BOUNDARY LAYER WORK AT
E. C. M. W. F.

BY

J. - F. LOUIS

1. Introduction

Faced with the problem of choosing a parameterisation scheme for the boundary layer, we started by studying in detail all the methods that have been used so far in general circulation and large scale prediction models.

I will not go into the details of these here, since other papers in this seminar have already dealt with them. It seems to us, however, that there is a good amount of similarities between the various models (a notable exception being the UCLA model). The main differences are confined to the vertical resolution in the boundary layer and to the values or functional relationships used for the various parameters.

We wanted to test these various parameterisation schemes, but the question of how to test them is a sticky one. It is obvious that testing each scheme in turn in a global prediction model would be prohibitively expensive, both in time and resources. Hence it is necessary to do some experiments either in limited area models or in one-column models.
One-column experiments are attractive because they are very cheap to do, but can they tell us what we really need to know to choose a parameterisation scheme ?

There are two things we are interested in. We are, of course, interested in predicting the boundary layer for its own sake because it is where we live and the weather associated with the PBL is significant. However, for the purpose of a 5 to 10 day forecast, this may not be our main concern. More important may be the interaction between the boundary layer and the large scale flow. We know that the large scale flow has a large influence on the boundary layer. It is partly the large scale flow that determines whether the boundary layer is stable or unstable, it is also the large scale flow that provides the kinetic energy for the turbulence in a mechanically driven boundary layer. Kuo has shown that if the atmosphere is more stable the boundary layer is shallower, but that its depth increases again if the large scale flow is oscillatory.

On the other hand we know from both theory and experimentation (see Hide's paper) that the presence of a boundary layer may completely alter the interior flow. The effect of the boundary layer may not be as drastic in the atmosphere as it is in some laboratory experiments, but some studies such as Charney and Eliassen's work on CISK have shown that the effect of the boundary layer can be quite important.

It seems to us that in a medium-range forecast model it is most important to simulate the large-scale flow correctly, thus the possible interactions between the boundary layer and the large scale flow should be our first concern. The "minimum sophistication" parameterisation scheme will be the one which simulates these interactions correctly, but most cheaply in terms of computing resources.

2. One-column model

In order to try to understand the interactions between the boundary layer and the large scale flow, we decided to set up a one-column model somewhat similar to those used by Kuo, Charney and Eliassen or Holton.

It should be noticed that in those studies the interaction was in one direction only. In the cases studied by Kuo the large scale flow was given - either steady state or a given neutral wave - , and its effect on the boundary layer computed. In the study of CISK by Charney and Eliassen the structure of the boundary layer was assumed to obey Ekman's theory and the effect on the large scale flow was computed.
In our case we want to study the 2-way feed-back between boundary layer and interior flow. We do it in the following way :

Let us write down the primitive equations:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \left[\omega_0 - \int_{p_0}^{p}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)dp\right]\cdot\frac{\partial u}{\partial p}$$
$$- fv + \frac{\partial}{\partial x}\left(\phi_0 - \frac{R}{p_0^{\kappa}}\int_{p_0}^{p}\theta p^{\kappa-1}dp\right) - \frac{g^2 p_0^{2}}{R^2}\frac{p^{1-\kappa}}{\theta}\frac{\partial}{\partial p}\left(\frac{p^{1-\kappa}}{\theta}\nu\frac{\partial u}{\partial p}\right) = 0$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \left[\omega_0 - \int_{p_0}^{p}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)dp\right]\cdot\frac{\partial v}{\partial p}$$
$$+ fu + \frac{\partial}{\partial y}\left(\phi_0 - \frac{R}{p_0^{\kappa}}\int_{p_0}^{p}\theta p^{\kappa-1}dp\right) - \frac{g^2 p_0^{2}}{R^2}\frac{p^{1-\kappa}}{\theta}\frac{\partial}{\partial p}\left(\frac{p^{1-\kappa}}{\theta}\nu\frac{\partial v}{\partial p}\right) = 0 \quad (1)$$

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} + \left[\omega_0 - \int_{p_0}^{p}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)dp\right]\cdot\frac{\partial \theta}{\partial p}$$
$$- \frac{g^2 p_0^{2}}{R^2}\frac{p^{1-\kappa}}{\theta}\frac{\partial}{\partial p}\left(\frac{p^{1-\kappa}}{\theta}\nu\frac{\partial \theta}{\partial p}\right) = 0$$

In this system we have made use of the continuity equation to express the vertical velocity in terms of the horizontal velocities, u and v and of the hydrostatic relation and equation of state to express the geopotential in terms of the potential temperature θ.

Let us now assume that the flow is composed of a basic flow which is steady but baroclinically unstable, and a perturbation which is small enough to allow us to linearise the equations by neglecting the products of perturbation quantities. We will use upper case letter for the basic flow variables, and lower case for the perturbation variables. The last term in each equation represents the vertical eddy flux divergence, using a diffusion coefficient ν.
Let us first look at the basic flow. We will choose it as simple as possible by taking

$$V = \Omega = 0$$
$$\frac{\partial}{\partial x} = \frac{\partial}{\partial t} = 0 \qquad (2)$$
$$\frac{\partial U}{\partial y} = \frac{\partial \Phi_o}{\partial y} = 0$$

Furthermore we will simplify the diffusion terms by assuming that the potential temperature which appears in them is some kind of average $\bar{\Theta}$, and by hiding all the constants in a new diffusion coefficient

$$\nu' = \frac{g^2 p_o^{2K}}{R^2 \bar{\Theta}^2} \nu$$

For the basic flow the system of equation (1) reduces to :

$$\frac{\partial}{\partial p}\left(p^{1-K} \nu \frac{\partial U}{\partial p} \right) = 0 \qquad (3.a)$$

$$f U - \frac{R}{p_o^K} \frac{\partial}{\partial y} \left[\int_{p_o}^{p} \Theta \, p^{K-1} \, dp \right] = 0 \qquad (3.b)$$

$$\frac{\partial}{\partial p}\left(p^{1-K} \nu \frac{\partial \Theta}{\partial p} \right) = 0 \qquad (3.c)$$

These equations are satisfied by the following :

$$U = U_* \left[1 - \left(\frac{p}{p_o} \right)^K \right] \qquad (4.a)$$

$$\Theta = \Theta_0 + \Gamma y + \Theta_* \left[1 - \left(\frac{p}{p_0}\right)^{\kappa}\right] \qquad (4.b)$$

with $\Gamma = \frac{\partial \Theta}{\partial y} = - \frac{f U_* \kappa}{R}$

We will now look at the perturbed flow by substituting (4) into (1) and linearising. We will also modify (1) somewhat by using the stream function ψ and the velocity potential ξ instead of u and v. They are defined such that

$$u = -\frac{\partial \psi}{\partial y} + \frac{\partial \xi}{\partial x}$$

$$v = \frac{\partial \psi}{\partial x} + \frac{\partial \xi}{\partial y}$$

After a little algebra we get:

$$\frac{\partial(\nabla^2 \xi)}{\partial t} + U \frac{\partial(\nabla^2 \xi)}{\partial x} + \frac{\partial}{\partial x}\left(\omega_0 - \int_{p_0}^{p} \nabla^2 \xi \, dp\right) \cdot \frac{\partial U}{\partial p} - f \nabla^2 \psi$$
$$+ \beta \frac{\partial \xi}{\partial x} + \nabla^2\left(\phi_0 - \frac{R}{p_0^{\kappa}} \int_{p_0}^{p} \theta p^{\kappa - 1} dp\right) - p^{1-\kappa} \frac{\partial}{\partial p}\left(p^{1-\kappa} \nu \frac{\partial(\nabla^2 \xi)}{\partial p}\right) = 0$$

$$\frac{\partial(\nabla^2 \psi)}{\partial t} + U \frac{\partial(\nabla^2 \psi)}{\partial x} - \frac{\partial}{\partial y}\left(\omega_0 - \int_{p_0}^{p} \nabla^2 \xi \, dp\right) \cdot \frac{\partial U}{\partial p} + f \nabla^2 \xi \qquad (5)$$
$$+ \beta \frac{\partial \psi}{\partial x} - p^{1-\kappa} \frac{\partial}{\partial p}\left(p^{1-\kappa} \nu \frac{\partial(\nabla^2 \psi)}{\partial p}\right) = 0$$

$$\frac{\partial \theta}{\partial t} + U \frac{\partial \theta}{\partial x} + \left(\frac{\partial \psi}{\partial x} + \frac{\partial \xi}{\partial y}\right) \cdot \frac{\partial \Theta}{\partial y} + \left(\omega_0 - \int_{p_0}^{p} \nabla^2 \xi \, dp\right) \cdot \frac{\partial \Theta}{\partial p}$$
$$- p^{1-\kappa} \frac{\partial}{\partial p}\left(p^{1-\kappa} \nu \frac{\partial \theta}{\partial p}\right) = 0$$

Now since we want to solve these equations in a vertical column, we have to assume a horizontal structure for the perturbed flow. We assume that each of the three variables ξ, ψ and θ has the form

$$A = A(p)\, e^{i(kx + ly - \sigma t)}$$

Since the flow is unstable, σ can be complex. Substituting into (5) we obtain the following system :

$$(iUk - i\sigma)\xi - ik\left(\frac{\omega_0}{k^2+l^2} + \int_{p_0}^{p}\xi\,dp\right)\cdot\frac{\partial U}{\partial p} - f\psi - \frac{i\beta k\xi}{k^2+l^2} + \phi_0 - \frac{R}{p_0^{\kappa}}\int_{p_0}^{p}\theta\, p^{\kappa-1}dp - p^{1-\kappa}\frac{\partial}{\partial p}\left[p^{1-\kappa}\nu\frac{\partial\xi}{\partial p}\right] = 0 \qquad (6.a)$$

$$(iUk - i\sigma)\psi + il\left(\frac{\omega_0}{k^2+l^2} + \int_{p_0}^{p}\xi\,dp\right)\frac{\partial U}{\partial p} + f\xi - \frac{i\beta k\psi}{k^2+l^2} - p^{1-\kappa}\frac{\partial}{\partial p}\left[p^{1-\kappa}\nu\frac{\partial\psi}{\partial p}\right] = 0 \qquad (6.b)$$

$$(iUk - i\sigma)\theta + ik\Gamma\psi + il\Gamma\xi + \left(\omega_0 + \int_{p_0}^{p}(k^2+l^2)\xi\,dp\right)\frac{\partial\Theta}{\partial p} - p^{1-\kappa}\frac{\partial}{\partial p}\left[p^{1-\kappa}\nu\frac{\partial\theta}{\partial p}\right] = 0 \qquad (6.c)$$

We chose the following boundary conditions :

(a) $\psi = \xi = 0$ at $p = p_0$ and $p = p_{top}$

(b) $\frac{d\phi}{dt} = w = 0$ at $p = p_0$

(c) $\omega = 0$ at $p = p_{top}$

(d) $\frac{\partial\theta}{\partial p} = 0$ at $p = p_{top}$

(e) either $\theta = 0$ or $\frac{\partial\theta}{\partial p} = 0$ at $p = p_0$

the condition (c) permits us to eliminate ω_0 from the equations because $\omega_0 = \int_{p_0}^{p_{top}}(k^2+l^2)\xi\,dp$, and condition (b) gives an equation for ϕ_0:

$$-i\sigma\phi_0 + \int_{p_0}^{p_{top}}(k^2+l^2)\xi\,dp \cdot \frac{R\Theta_0}{p_0} = 0 \qquad (7)$$

It can be seen that equations (6a,b,c) and (7) form a complete system for ξ, ψ, θ and ϕ_0 and can be considered as an eigenvalue problem, the vertical structure of the variables being determined by the eigen vectors corresponding to the eigen value $i\sigma$.

Notice that no distinction has been made between the boundary layer and the interior flow. They obey the same equations, but the boundary layer will emerge automatically because it is only near the ground that viscous terms in equation (6a,b,or c) are large.

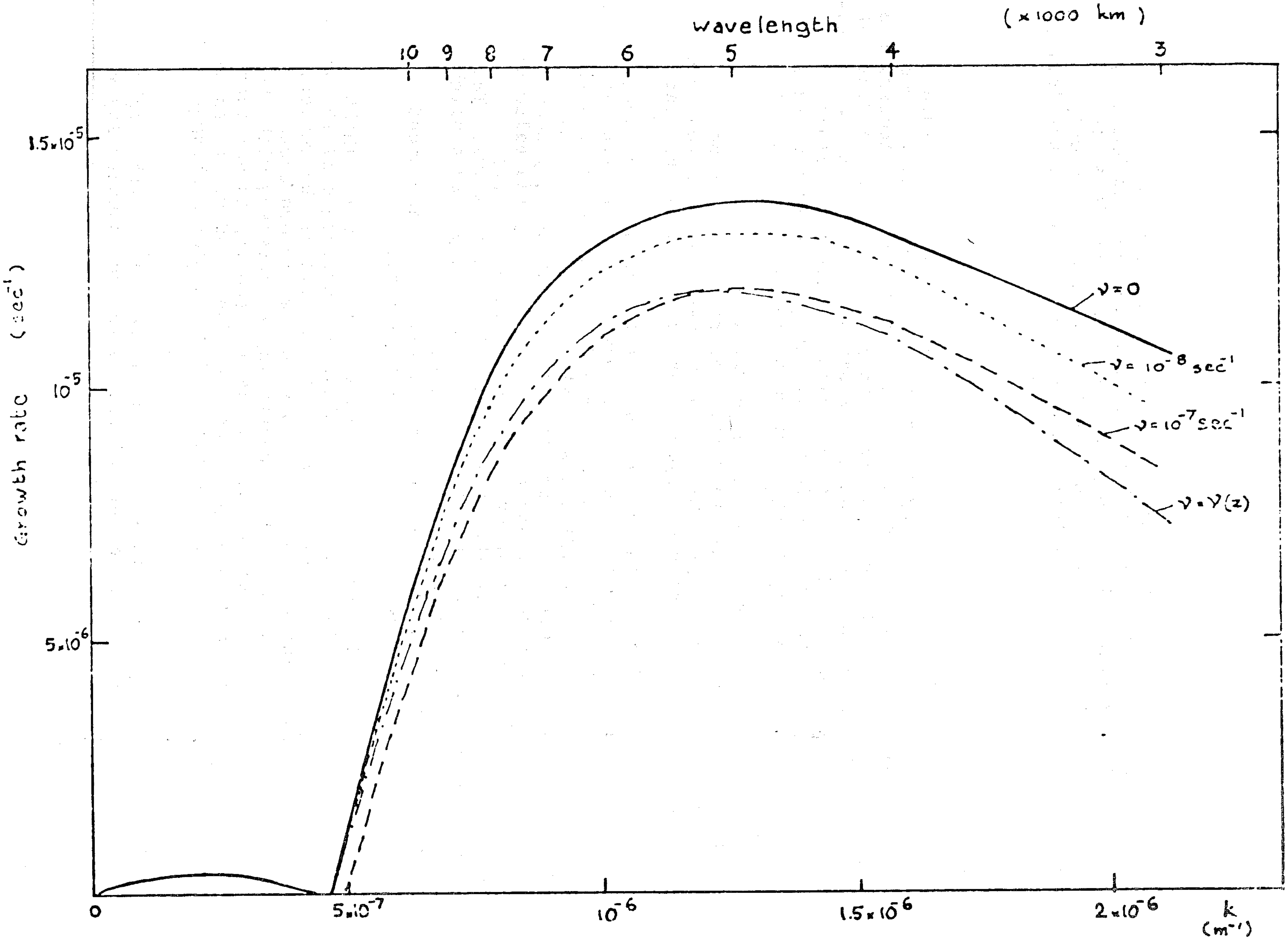
3. Results

Let us look at the first results we have obtained with this system of equations. In solving it we have departed slightly from the above discussion in that we have used a more realistic temperature distribution for the basic flow than that given by (4.b) : a constant lapse rate in the troposphere and a constant temperature in the stratosphere. Even though (3.c) is not satisfied by this temperature distribution, it is not very serious. It can be assumed that this vertical structure is kept in equilibrium by radiation terms not included in (3.c).
In the computations we have performed so far we have taken the meridional wave number ? equal to zero. The results which we presented below are quite preliminary and we have not examined them in detail yet. This is more a progress report than anything else !

Let us first look at the growth rate of the baroclinic waves as a function of wave length. The solid curve shows the value of the growth rate for the inviscid case ($\nu = 0$). It can be seen that with constant diffusion coefficients ($\nu = 10^{-7}$ sec^{-1} in one case, 10^{-8} sec^{-1} in the other), the growth rate is decreased by about 5 to 10 %. The decreased growth rate is controlled by the value of in the lowest levels as shown by the curve labelled $\nu = \nu(z)$ for which $\nu = 10^{-7}$ sec^{-1} in the first km and 10^{-8} sec^{-1} above. This curve is very close to the $\nu = 10^{-7}$ sec^{-1} curve, except for a slight shift in the size of the most unstable wave.
At the same time as the growth rate is lowered, the phase speed (not shown) is increased, also by about ten percent.

Figure 2 shows the vertical structure of the wave for which $k = 10^{-6}$ m^{-1} ($\lambda \approx 6000$ km). The figure shows the structure of the waves for two values of the diffusion coefficient as well as the inviscid case. The boundary layer is quite obvious in the viscous cases. As expected the thickness of the boundary layer increases with the value of the diffusion coefficient. It is interesting to see that the boundary layer for θ is deeper than that for ξ and ψ. It can also be seen that, in the interior, the ratio of θ to ψ or ξ changes considerably with ν.

Figure 3 shows the variation of the phase angles with height. Here the effect does not seem very marked above the boundary layer, except for ξ.



<u>FIGURE 1:</u> Growth rate of baroclinic wave for various diffusion coefficients.

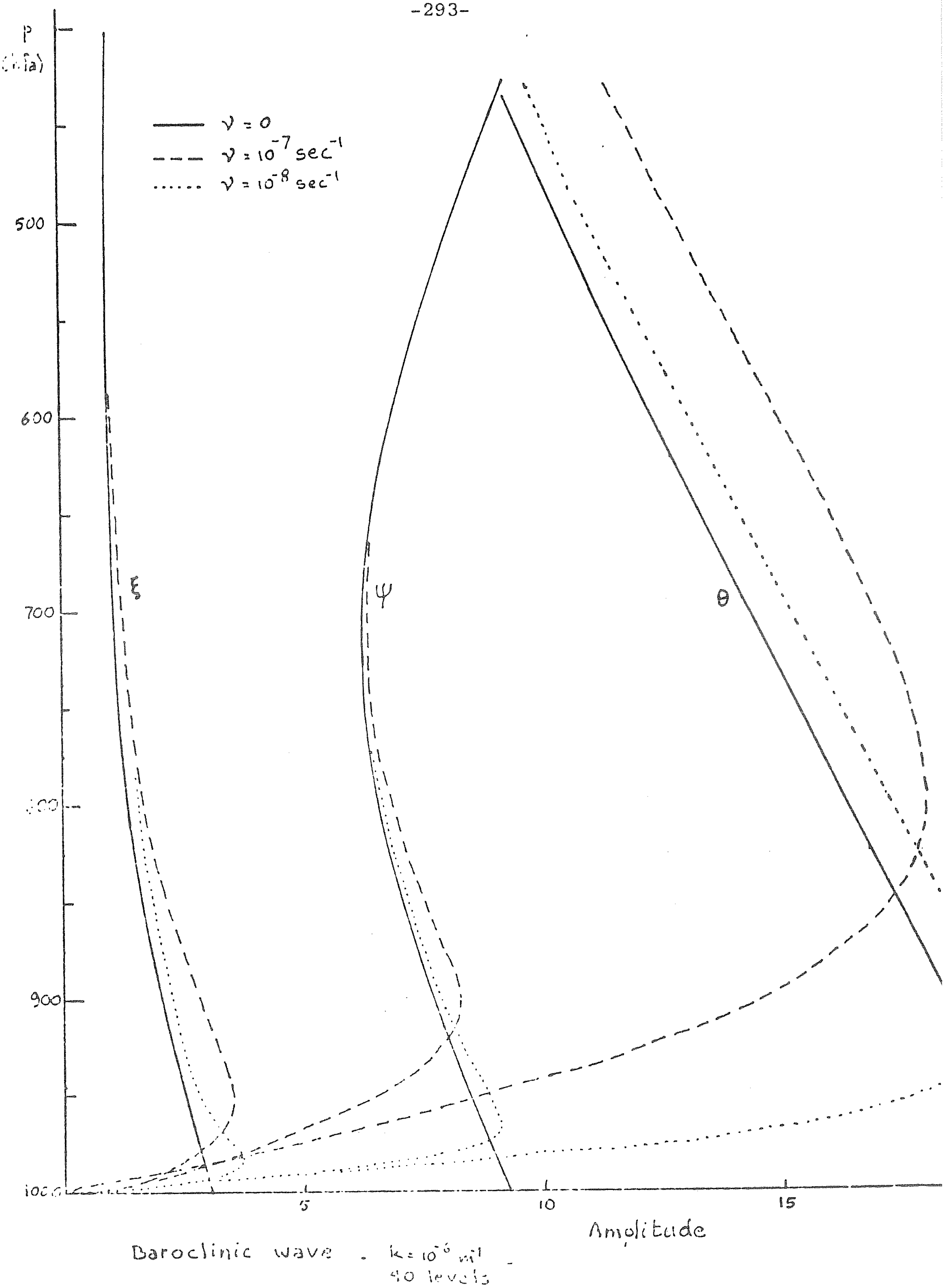


FIGURE 2: Vertical structure of baroclinic wave.

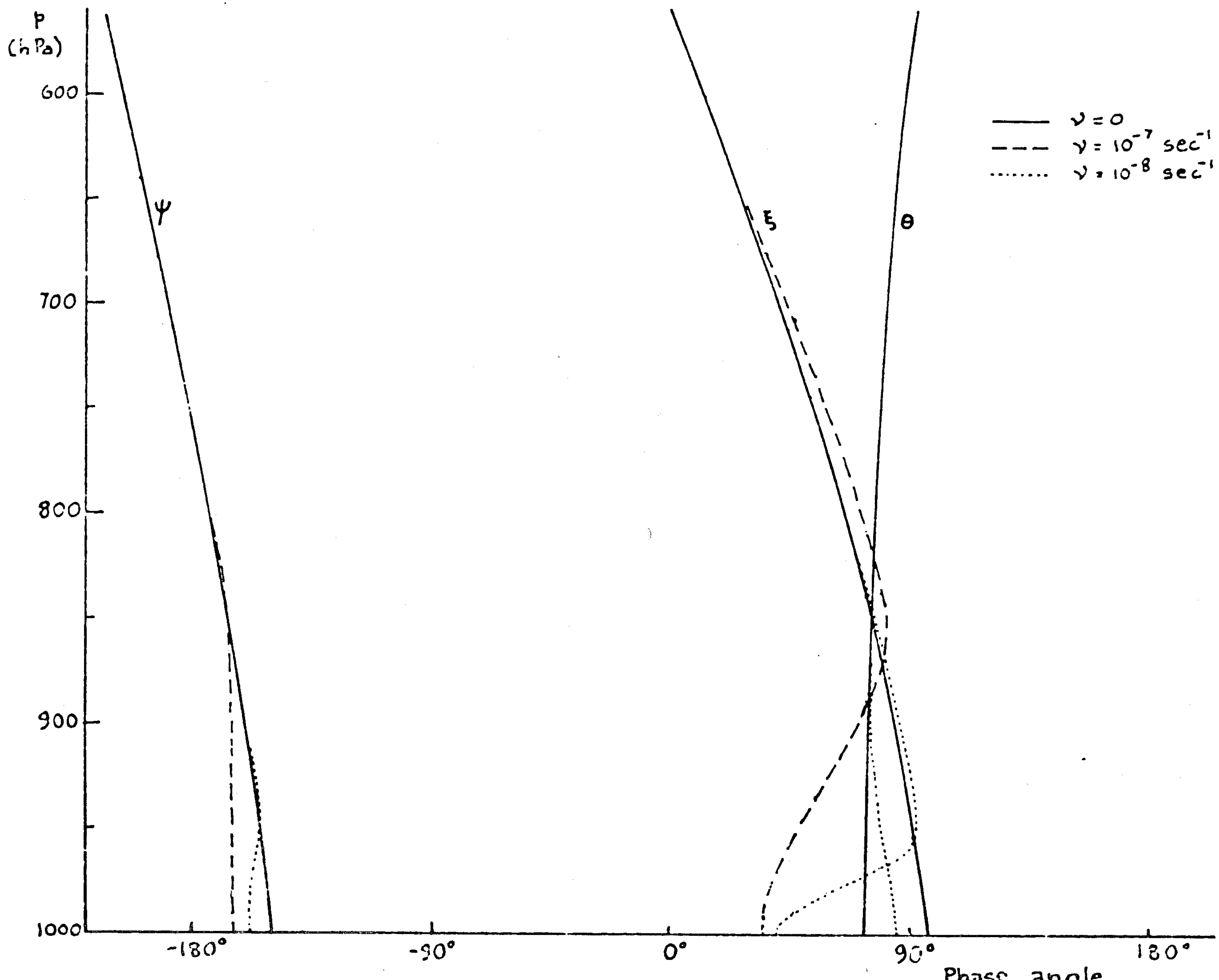


FIGURE 3: Phase angle of baroclinic wave.

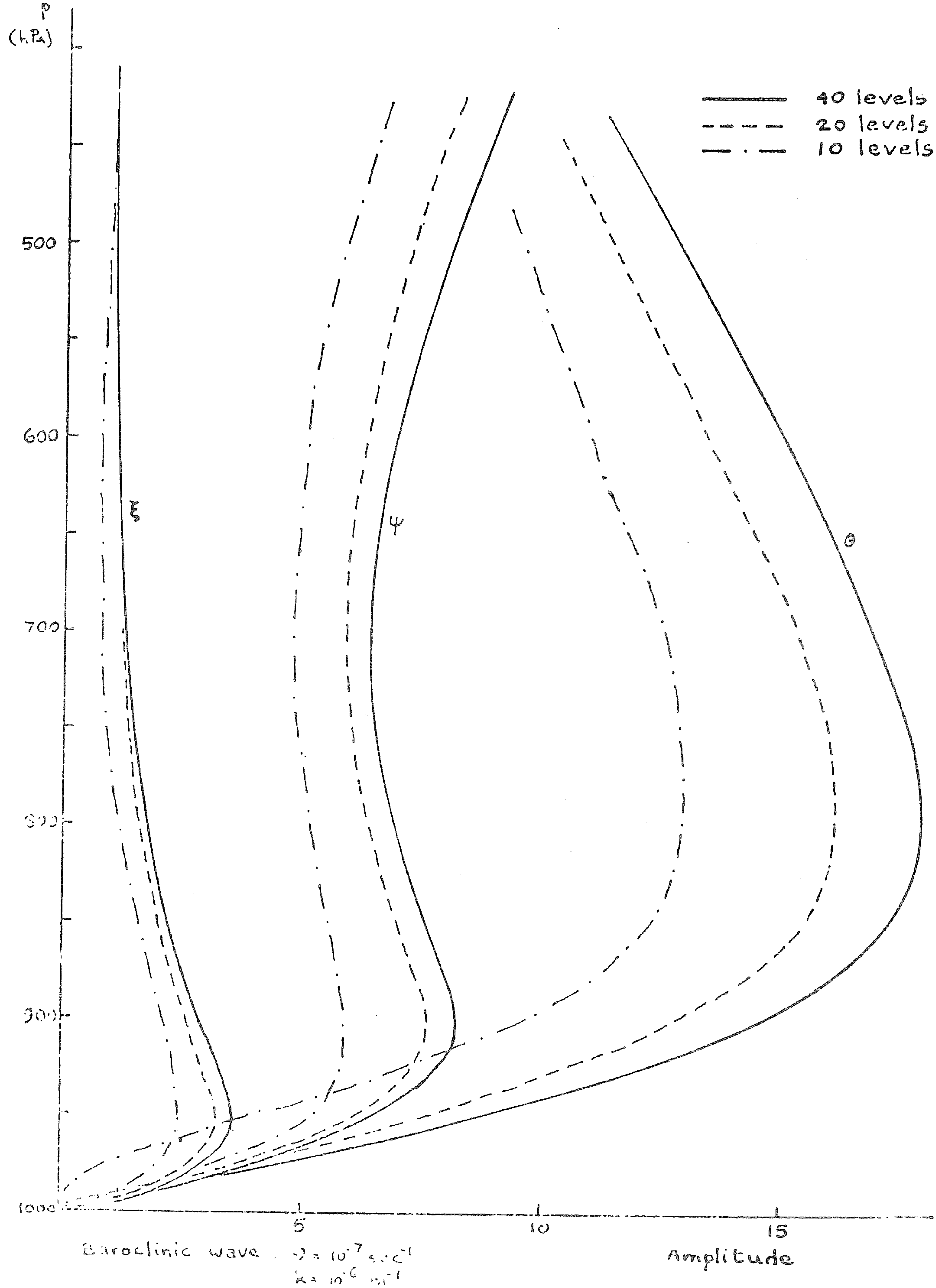


FIGURE 4: Effect of vertical resolution on the vertical structure of baroclinic wave.

Finally figure 4 shows the effect of the vertical resolution on the structure of the waves. In figures 1 to 3 we had used 40 levels unevenly distributed so that the levels were concentrated toward the bottom. There were about 10 levels in the boundary layer. In figure 4 we again show the same curves as in figure 2 for 40 levels and $\nu = 10^{-7}$ sec^{-1} (solid lines) but we also show the curves corresponding to the same diffusion coefficient, but using only 20 and 10 levels respectively. The differences are quite obvious, not only in the boundary layer but in the interior as well.

As I said before, these results are only preliminary and a lot more tests have to be run. Furthermore we are also planning to solve equation (5) as an initial value problem, using the same assumptions for the horizontal structure of the waves. This will then permit us to test more complicated, non linear formulations for the boundary layer, as well as the effect of the diurnal cycle forcing.

References:

Charney, J.G., A. Eliassen	1964	On the growth of the hurricane depression. J. Atmos. Sci., 21, 68 - 75
Charney, J.G.	1973	Moveable CISK. J. Atmos. Sci. 30, 50-52
Holton, J.R., J.M. Wallace, J.A.Young	1971	On boundary layer dynamics and the ITCZ. J. Atmos.Sci. 28, 275-280
Kuo, H.L.	1973	Planetary boundary layer flow of a stable atmosphere over the globe. J. Atmos. Sci. 30, 53-65