity at the first level $\mathbb{V}_1$ ($\mathbb{V}_o$ =0 because of the no-slip condition). Similarly the momentum fluxes

$$F_u = \rho \overline{w'u'} \qquad (2)$$

must be expressed in terms of $\mathbb{V}_1$.

2. Mixing Length Hypothesis

In order to derive a relation between the eddy fluxes and the large scale variables, let us first consider the very simple situation where the atmosphere is neutrally stable ($\bar{\theta}$ constant) so that there is no gravitational effect, and the mean velocity depends on the height z only. The turbulence, which is driven by the shear in the mean wind, is assumed to be homogeneous and isotropic in any horizontal plane. If the flow is fully turbulent the three components of the wind perturbations have the same order of magnitude :

$$u' \sim v' \sim w' \tag{3}$$

In order to estimate u' we introduce the mixing length hypothesis as follows. We can always say that the perturbation quantity u' at height z is the difference between the value of the mean quantity $\bar{u}$ at z and its value some distance away:

$$u' = \bar{u}(z+l) - \bar{u}(z), \tag{4}$$

and, by a simple Taylor expansion limited at the first term:

$$u' \simeq l \frac{\partial \bar{u}}{\partial z} . \tag{5}$$

This length l is, like u', a stochastic variable, but we make the hypothesis that in the mean it has a unique value, called the mixing length, which is representative of the local intensity of the turbulence, so that we can write similarly

$$\theta' \simeq l \frac{\partial \bar{\theta}}{\partial z} \quad \text{and} \quad q' \simeq l \frac{\partial \bar{q}}{\partial z} \tag{6}$$

Furthermore, because of (3) , we also have

$$w' \simeq l \frac{\partial \bar{u}}{\partial z} \tag{7}$$

so that (2) becomes

$$F_u = \rho \overline{w'u'} \simeq - \rho l^2 \frac{\partial \bar{u}}{\partial z} \cdot \left| \frac{\partial \bar{v}}{\partial z} \right| \tag{8}$$

(The minus sign is introduced so that the flux be down the gradient).

We have progressed a little bit since now the eddy flux is expressed in terms of the gradient of a large scale quantity, but we have to go one step further because $\partial\bar{u}/\partial z$ is a local gradient which cannot be represented accurately by differences since the wind varies rapidly with height near the ground. It is then necessary to know the wind profile.

3. Wind Profile and Drag Law

The wind profile can be derived from (8) if we make one more assumption which is, in fact, related to the mixing length hypothesis. We assume that, near the ground, the mixing length depends only on the height z, and not on any scale length of the large scale flow. Hence we write:

$$l = kz \qquad (9)$$

where the coefficient of proportionality k is von Karman's constant. If we introduce the auxiliary variable u_* (the friction velocity) :

$$u_* = \frac{1}{\rho} |F_{u_0}|^{\frac{1}{2}} \qquad (10)$$

where F_{u_0} is the surface stress,

then (8) can be written :

$$kz \frac{\partial \bar{u}}{\partial z} = u_* \qquad (11)$$

Since the stress varies slowly with height (typically 10 per cent in 100 m) and we are interested in the profile of $\bar{u}$ near the ground, we take u_* constant and can then integrate (11) :

$$\bar{u}(z) = \frac{u_*}{k} \ln \frac{z}{z_0} \qquad (12)$$

The roughness length z_0 is a constant characteristic of the ground surface.

Hence, if our assumptions are correct, there is a universal relation between the momentum flux at the ground and the mean wind profile. Numerous observations of the wind profile near the ground have shown that this relationship is indeed valid. In principle wind observations at only two levels near the ground are enough to determine both the surface flux of momentum and the roughness length. Furthermore, if z_0 is known, the surface flux can be determined from the velocity at the first level of the model above the ground, using the following formula :

$$F_u = -\rho \left[\frac{k}{\ln\left(\frac{z_1}{z_0}\right)} \right]^2 |\bar{v}_1| \, \bar{u}_1 = -\rho C_D |\bar{v}_1| . \bar{u}_1 \qquad (13)$$

$$\text{with } C_D = \left[k / \ln\left(\frac{z_1}{z_0}\right) \right]^2$$

Eq. (13) is commonly referred to as the drag law. It is important to recognise that the drag coefficient C_D is not a constant, but depends on the height z_1 of the level where u_1 is taken. Also, this height z_1 must be small enough (a few tens of meters, at most) so that the Coriolis parameter is not important and the wind direction does not change with height (stress parallel to shear).

Finally, it must be remembered that (13) was derived for neutral static stability of the atmosphere. In many models, however, this formula is used also for non-neutral conditions, together with similar expressions for the heat flux and momentum flux :

$$F_H = -\rho c_p C_D |\bar{v}_1| (\theta_1 - \theta_0) \tag{14}$$

$$F_q = -\rho C_D |\bar{v}_1| (q_1 - q_0) \tag{15}$$

Observations show that if one wishes to use eq. (13), (14) and (15) for non-neutral conditions, the drag coefficient must then depend on the static stability. Let us see how the theory can be extended to non-neutral cases.

4. Similarity Theory

First we note that we do not really need the mixing length theory to derive (11). It could have been obtained directly from dimensional arguments. If we assume that the only velocity scale near the ground is u_* and the only length scale is z, then it is likely that $\partial\bar{u}/\partial z$ should be proportional to u_*/z. This argument no longer holds in the stable or unstable cases ($\partial\bar{\theta}/\partial z \neq 0$) because, as we shall see, another length scale besides z can now be defined.

When $\partial\bar{\theta}/\partial z \neq 0$ the effect of gravity cannot be neglected because of the buoyancy forces which are characterised by the expansion parameter $\beta (= g/T_0)$. Thus instead of two dimensional parameters (u_* and z) characteristic of the flow, we now have four: u_*, z, β and θ_* where the scaling temperature θ_* is defined such that

$$F_{H_0} = -\rho c_p u_* \theta_* . \tag{16}$$

These four parameters, however, involve only three dimensional units: length, time and temperature. Thus we can construct one non-dimensional parameter, using the dimensional ones:

$$\zeta = \frac{z \beta \theta_*}{u_*^2} \tag{17.a}$$

The combination $L = u_*^2 / \beta \theta_*$ (17.b)

is an internal length scale for the turbulence and is generally called the Monin-Obukhov length. It is positive in stable cases and negative in unstable ones.

Now the similarity principle states that, when non-dimensionalised, the various large scale quantities of the flow must be universal functions of the non-dimensional numbers formed by independent combinations of the dimensional parameters (in this case there is only one such combination). Thus if this principle holds for the surface layer, we must have :

$$\frac{kz}{u_*}\frac{\partial \bar{u}}{\partial z} = \phi_m\left(\frac{kz}{L}\right) \qquad (18.a)$$

and

$$\frac{kz}{\theta_*}\frac{\partial \bar{\theta}}{\partial z} = \phi_h\left(\frac{kz}{L}\right) \qquad (18.b)$$

Notice that (11) is simply a particular case of (18.a), when $L = \infty$ because $\theta_* = 0$.

5. Flux Profile Relationships

Much experimental work has gone into trying to determine the form of these functions ϕ_m and ϕ_h (the so-called "flux-profile relationships"), and several analytical formulas have been proposed to describe them. A review of these relationships can be found in Dyer (1974). As an example I will present here the results of one of the most complete and careful experimental determinations of these relationships, by Businger, et al. (1971). Fig. 1 and 2 compare experimental data with suggested analytical forms for the non-dimensional wind shear and temperature gradient respectively. One of these forms ($\phi_m^4 - 9\zeta\phi_m^3 = 1$) is the KEYPS profile, so baptised by Panofsky (1963) because it was derived in different ways by Kazanski and Monin (1956), Ellison (1957), Yamamoto (1959), Panofsky (1961) and Sellers (1962). In these figures ζ is defined (in our notation) as kz/L.

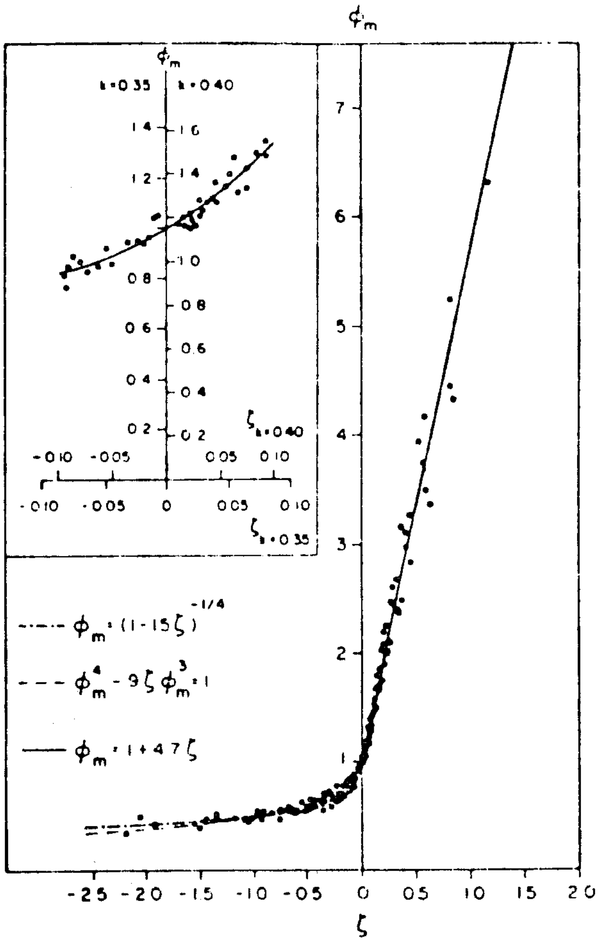

FIG. 1. Comparison of dimensionless wind shear observations with interpolation formulas.

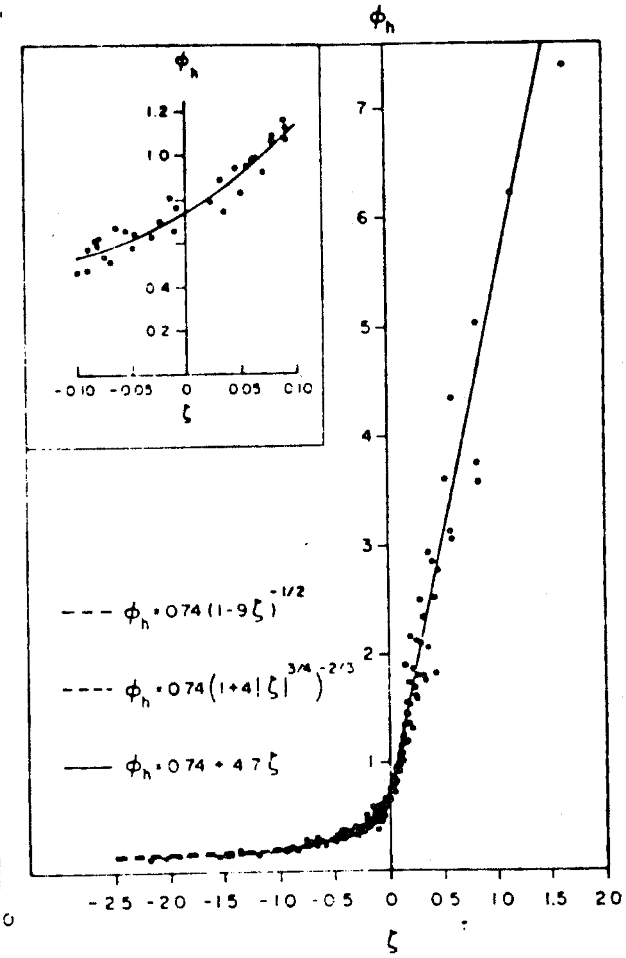

FIG. 2. Comparison of dimensionless temperature gradient observations with interpolation formulas.

(from Businger, et al., 1971)

6. Use of Flux-Profiles in Atmospheric Models

Having determined the flux-profile relationships, we are again faced with the problem that the equations (18.a and b) involve local gradients and that they have to be integrated before we can use them in a model. It can be seen that (18.a), (18.b) and (17.b) form a complete system in which the unknown u_*, θ_* and L can be related to the model variables $\bar{u}$ and $\bar{\theta}$. This is not easy to do because of the complicated analytical forms suggested for ϕ_m and ϕ_h . If, by analogy with (12), we write

$$\bar{u} = \frac{u_*}{k}\left[\ln\left(\frac{z}{z_0}\right) - \Psi_m\right] \tag{19.a}$$

and

$$\bar{\theta} - \bar{\theta}_0 = \theta_*\left[\ln\left(\frac{z}{z_0}\right) - \Psi_h\right] \tag{19.b}$$

the new functions Ψ_m and Ψ_h can be determined in terms of $\frac{z}{L}$ (see Paulson, 1970, and Barker and Baxter, 1975). Then L can be eliminated, using its definition (17.b). Barker and Baxter showed that, in principle, L can be reduced to an expression involving only a stability parameter (the bulk Richardson number) :

$$Ri_B = \frac{g}{\bar{\theta}} \frac{z_1(\bar{\theta}_1 - \bar{\theta}_0)}{\bar{u}_1^{\,2}} , \tag{20}$$

and the ratio z_1/z_0 (where z_1 is the height of the first level in the model, and z_0 the roughness length). In practice this has to be done by numerical methods, either every time the surface fluxes are computed in the model, or once and for all, constructing tables or nomograms of Ψ_m and Ψ_h for all possible values of Ri_B and z_1/z_0 (see Clarke, 1970 or Deardorff, 1968). This being done we can now use the same expressions as before for the surface fluxes:

$$F_m = -\rho C_D |\bar{V}_1| \bar{u}_1 \tag{21.a}$$

$$F_h = -\rho c_p C'_D |\bar{V}_1| (\bar{\theta}_1 - \bar{\theta}_0) \tag{21.b}$$

$$F_q = -\rho C'_D |\bar{V}_1| (\bar{q}_1 - \bar{q}_0) \tag{21.c}$$

but now the drag coefficients

$$C_D = \left\{ k / \left[\ln\left(\frac{z_1}{z_0}\right) - \Psi_m\left(Ri_B, \frac{z_1}{z_0}\right)\right]\right\}^2 \tag{22}$$

and

$$C'_D = \left\{ k^2 / \left[\ln\left(\frac{z_1}{z_0}\right) - \Psi_h\left(Ri_B, \frac{z_1}{z_0}\right)\right] \cdot \left[\ln\left(\frac{z_1}{z_0}\right) - \Psi_m\left(Ri_B, \frac{z_1}{z_0}\right)\right]\right\}$$

are not only functions of the height of the first level in the model, but also functions of the stability parameter Ri_B.

7. Final Remarks

As a conclusion I should like to make a few comments about the validity of this theory. First of all we must remember the assumptions made about the turbulence.

(a) It must be stationary, i.e. it must not change with time. It can be argued that the theory is still valid if the turbulence adjusts instantly to changes in the large-scale forcing. It is doubtful, however, that such instantaneous adjustment happens for example when the regime changes from a stable to an unstable boundary layer in the morning.

(b) The turbulence was assumed to be homogeneous in a horizontal plane. What happens to the fluxes when the characteristics of the ground are highly variable is not well known.

Finally it must be realised that the flux-profile relationships are only valid near the ground. Higher up other length scales become important, especially the height of the boundary layer, and the Coriolis parameter also becomes important, introducing another time scale $1/f$.

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